ϕ is given in terms of the fields **E**; the use of the vector potential **A** is inessential. All could be expressed in terms of

$$\mathbf{K}(x) = \int_{t_0}^{t_1} \mathbf{E}(x,t)dt,$$

which happens to be equal to $A(x,t)$ for $t > t_1$ in this specified gauge.

Also, it is not possible to determine during the "acceleration time" (say, time interval $t_0 - t_1$) on which side of the core an electron has traveled. That is, to determine this one would have to measure its energy with an accuracy of the order $\epsilon_1 + \epsilon_2$. To achieve this accuracy one needs a time $\Delta t \gtrsim (\epsilon_1 + \epsilon_2)^{-1}$. During this time the fringe pattern would shift over at least one period and the interference would have been washed out.

Finally, the existence of a phase shift here is neither more nor less mysterious than the influence of the flux through the middle of a Josephson loop on the current in the loop. And, any experimental (as contrasted with theoretical) nonobservance of the Bohm–Aharonov effect would indeed destroy the basis of quantum mechanics and require the construction of a completely new theory. Nothing like that seems to be called for (see, e.g., Ref. 3).

[1]S. M. Roy, Phys. Rev. Lett. **44**, 111 (1980).
[2]L. J. Tassie and M. Peshkin, Ann. Phys. (NY) **16**, 177 (1961).
[3]H. A. Fowler, L. Marton, Y. A. Simpson, and J. A. Suddeth, J. Appl. Phys. **32**, 1153 (1961); G. Möllenstedt and W. Bayh, Naturwiss. **48**, 400 (1961); **49**, 81 (1962).

Students' preconceptions in introductory mechanics

John Clement
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003

(Received 14 August 1980; accepted for publication 18 March 1981)

Data from written tests and videotaped problem-solving interviews show that many physics students have a stable, alternative view of the relationship between force and acceleration. This "conceptual primitive" is misunderstood at the qualitative level in addition to any difficulties that might occur with mathematical formulation. The misconception is highly resistant to change and is remarkably similar to one discussed by Galileo, as shown by comparison of his writings with transcripts from student interviews. The source of this qualitative misunderstanding can be traced to a deep-seated preconception that makes a full understanding of Newton's first and second laws very difficult. In such cases learning becomes a process in which new concepts must displace or be remolded from stable concepts that the student has constructed over many years.

INTRODUCTION

Physics is commonly considered to be a difficult subject. When one searches for sources of the difficulty that students encounter in physics, one can identify many contributing factors such as abstractness of the material, degree of logical precision required in problem solving, sophistication in the types of reasoning required (including formal reasoning in the Piagetian sense), and mathematical skills required. This paper discusses another source of difficulty that has been acknowledged but that has been insufficiently analyzed in the past, namely, the presence in physics of inherently difficult *conceptual primitives*. These include: (i) *key concepts* such as mass, acceleration, momentum, charge, energy, potential difference, torque, etc.; and (ii) *fundamental principles* and *models* such as Newton's laws, conservation laws, the atomic model, electron flow models for circuits, etc. The term conceptual primitive will be used here to refer to a mental construct, the understanding of which is a basic prerequisite for many higher-order concepts. It is easy to underestimate the learning difficulties that these "root concepts" present to the student. Many science-oriented students have difficulty understanding these concepts at the qualitative level in addition to any difficulties that occur with quantitative formulation. Difficulties at the qualitative level may go undetected, however, because a student's superficial knowledge of formulas and formula manipulation techniques can mask his or her misunderstanding of underlying qualitative concepts.

In some cases, difficulties with conceptual primitives appear to originate in intuitive preconceptions that the student develops on his own before entering courses. This paper discusses a particularly strong qualitative preconception in the area of force and motion. A particularly difficult conceptual primitive is the relationship between force and acceleration, summarized in the equation $F = ma$. An understanding of $F = ma$ is made more difficult because it conflicts with the beginner's intuitive preconceptions about motion. In the real world, where friction is present, one must push on an object to keep it moving. Since friction is often not recognized as a force by the beginner, the student may believe that continuing motion implies the presence of a continuing force in the same direction, as a necessary cause of the motion. Empirical evidence will be presented indicating that many beginners apply this point of view to various simple mechanics problems. In fact, the misconception shows up in a wider diversity of problem situations than one would expect, and appears to

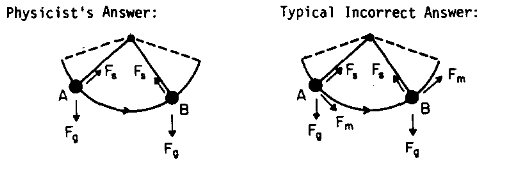

Fig. 1. Correct and incorrect answers to pendulum problem.

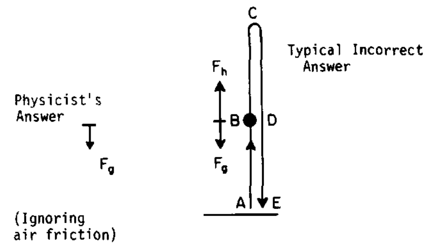

Fig. 2. Correct and incorrect answers to coin problem.

still be present in many students after they have completed a course in mechanics. It therefore appears to be a major stumbling block in the physics curriculum. Related misconceptions have been studied by Driver,[1] Viennot,[2] Lawson *et al.*,[3] and DiSessa.[4] It is shown here that preconceptions can be studied using problems of minimum complexity that help to isolate the source of the errors.

"MOTION IMPLIES A FORCE" PRECONCEPTION

The error pattern described in Example 1 below was observed in a large number of course laboratory write-ups from students taking introductory mechanics after they had worked with pendulums in the lab. A typical incorrect solution to the pendulum problem is shown in Fig. 1.

Example 1: pendulum problem

(a) A pendulum is swinging from left to right as shown below. Draw arrows showing the direction of each force acting on the pendulum bob at point A. Do *not* show the total net force and do not include frictional forces. *Label each arrow* with a name that says what kind of force it is.

(b) In a similar way, draw and label arrows showing the direction of each force acting on the pendulum bob when it reaches point B.

Typical incorrect explanation: F_m is the force that makes the pendulum swing upward. If F_m weren't there, the pendulum could never move up to the top of its swing.

Here, F_m is seen as one of the forces acting on the bob and is often described as the force that "makes the pendulum go up on the other side." We also noticed that students drawing force diagrams for an object sliding down a track, or for an object in orbit, would often include a force in the direction of motion. These classroom observations led us to suspect that many students were applying the idea that continuing motion implies the presence of a continuing force in the same direction as the motion. We call this the "motion implies a force" misconception. This type of belief shows up in pre-Newtonian theories of motion such as an impetus force injected into an arrow and traveling with it, or the Aristotelian explanation of the horizontal motion of an arrow after release from the bow via forward forces from air currents.[5] What is surprising is the pervasiveness of the belief and the wide diversity of situations in which it shows up, once one begins to listen to students' common sense theories.

In an effort to further isolate the source of this type of error, we designed the problem shown in Fig. 2, and predicted that it might produce a similar type of error in spite of its extreme simplicity.

Example 2: coin problem

A coin is tossed from point A straight up into the air and caught at point E. On the dot to the left of the drawing draw one or more arrows showing the direction of each force acting on the coin when it is at point B. (Draw longer arrows for larger forces.)

Typical incorrect answer: While the coin is on the way up, the "force from your hand" F_h pushes up on the coin. On the way up it must be greater than F_g, otherwise the coin would be moving down.

The coin problem was given to a group of engineering students on a diagnostic test early in their first semester in a class required of all engineering majors.[6] These students had not had college physics, but most had had high school physics. As shown in Table I, the students did very poorly, with 88% giving an incorrect answer. Virtually all (90%) of the errors in this case involved showing an arrow labeled as a force pointing upwards at position B. Eleven students were interviewed while solving this problem aloud, five of whom had taken a physics course in mechanics for scientists and engineers. Three students solved this problem correctly, while seven students drew an upward arrow at point B, referring to it as the "force of the throw," the "upward original force," the "applied force," the "force that I'm giving it," "velocity is pulling upwards, so you have a net force in this direction (points upwards)," "the force up from velocity," and "the force of throwing the coin up." Another student gave a questionable response, referring to "a momentum force...acting up" that doesn't belong in "a formal free-body diagram" but "is definitely a force." The latter three responses were from students who had taken the mechanics course. All of these students were engineering majors. Again, we see that it is difficult for the student to think about an object continuing to move in one direction with the total net force acting in a different direction. These findings supported our hypothesis that the "motion implies a force" preconception was involved in the students' responses to these problems.

Another example is provided by the rocket problem shown in Fig. 3.

Example 3: rocket problem

(a) A rocket is moving along sideways in deep space, with

Table I. Performance on coin and rocket problems.

		% correct before course	% correct after course	% correct[a]
Coin problem		12% ($N = 34$)	28% ($N = 43$)	30% ($N = 37$)
Rocket problem	(Part a)	11% ($N = 150$)	23% ($N = 43$)	35% ($N = 37$)
	(Part b)	38% ($N = 150$)	72% ($N = 43$)	65% ($N = 37$)

[a]For engineers with two semesters of physics at a second institution.

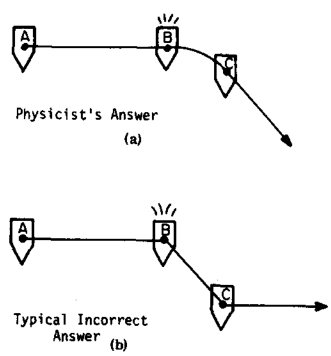

Fig. 3. Correct and incorrect answers to rocket problem.

its engine off, from point A to point B. It is not near any planets or other outside forces. Its engine is fired at point B and left on for 2 sec while the rocket travels from point B to some point C. Draw in the shape of the path from B to C. (Show your best guess for this problem even if you are unsure of the answer.)

(b) Show the path from point C after the engine is turned off on the same drawing.

Typical incorrect answer: The force of the rocket engine combines with whatever was making it go from A to B to produce path BC. After C, whatever made it go from A to B will take over and make it go sideways again, causing the rocket to return to its original direction of motion.

Results from written testing on this problem with a representative group of 150 pre-physics engineering students are shown in Table I. 89% drew an incorrect path for part (a) of the rocket problem while 62% missed part (b). A summary of the responses to the rocket problem is given in Table II.

The curved path from B to C is a detailed aspect of the motion that the uninitiated student will rarely reproduce. A more surprising and significant difficulty than this, however, is the tendency in many students to actually draw the rocket's motion returning to a horizontal direction after the engine is shut off at point C. The student's prediction that the rocket will return to a horizontal path is usually accompanied by a reference to some influence acting on the rocket from A to B that "takes over" again after C. This behavior can be explained by assuming that, for the student, the presence of constant motion from A to B implies the presence of a continuing force in the same direction, even though the problem states that no outside forces are present. Note also that students usually show the direction of motion changing instantaneously in a noncontinuous manner, apparently to correspond to instantaneous changes in the direction of applied force.

Taped interviews were conducted with 18 of the above students. Five of the seven students who had responses of type 3 or 4 in Table II made a specific reference to the idea that "whatever was making it go to the right before will take over again after point C." (See Appendix for example of a rocket problem transcript.) These interview results, and the consistent error pattern both within each problem and across the problems indicate that most errors are not due to random mistakes but rather are based on a stable misconception that is shared by many individuals.

DISCUSSION OF SIMILAR ARGUMENTS IN GALILEO'S WRITINGS

Two typical transcript excerpts from freshman engineering students working on the coin problem are shown below:

Transcript of S1

S1: *So there's a force going up* and there is *the force of gravity pushing* it down. And *the gravity is less* because the coin is still going up until it gets to C. (Draws upward arrow labeled ("force of the throw") and shorter downward arrow labeled "gravity" at point B in Fig. 2.) ...if the dot goes up *the force of throw gets to be less and less because gravity is pulling down on it, pulling down*.

Interviewer: Okay, what about the length of this arrow ("force of the throw"). If we use that to represent how strong the force is, would it be stronger than gravity at point B?

S1: Yeah, because the ball is still going up, so the force of the throw is still overcoming the force of the gravity that wants to make it go down.

Transcript of S2

S2: At B there'd be two—that I could think of. The upward force—the *upward original force* that was given to the coin to make it fly in the air... (draws upward arrow at B)...and the gravitational. (Draws downward arrow at B.) But the *reason that the coin is going up is because the*

Table II. Response categories for rocket problem.

	$n = 150$ Entering freshman engineers		$n = 43$ After mechanics course	
(1) Correct	14	9%	8	19%
(2) Partially correct	40	27%	16	37%
(3) Returns to horizontal	62	41%	9	21%
		46%		26%
(4) Returns partially to horizontal	8	5%	2	5%
(5) Other	26	17%	8	19%

original is greater than the gravitational.

The italicized statements indicate that the students believe that a force is acting upwards on the coin at point *B*, and that the coin is continuing to rise because the upward force is greater than the gravitational. This is evidence for the "motion implies a force" belief, in this case with reference to the sum of forces acting on the body.

After these student explanations were analyzed we discovered that Galileo had made some similar arguments in his manuscript *De Motu* (*On Motion*). In explaining the motion of an object thrown upwards he states:

> The body moves upward, provided the impressed motive force is greater than the resisting weight. But that force, as has been shown, is continuously weakened; it will finally become so diminished that it will no longer overcome the weight of the body and will not impel the body beyond that point... .
>
> As the impressed force characteristically continues to decrease, the weight of the body begins to be predominant, and consequently the body begins to fall... .
>
> This is what I consider to be the true cause of the acceleration of motion.[7]

His explanation that "the impressed motive force is greater than the resisting weight" is similar in many ways to the students' explanations. S2 explains that the "upward original force...is greater than the gravitational," and S1 explains that the "force of the throw...is...overcoming the force of gravity." Indeed, it is remarkable how similar the statements are, given the fact that the speakers are separated culturally by over 300 years. In each case, they describe a continuing upward force acting on the coin as a cause of motion, and state that the upward motion requires that this force be larger than the force of gravity.

Of course, Galileo thought much more deeply about these issues in his ingenious thought experiments than students do. When he published *Two New Sciences* much later in his career, Galileo presented essentially the above argument, but was unwilling to endorse or refute it.[8] He assigned the argument to Sagredo, the "middleman" in the dialogs, rather than to either Salviati, the spokesman representing himself, or to Simplicio, whose views are closest to Galileo's Aristotelian adversaries. Following Sagredo's presentation, in *Two New Sciences*, Salviati says:

> The present does not seem to me to be an opportune time to enter into the investigation of the cause of the acceleration of natural motion...it suffices our Author that we understand him to want us to investigate and demonstrate some attributes of a motion so accelerated (whatever be the cause of its acceleration) that the momenta of its speed go increasing, after its departure from rest, in that simple ratio with which the continuation of time increases... .[9]

One of Galileo's strengths, in contrast to the philosophical generalists of his age, was that he was able to make deep progress by intentionally restricting his field of inquiry (in this case to kinematics). But the quotations from Galileo indicate that real conceptual change in this area is an extremely difficult task that should not be underestimated.[10] The fact that Galileo propounded a careful and well-articulated impetus theory during part of his career, and the fact that present-day students give explanations that are very similar in their basic aspects to that theory, is supporting evidence for the strong, intuitive attraction of the "motion implies force" belief. The students' errors appear not to be simply capricious; the belief appears to be *plausible theory* that has been constructed by students on the basis of experience. This historical comparison makes the high error rates for students on these problems somewhat less surprising.

SUMMARY OF CHARACTERISTICS FOR THE "MOTION IMPLIES A FORCE" PRECONCEPTION

By studying the error patterns discussed so far, we can summarize what appear to be the most common characteristics of the "motion implies a force" preconception:

(1) Continuing motion, even at a constant velocity, can trigger an assumption of the presence of a force in the direction of motion that acts on the object to cause the motion.

(2) Such invented forces are especially common in explanations of motion that continues in the face of an obvious opposing force. In this case the object is assumed to continue to move because the invented force is greater than the opposing force.

(3) The subject may believe that such a force "dies out" or "builds up" to account for changes in an object's speed.

The diversity of situations in which this preconception surfaces suggests that it is a major source of the difficulties encountered by students in understanding the physical principles associated with the equation $F = ma$.

POST-COURSE RESULTS

In order to determine the effect of a physics course on these misconceptions we also tested two groups of students who had taken mechanics. The students in post group A were paid volunteers who agreed to take a diagnostic test before their final exam in a standard, one-semester introductory mechanics course for engineers and science majors. Most of these students were sophomores and they were from the same institution as the freshman group reported on earlier. The teacher of the course has received consistently high praise in written evaluations from students for his clarity of presentation, helpfulness, and genuine interest in teaching. The average grade in the course for these volunteers happened to be significantly higher than the course mean. The students in post group B were sophomore, junior, and senior engineering majors enrolled in an upper-level engineering course at a second institution. All had previously taken mechanics.

Scores of the post-course students were somewhat better than those of the pre-physics students, but an alarmingly high number of students still gave wrong answers of the same kind on these very basic problems, as shown in Table I. This was in spite of the fact that none of the problems require advanced mathematical skills. What they do require is an adequate knowledge of the basic qualitative model for how forces affect motion.

On the rocket problem, these students did somewhat better in avoiding the most blatant error: the misconception that the rocket will return to a horizontal path. However, on the coin problem, the percentage of error only changed from 88% to 75% for group A, a rather disturbing result. In this problem, almost all errors were again in the form of an upward arrow. Additional data for this group show

44% drawing forces incorrectly on the pendulum problem, with a 51% error rate at the second institution. 68% and 78% of these errors, respectively, included arrows drawn horizontally or tangential to the motion. Possibly, these error rates are lower than for the coin problem because the opposition between the direction of motion and the gravitational force is more pronounced in the coin problem. This is consistent with the fact that more invented forces were shown on the upswing of the pendulum than on the downswing.

Although the pre-course and post-course tests were given to different groups, the two independent results indicate what can be expected of students before and after the introductory course, and the fact that post group A was an above average sample from the course leads us to be concerned about the level of understanding that is generally attained. In conclusion, the data support the hypothesis that for the majority of these students, the "motion implies a force" preconception was highly resistant to change. This conclusion applies to the extent that the students could not solve basic problems of this kind where the direction of motion does not coincide with the direction of the net force.

IMPLICATIONS FOR INSTRUCTION

The above findings lead us to suspect that it may be necessary to devote more attention to fundamental principles underlying the Newtonian view than is currently practiced, and that teaching strategies limited to expository presentation may be unlikely to succeed in this area. The "motion implies a force" preconception is not likely to disappear simply because students have been exposed to the standard view in their physics courses. More likely, Newtonian ideas are simply misperceived or distorted by students so as to fit their existing preconceptions; or they may be memorized separately as formulas with little or no connection to fundamental qualitative concepts. Discouraging as these implications may seem, it should be remembered that historically, pre-Newtonian concepts of mechanics had a strong appeal, and scientists were at least as resistant to change as students are.

Serious difficulties with conceptual primitives have also been documented for undergraduates in several other areas of physics, including relative motion, torque,[11] simple circuits,[12] and acceleration.[13] In addition, preconceptions producing consistent error patterns have been identified in the areas of Newton's third law, centrifugal force,[14] velocity,[15] elastic forces, and curvilinear motion.[16] These involve beliefs such as assuming that a stronger person experiences a smaller force than a weaker person when they push away from each other on an ice rink, drawing radially outward forces in circular motion, assuming that an object passing another moving object is traveling at the same speed when it is next to the object, believing that passive objects like tables cannot be sources of force, and believing that objects projected from a curved tube will continue to follow a curved path. Not all of these error patterns are as strong as the ones discussed here, but they do show up in a significant percentage of students.

Preconceptions need not be viewed exclusively as obstacles to learning, however. Since they ordinarily have some predictive power in certain practical situations, they can be thought of as "zeroth-order models" that the students possess; models that can be modified in order to achieve greater precision and generality.[17] In this approach to attacking the problem, the goal is to find teaching strategies that encourage students to articulate and become conscious of their own preconceptions by making predictions based on them. A second goal is then to encourage them to make explicit comparisons between these preconceptions, accepted scientific explanations, and convincing empirical observations. Similar strategies have been advocated by Fuller, *et al.*,[18,19] and Arons,[20] among others. In one attempt to develop this approach we are designing laboratory activities for introductory mechanics in which students are asked to give a large number of qualitative predictions and explanations about elementary phenomena such as the motion of the simple pendulum or of the tossed coin.[21] We have found that questions about the direction and relative magnitudes of forces, velocities, and accelerations at different points of the motion are quite challenging to introductory students. In the absence of formulas to "plug into," such questions are an effective way of getting students to think about their own preconceptions. In general, when qualitative misconceptions arise, it is necessary for students to express them and to actively work out their implications in order to see the advantages of the Newtonian point of view. Class discussions and arguments between pairs of students are especially helpful in this regard. Further development of innovative instruction techniques that emphasize rigorous understanding of qualitative principles should be encouraged.

Galileo was apparently aware of this type of teaching strategy, for his dialogs represent a marvelous attempt to deal directly with the common preconceptions and prevailing theories of his time at a qualitative level. The enormous conceptual breakthroughs that were achieved by Galileo were not easy to communicate to his peers. His writings appeal to the reader's intuitions by using concrete, practical situations to illustrate his theories.[22,23] One might do worse than to take these aspects of Galileo's teaching technique as a model for pedagogy today.

Apparently one cannot consider the student's mind to be a "blank slate" in the area of force and motion. Many of the concepts presented in this area must displace or be remolded from stable intuitive concepts that the student has constructed over a number of years. Increased awareness of such preconceptions should allow the development of new instructional strategies that take student's beliefs into account and that foster a deeper level of understanding than is currently the norm.

ACKNOWLEDGMENTS

I wish to thank S. Drake, A. Arons, L. McDermott, J. Lochhead, and F. Byron for their comments on an earlier draft of this article. Research reported in this paper was supported by NSF Award No. SED 78-22043 in the Joint National Institute of Education/National Science Foundation Program of Research on Cognitive Processes and the Structure of Knowledge in Science and Mathematics. Parts of this paper were presented in a symposium, Research in Physics Education, sponsored jointly by the AAPT and the National Association for Research in Science Teaching at the AAPT meeting, January 1980.

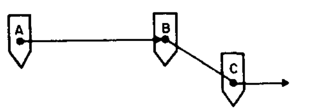

Fig. 4. Student's drawing for rocket problem.

APPENDIX: EXAMPLE OF A TRANSCRIPT FROM THE ROCKET PROBLEM

Student S3

One student answered the rocket problem (Fig. 4) as follows:

I: OK, can you describe the motion and tell me what the rocket did?

S: OK. The rocket was moving towards here (points to right)—a force acting upon it here (points to *B*) to drive it down—so in effect it would be driving it at an angle because there's two forces acting upon it.

I: And after the engine shuts off?

S: Right here (points to *C*)—and with the same force acting upon it—motion—it'd continue along this path (horizontally to the right).

This subject apparently believes that a force is needed to cause the initial movement at a constant velocity with the engine off. After the engine is fired and turned off, this "same force acting upon it" horizontally causes the rocket to return to the horizontal path. Notice that the student's ideas are quasiconsistent in this case. A belief that a constant force causes a constant velocity implies that there must be a constant horizontal force; these two ideas then predict both the straight diagonal path during the burn, and the return to a horizontal path afterwards.

[1]R. P. Driver, Ph.D. dissertation, University of Illinois, Urbana-Champaign, 1973.

[2]L. Viennot, Eur. J. Sci. Educ. **1**, 205 (1979).

[3]R. A. Lawson, D. E. Trowbridge, and L. C. McDermott, AAPT Announcer **9**, 87 (1979).

[4]A. DiSessa, in *Cognitive Process Instruction*, edited by J. Lochhead and J. Clement (Franklin Institute, Philadelphia, 1979).

[5]For a summary of the history of different impetus theories, see A. Franklin, Phys. Teach. **16**, 4 (1978). A more thorough discussion is given in E. J. Dyksterhuis, *The Mechanization of the World Picture* (Clarendon, Oxford, 1961).

[6]This sample was chosen in part because engineering students comprise the largest clientele of physics departments at many universities.

[7]G. Galileo, *De Motu*, translated by I. E. Drabkin (University of Wisconsin, Madison, WI, 1960), p. 89.

[8]Sagredo: "...it seems to me that a very appropriate answer can be deduced for the question agitated among philosophers as to the possible cause of acceleration of the natural motion of heavy bodies. For let us consider that in the heavy body hurled upwards, the force impressed upon it by the thrower is continually diminishing, and that this is the force that drives it upward as long as this remains greater than the contrary force of its heaviness... . The diminutions of this alien impetus then continuing, and in consequence the advantage passing over to the side of the heaviness, descent commences... . And since this [force] continues to diminish, and comes to be overpowered in ever greater ratio by the heaviness, the continual acceleration of the motion arises therefrom." G. Galilei, *Two New Sciences*, translated by S. Drake (University of Wisconsin, Madison, WI, 1974), p. 157–158.

[9]Reference 8, p. 159.

[10]Although there is wide agreement on the fact that Galileo never stated Newton's second law, the extent to which he progressed toward a statement of the first law of inertia has been a point of discussion. See (a) S. Drake, Am. J. Phys. **32**, 601 (1964), (b) J. Losee, Am. J. Phys. **34**, 430 (1966), (c) S. Drake, Sci. Am. **243**, 151 (1980).

[11]W. Barowy and J. Lochhead, AAPT Announcer **10**(2), 74 (1980).

[12]N. Fredette and J. Lochhead, Phys. Teach. **18**, 194 (1980).

[13]D. Trowbridge and L. McDermott, Am. J. Phys. **49**, 2242(1981).

[14]Reference 2.

[15]D. Trowbridge and L. McDermott, Am. J. Phys. **48**, 12 (1980).

[16]M. McCloskey, A. Caramazza, and B. Green, Science **210**, 4474 (1980).

[17]Impetus theory, for example, can be seen historically as an important intermediate step between Aristotle's antipersperis theory and the modern concept of inertia. For a discussion of how more formal physical principles may be connected to physical intuitions, see J. Clement, in *Cognitive Process Instruction*, edited by J. Lochhead and J. Clement (Franklin Institute, Philadelphia, 1979).

[18]R. G. Fuller, R. Karplus, and A. Lawson, Phys. Today **30**, 23 (1977).

[19]R. Fuller, *The ADAPT Book* (ADAPT Program, University of Nebraska, Lincoln, 1977).

[20]A. Arons, *The Various Language* (Oxford University, New York, 1977).

[21]Draft available from the author on request.

[22]Reference 7.

[23]G. Galilei, *Dialogue Concerning the Two Chief World Systems*, translated by S. Drake (University of California, Berkeley, 1962).

SOLUTION TO THE PROBLEM ON PAGE 47

Needed results of derivations available in many introductory physics or intermediate mechanics texts will be quoted to save space in presenting the solution, and references made to only one of many possible sources.[1]

(a) For the circular orbit, the radius was to be $(R + h)$, so the potential energy is[2] $U = -GMm/(R + h)$. Newton's second law gives $GMm/(R + h)^2 = mv^2/(R + h)$, because the acceleration is centripetal. This gives for the kinetic energy $T = \frac{1}{2}mv^2 = GMm/2(R + h) = \frac{1}{2}|U|$, and for the total energy $E = T + U = -GMm/2(R + h)$. Since the potential energy is determined by the location of P, and T has the value intended for the circular orbit, the energy of the actual orbit is $E < 0$, which shows the orbit is an ellipse.[2] The energy for a particle in an inverse-square central force field, moving in an elliptical orbit with semimajor axis a is given by[3] $-GMm/2a$. Thus we see that $a = R + h$, is the semimajor axis.

[1]Murray R. Spiegel, *Theory and Problems of Theoretical Physics*, Schaum's Outline Series (McGraw-Hill, New York, 1967).

[2]Reference 1, p. 121.

[3]Reference 1, p. 135, Eq. (2).