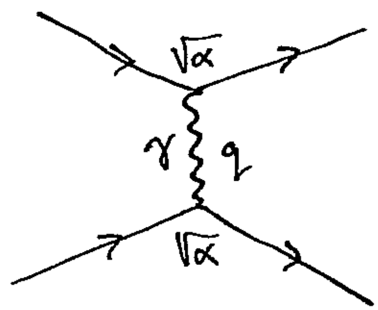
Scattering experiments to investigate

Nuclear size, shape and structure

Nucleon size shape and structure

$\frac{d\sigma}{d\Omega} \rightarrow$ probability of scattering into solid angle element $d\Omega$



$P(\theta) = P(\vec{q}^2) = |A(\vec{q}^2)|^2$ One to one relationship between θ and $\vec{q}^2$

Rutherford scattering: spinless charged particle scattering from heavy spinless pointlike charge:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ruth} = \frac{4m^2(Ze^2)^2}{p^4 \sin^4 \theta/2} = \frac{4m^2(Ze^2)^2}{q^4}$$

Mott scattering includes (relativistic) effects due to electron spin

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = 4(Ze^2)^2 \frac{E^2}{(|\vec{q}|c)^4}\left(1-\beta^2 \sin^2 \theta/2\right)$$

Reduces to Rutherford scattering for $\beta \to 0$, $E \to mc^2$

Proton still treated as pointlike and spinless…….

Elastic scattering off an extended charge distribution (still spinless)

$$\left(\frac{d\sigma}{d\Omega}\right)_{Exp} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left|F(\vec{q}^{\,2})\right|^2 \qquad \text{with} \qquad F(\vec{q}^{\,2}) = \int d^3\vec{r}\, e^{i\vec{q}\cdot\vec{r}/\hbar} \rho(\vec{r})$$

Can get $\rho(r)$ from Fourier transform of experimentally measured form factor

Gives for instance the density distributions for atomic nuclei

Next looked at magnetic moments (associated with particle spin)

$$\vec{\mu} = g \frac{e}{2mc} \vec{J}$$

For fundamental fermions (Dirac particles) $g = 2, \quad \vec{J} = \vec{S} = 1/2$

Small anomalous magnetic moments of electron and muon accounted for by higher order corrections.

Anomalous magnetic moments of the proton and neutron are large:

$$\mu_p = 2.79\mu_N$$

$$\mu_n = -1.91\mu_N$$

$\mu_N \equiv$ nuclear magneton $\frac{e\hbar}{2m_N c} \ll \mu_B = \frac{e\hbar}{2m_e c}$

↑ Bohr magneton

Evidence for substructure in the proton and neutron

Relative values well described in the (static) quark model

Nucleon Elastic Form Factors: need also to account for nuclear spins

Electron proton scattering (ep → ep)

Electron-deuteron scattering (subtract proton contribution for en scattering)

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rosenbluth} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[\frac{G_E^2 + bG_M^2}{1+b} + 2bG_M^2 \tan^2 \theta/2\right]$$

$$G_E = G_E(\vec{q}^{\,2})$$
$$G_M = G_M(\vec{q}^{\,2})$$

$$b = \frac{-\vec{q}^{\,2}}{4M_N^2 c^2}$$

$G_E(0) = Q/e$ = 1 (proton) or 0 (neutron)

$G_M(0) = \mu/\mu_N$ = 2.79 (proton) or -1.91 (neutron)

We have seen the distributions of differential cross-section as a function of scattering angle. Can extract the form factors from these distributions:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rosenbluth} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left[\frac{G_E^2 + bG_M^2}{1+b} + 2bG_M^2 \tan^2\theta/2\right] = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left[I + S\tan^2\theta/2\right]$$

Evaluate at fixed values of momentum transfer and extract G_E, G_M

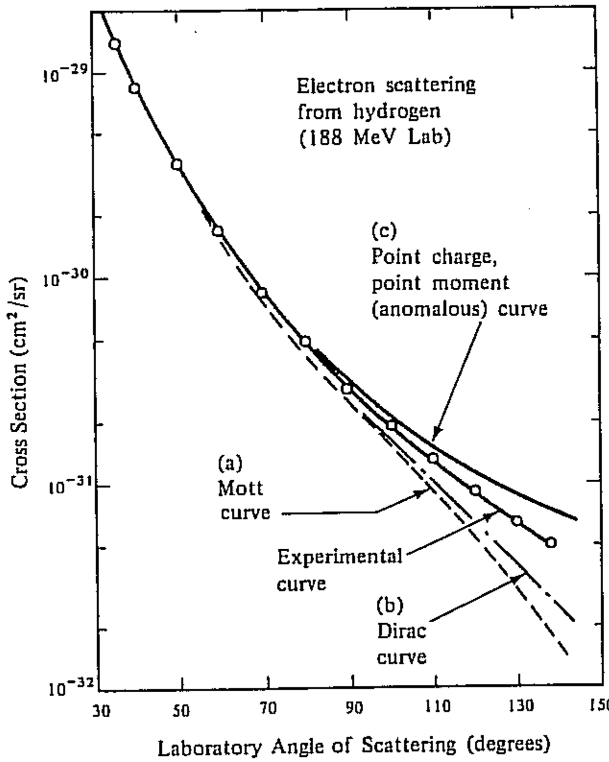


Fig. 6.13. Electron–proton scattering with 188 MeV electrons. [R. W. McAllister and R. Hofstadter, *Phys. Rev.* **102**, 851 (1956).] The theoretical curves correspond to the following values of G_E and G_M: Mott (1; 0), Dirac (1;1), anomalous (1; 2.79).

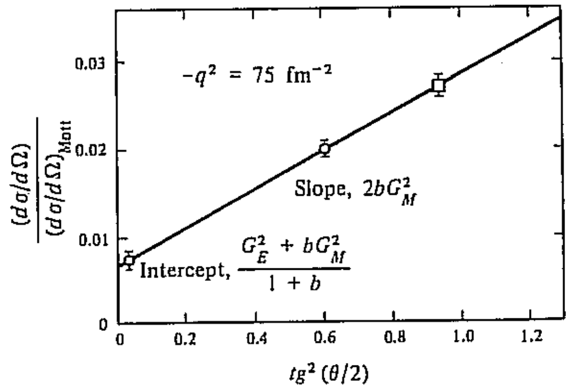
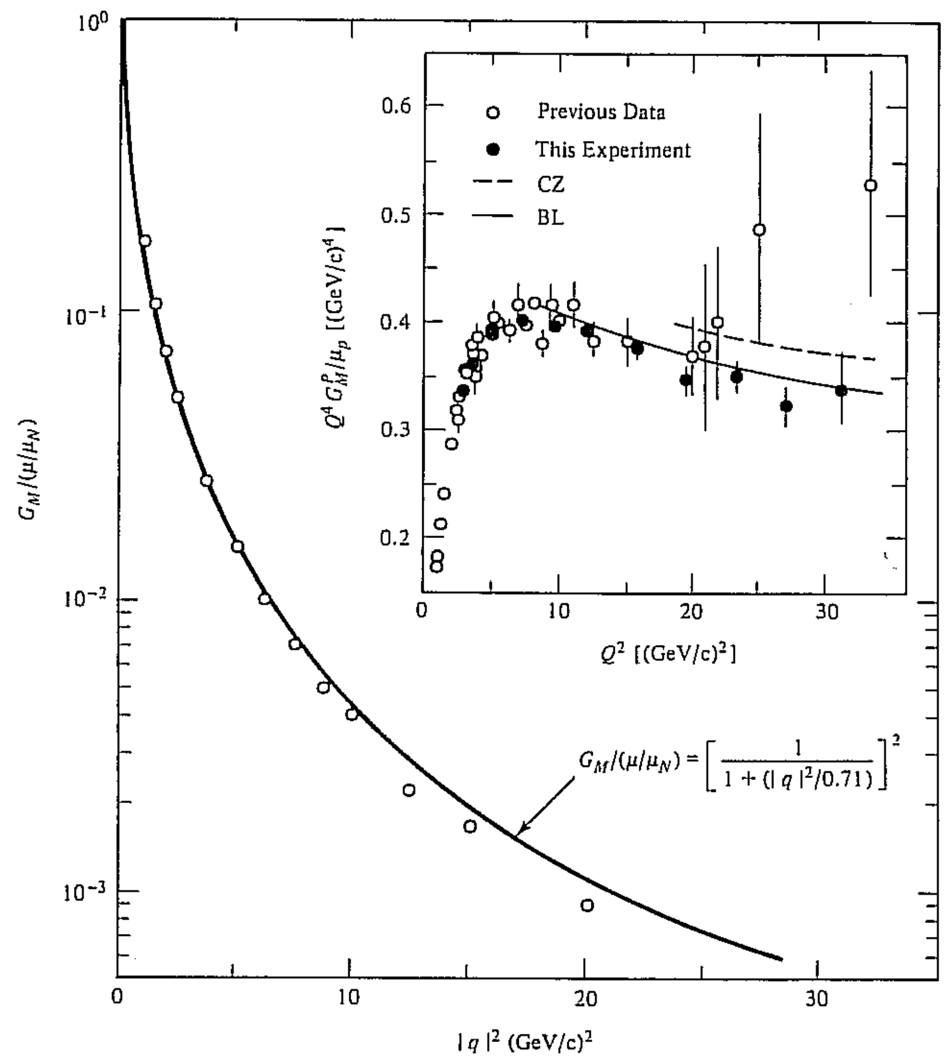


Fig. 6.14. Rosenbluth plot. See the text for description.

⇒ Extraction of nucleon elastic form factors

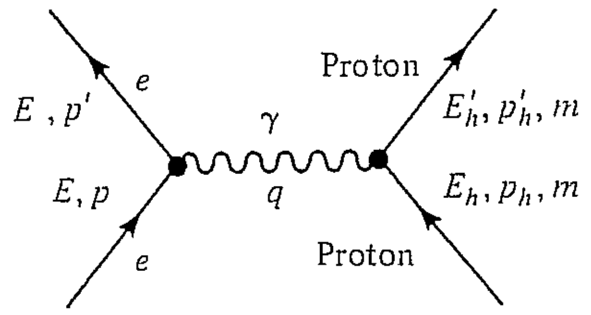
"dipole" fit

$$G_E^p(\vec{q}^2) \approx \frac{G_M^p(\vec{q}^2)}{(\mu_p/\mu_N)} \approx \frac{G_M^n(\vec{q}^2)}{(\mu_n/\mu_N)} \approx \frac{1}{(1+|\vec{q}|^2/q_0^2)^2} \qquad q_0^2 \sim 0.71\,(GeV/c)^2$$

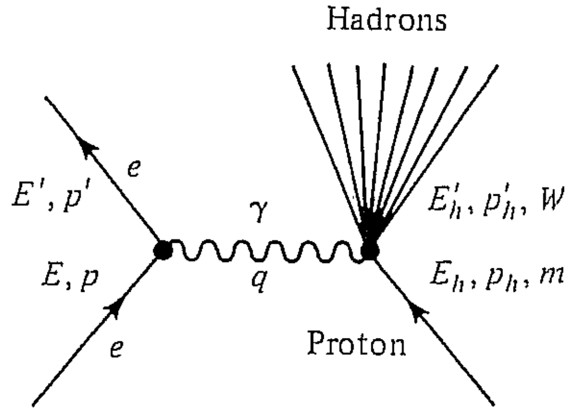


Provides information on the size of the nucleons:

$$\langle r_E^2(proton)\rangle \approx \langle r_M^2(proton)\rangle \approx \langle r_M^2(neutron)\rangle \approx 0.74\,fm^2$$



(a) Elastic



(b) Inelastic

Inelastic scattering

$$\nu = E - E' \qquad q^2 = \left(\frac{\nu}{c}\right)^2 - (\vec{p} - \vec{p}')^2 = \frac{\nu^2}{c^2} - (P_h')^2$$

$$W^2c^2 = \left(P_h'\right)^2 = (P_h + q)^2 = M^2c^2 + 2P_h \cdot q + q^2 \quad = M^2c^2 + 2M\nu + q^2 \quad \nu = \frac{P_h \cdot q}{M}$$

$M \equiv$ proton mass

clearly Lorentz invariant

$$\nu = \frac{P_h \cdot q}{M} = \frac{1}{M}\left(Mc, \vec{0}\right) \cdot \left(\frac{E - E'}{c}, \vec{p} - \vec{p}'\right) = E - E'$$ in the lab frame

or ……. consider a nucleus (or nucleon) with initial 4-momentum P_N that absorbs four-momentum q in the scattering process:

$$(P_N + q)^2 = M_N^2 c^2 \Rightarrow \cancel{P_N^2} + 2P_N \cdot q + q^2 = \cancel{M_N^2} c^2$$

$$\Rightarrow \left|q^2\right| = 2P_N \cdot q = 2\left(M_N c, \vec{0}\right) \cdot \left(\nu, \vec{q}\right) = 2M_N \nu$$

For quasi-elastic scattering $\quad \nu = \left|q^2\right| / (2M_N)$

Scattering from nucei

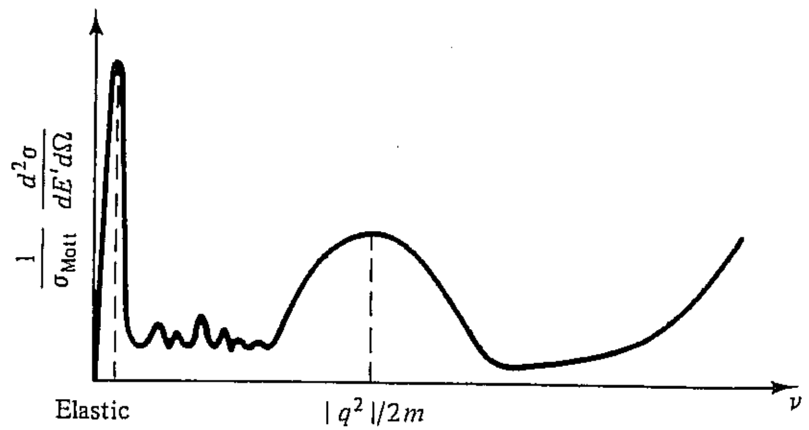


Fig. 6.17. Typical double differential cross section, normalized by dividing through by the Mott cross section, for inelastic electron scattering from a nucleus. The final rise shown is due to the onset of pion production.

Scattering from nucleon

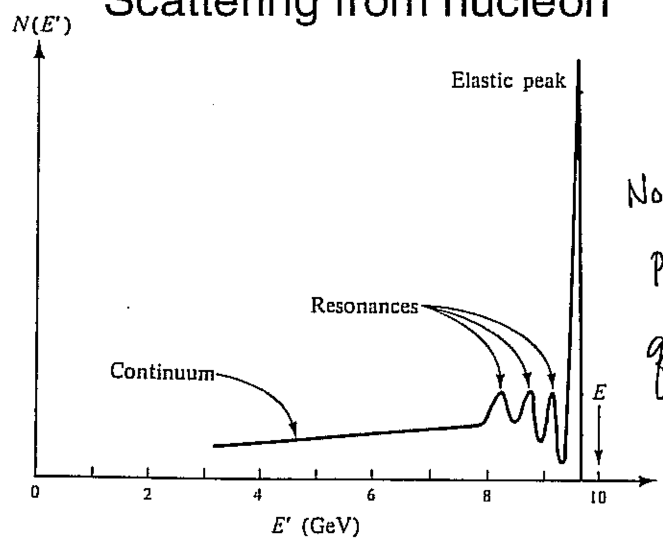


Fig. 6.18. Inelastic electron scattering from protons. $N(E)$ gives the number of scattered electrons with energy E. Note that this figure is backwards relative to Fig. 6.17.

No quasi-elastic peak !

quarks are confined.

In elastic scattering the one to one relationship between the three-momentum transfer squared and the scattering angle θ means there is only one kinematic variable describing the scattering (the final state energy is fixed, only the direction of the momentum vector changes)

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rosenbluth} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left[\frac{G_E^2 + bG_M^2}{1+b} + 2bG_M^2 \tan^2\theta/2\right]$$

So form factors are also functions of only one kinematic variable

$$(p' - p)^2 = q^2 = M^2c^2 + M^2c^2 - 2p' \cdot p$$

In the lab frame:

$$\Rightarrow q^2 = 2M^2c^2 - 2(Mc, \vec{p}') \cdot (Mc, \vec{0}) = 2M^2c^2 - 2M^2c^2 = 0$$

$$q^2 = \frac{\nu^2}{c^2} - |\vec{q}|^2 = 0 \qquad \nu, \vec{q}^{\,2}$$ are not independent variables

In inelastic scattering, the equivalent analysis leave you with:

$$q^2 = 2M^2c^2 - 2MWc^2 \neq 0 \quad (= \frac{\nu^2}{c^2} - |\vec{q}|^2)$$

q^2, ν, W inter-related

Now ν and $\vec{q}^{\,2}$ can vary independently. Both quantities (or two equivalent kinematic variables, for instance E' and θ, which are typically used in the lab frame) are required to describe inelastic scattering.

$$\left(\frac{d^2\sigma}{dq^2 d\nu}\right)_{Rosenbluth} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[W_2(q^2,\nu) + 2bW_1(q^2,\nu)\tan^2\theta/2\right]$$

↓ ↓

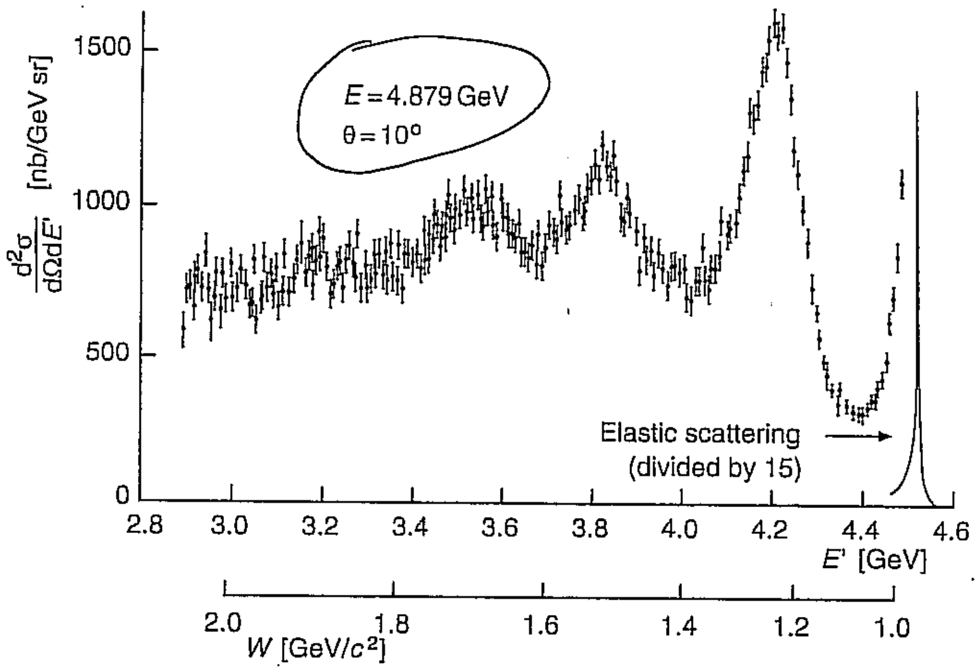
Structure functions of the proton

As we did for elastic electron-nucleon scattering, we can extract the structure function from scattering data:

Someone asked about this kind of distribution:

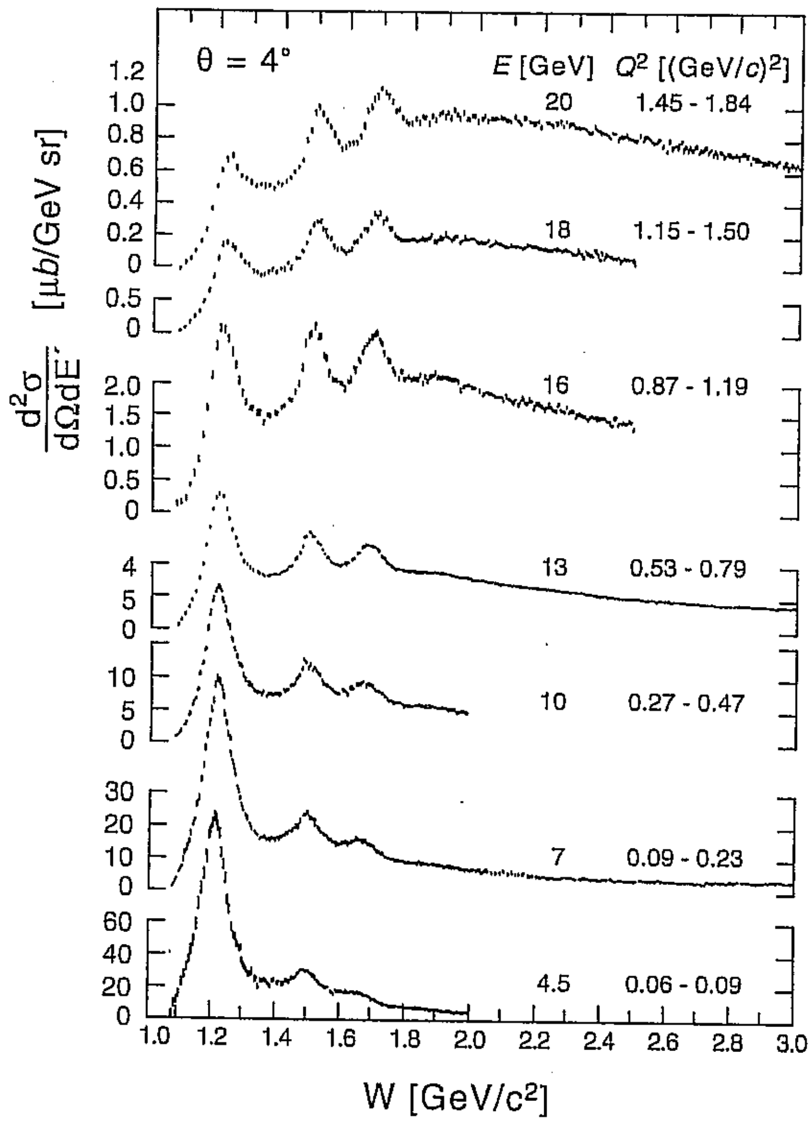
This is a ratio of the (doubly) differential cross-section relative to the Mott scattering cross-section, but this is NOT a form factor.....

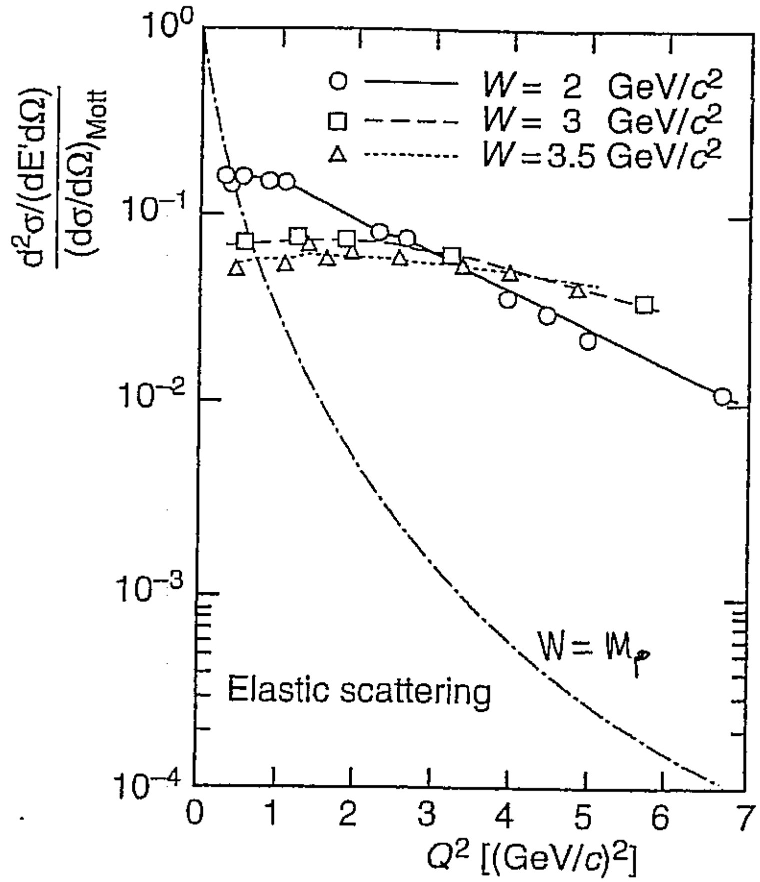
Remember (in the elastic case) the form factor is a Fourier Transform of the spatial charge (or magnetic moment) distribution. So the form factor is the ratio of the cross-sections as a function of the momentum transfer.



Pick specific beam energy and angle. Then E', W related. Can plot wrt either

Look at doubly differential cross-section as a function of W (mass of the hadronic final state) for different beam energies (four-momentum transfers). Scattering angle θ is fixed (here at 4°)





THIS is the plot that delivers the form factors (or rather the structure functions) as a function of q^2 (at fixed W)

So for deep-inelastic electron-proton scattering, the differential cross-section shows little dependence on q^2. This is the behaviour (for a form factor) that is typical of scattering from a *pointlike* object.

Form factors (structure functions) which are in principle function of both ν and q^2 can thus be written in terms of a single variable:

$$W_1(\nu, q^2) \to W_1(\nu) \Rightarrow W_1(x)$$
$$W_2(\nu, q^2) \to W_2(\nu) \Rightarrow W_2(x)$$

$$x = \frac{-q^2}{2M\nu}$$ (more later on this choice)

Usually use dimensionless structure functions

$$F_1(x) = Mc^2 W_1(x)$$
$$F_2(x) = \nu W_2(x)$$

"electric" structure function $F_2(x) = \sum_i q_i^2 x f(x)$

q_i^2: parton charges

$f(x)$: Parton fractional momentum distributions (saw these on assignment 3)

Consider a reference frame in which the proton momentum is very large (the Breit frame) Write the momentum of a parton as:

$$x = \frac{quark\,4-momentum}{proton\,4-momentum}$$

That is, we are ignoring the proton mass and the transverse components of the parton momentum.

So each parton carries a (different) fraction x of the proton four momentum *P*

Suppose a parton absorbs four momentum *q* from a scattering event:

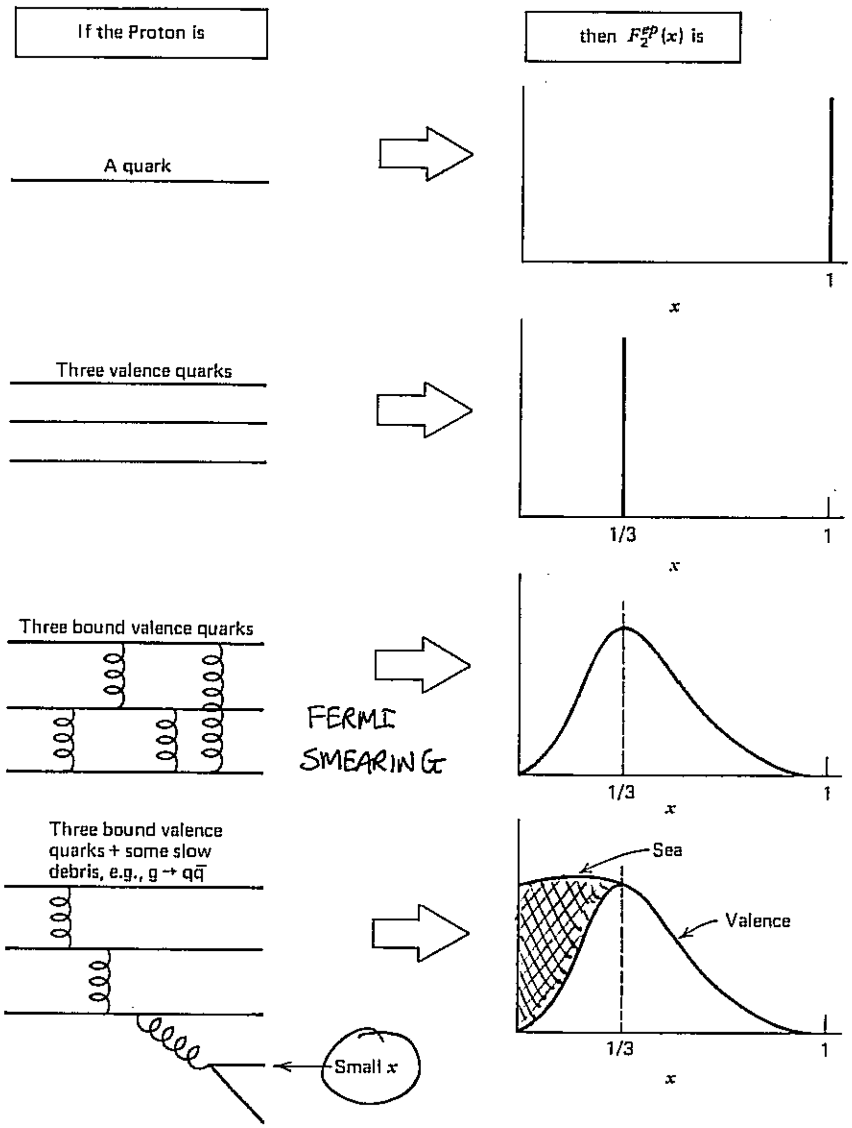
$$(xP+q)^2 = m_{parton}^2 c^2 \approx 0 \Rightarrow x^2P^2 + q^2 + 2xP\cdot q = 0$$

Partons behave as ~ massless

$$\Rightarrow q^2 + 2xP\cdot q = 0 \quad \Rightarrow \boxed{x = \frac{-q^2}{2M\nu}}$$

Hence that choice of variable earlier (when ν would have done just fine…….)

Back to $F_2(x) = \sum_i q_i^2 x f(x)$



$$F_2(x) = x\sum_i q_i^2 [f(x) + \bar{f}(x)]$$

distribution of anti-quark

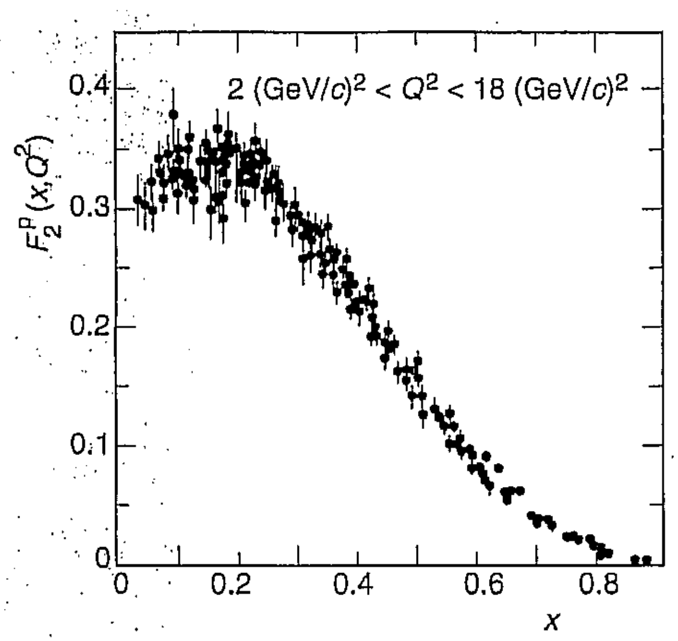
Momentum distribution of quarks and anti-quarks inside the proton cannot be calculated. Needs to be measured.

What about $F_1(x)$?

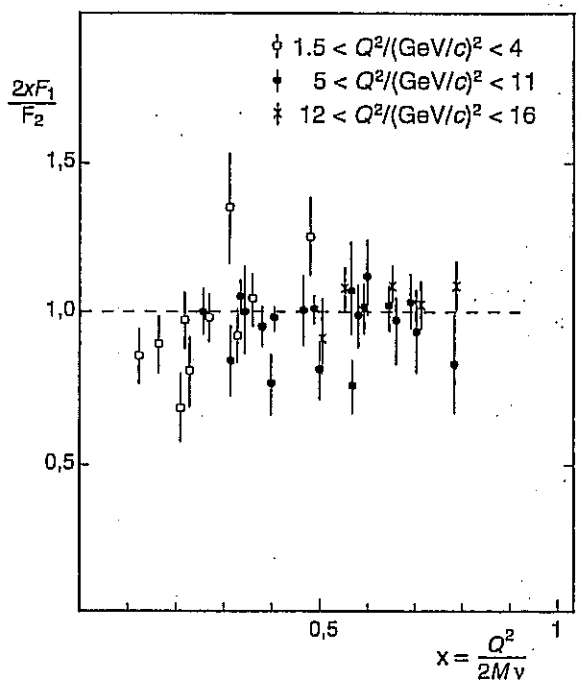
This is the magnetic contribution, which depends on the spin of the partons:

$$F_1(x) = 0 \qquad [spin-0 \;\; partons]$$

$$2xF_1(x) = F_2(x) \qquad [spin-1/2 \;\; partons] \qquad \text{Callan-Gross relation}$$

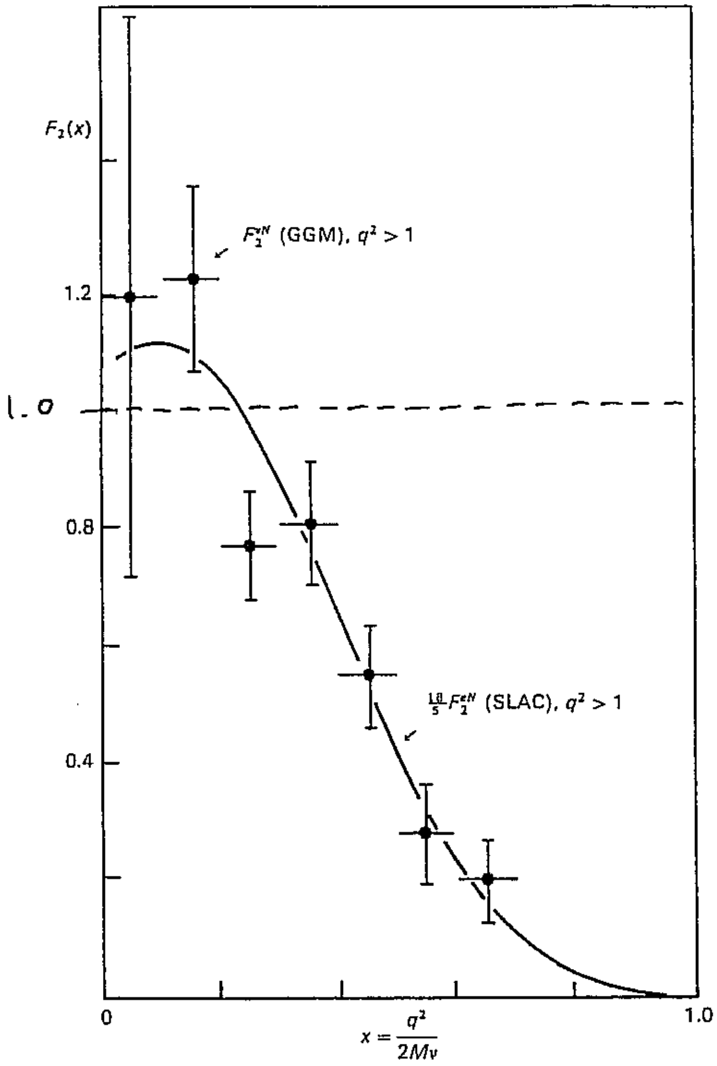


Demonstrates q^2 independence of F_2
(Normalization ?)



Establishes that partons are spin – 1/2

Note that for high momentum transfer scattering also have to account for weak interaction effects. Can also measure $F_2(x)$ using neutrino-nucleon scattering:



$$\int_0^1 F_2(x)dx \sim 0.5$$

Total fraction of proton momentum carried by quarks is about 50%. The rest is attributed to gluons.

Quark / anti-quark components

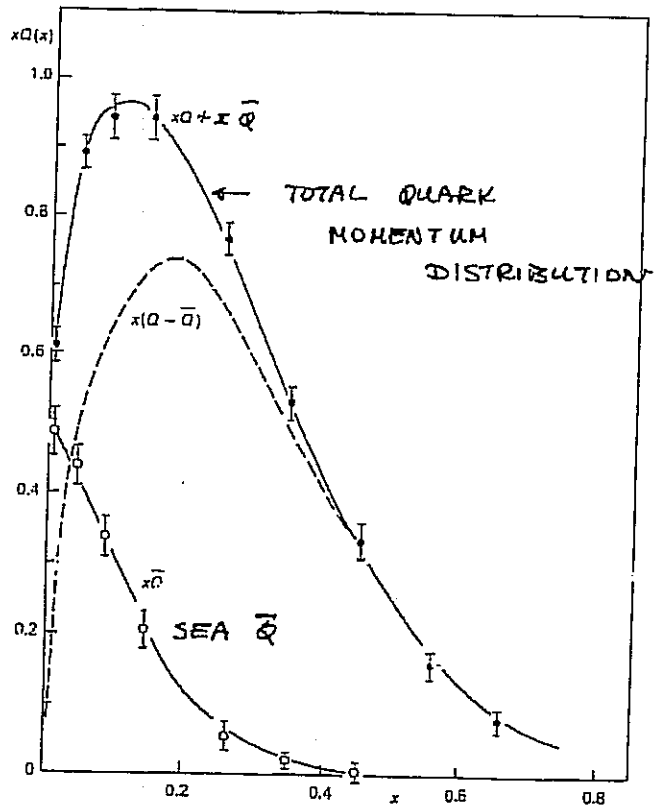


Figure 8.12 (b) Momentum distributions of quarks (Q) and antiquarks ($\bar{Q}$) in the nucleon, at a value of q^2 of order 10 GeV^2, obtained from results on neutrino and antineutrino scattering in experiments at CERN and Fermilab. The neutrino and antineutrino differential cross-sections measure the structure functions F_2 and F_3 in Eq. (8.17), and the difference and sum of these, through Eq. (8.23), give the quark and antiquark populations weighted by the momentum fraction x. The antiquarks ($\bar{Q}$) are concentrated at small x, the region of the so-called quark-antiquark "sea." The "valence" quarks of the static quark model ($Q - \bar{Q}$) are concentrated toward $x = 0.2$.