Phy489 Lecture 9

Discussion is of baryon wavefunctions because baryons are made up of three quarks (which are treated here as identical) while mesons consist of distinguishable particles since they are made of quark-antiquark pairs.

Slide from Lecture 2

Baryons

- There are 10 three quark combinations:

$$uuu, uud, udd, ddd, uus, uds, dds, uss, dss, sss$$

- Spin combinations:
  - $\uparrow\uparrow\uparrow$ spin 3/2 (get 10 states)
  - $\uparrow\uparrow\downarrow$ spin 1/2 (get 8 states – see below)

all baryons are fermions

- Different numbers of states for the two spin configurations arises due to requirements on the symmetry properties of the baryon wavefunction which must be anti-symmetric in the exchange of any two quarks.

$$\psi = \psi(space)\psi(spin)\psi(flavour)[\psi(colour)]$$

See § 5.9 (we will not cover chapter much of chapter 5, but will briefly discuss the issue of hadronic wavefuntions in a future lecture.)

Baryon Wavefunctions

We will discuss things in terms of baryons, which are made up of three quarks. But clearly the same applied to anti-baryons.

The spin-state of a baryon is $\frac{1}{2} \oplus \frac{1}{2} \oplus \frac{1}{2}$

$$\left(\frac{1}{2} \oplus \frac{1}{2}\right) \oplus \frac{1}{2} = (0,1) \oplus \frac{1}{2} = \frac{1}{2}, \frac{3}{2}$$

for the case of no relative orbital angular momentum (more on slide 4).

Baryons in the uds Quark Model

In the uds quark model, there are three flavours and so $3^3 = 27$ qqq flavour combinations (e.g. now counting uud, udu and duu separately).

What are their symmetries (e.g. under exchange of two quarks)?

There are also (similarly) 3^3 colour combinations to consider.

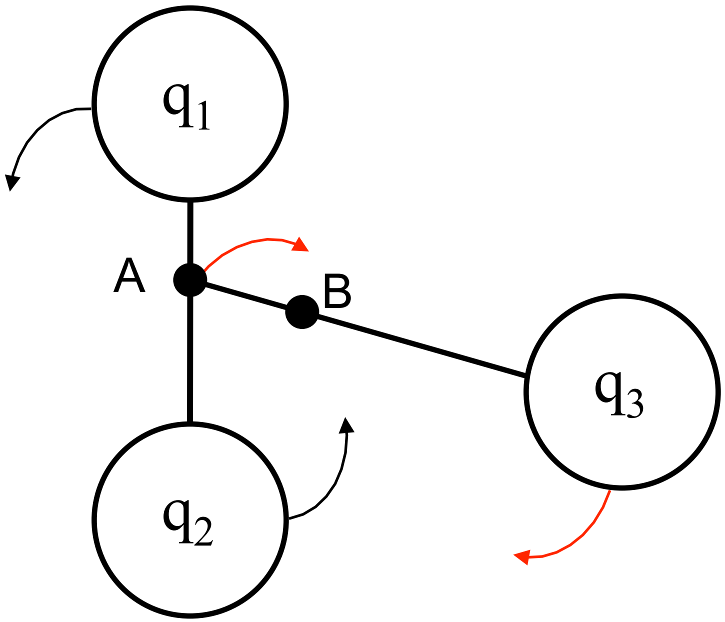
Will see that for a system of three objects each taking on one of three values (here either colour or flavour) the 27 combinations consist of:

- 10 symmetric states
- 2 x 8 states with mixed symmetry (anti-symmetric in one pair)
- 1 fully anti-symmetric

Where the symmetries referred to are under the exchange of any two quarks.

When discussing orbital angular momentum (so far) we have considered only the relatively simple case of mesons, where there are only two objects (a quark and an anti-quark) which can have relative angular momentum. Baryons are more complicated.

Angular momentum in three quark states



q_1, q_2 have ℓ about point A (CM_{12})

(q_1,q_2),q_3 have ℓ' about point B (CM_{123})

Consider the ground-state $\left(\ell = \ell' = 0\right)$ so $j = \dfrac{1}{2}, \dfrac{3}{2}$ as above.

Baryon wavefunction symmetries

We have said before that the total wavefunction for a fermion (and hence for all baryons) must be anti-symmetric in the exchange of identical particles, e.g.

$$\psi(1,2,3,.....) = -\psi(2,1,3,.....)$$

This is not the case for mesons (since these are made of quark-antiquark pairs) but for baryons (in the case where we treat all quarks as identical – this is a subtle point) we require that the TOTAL wavefunction

$$\psi(total) = \psi(space)\cdot\psi(spin)\cdot\psi(flavour)\cdot\psi(colour)$$

be anti-symmetric in the exchange of any two quarks. For the ground-state the spatial part of the wavefunction is always symmetric [in the non-relativisitic case the spatial wavefunction is just given in terms of spherical harmonics – the lowest order SH (e.g. for $\ell = 0$) is just a constant (e.g. uniform, which can only be symmetric).

Baryon spin states

For the spin states (which give the total spin in the case we are considering) there are $2^3 = 8$ possibilities:

$\uparrow\uparrow\uparrow$			symmetric
$\uparrow\uparrow\downarrow$	$\uparrow\downarrow\uparrow$	$\downarrow\uparrow\uparrow$	
$\downarrow\downarrow\uparrow$	$\downarrow\uparrow\downarrow$	$\uparrow\downarrow\downarrow$	
$\downarrow\downarrow\downarrow$			symmetric

However, we want these in linear combinations with are eigenstates of total angular momemtum (as we did in the previous lecture for the case of $\frac{1}{2} \oplus \frac{1}{2}$).

Symmetric eigenstates of total angular momentum

$$\left|\frac{3}{2}\ \frac{3}{2}\right\rangle = \uparrow\uparrow\uparrow$$

$$\left|\frac{3}{2}\ \frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow\right)$$

$$\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left(\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow\right)$$

$$\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle = \downarrow\downarrow\downarrow$$

By symmetric, here, I mean under the exchange of any two quarks

Mixed symmetry states

$$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_{12} = \frac{1}{\sqrt{2}}\left(\uparrow\downarrow - \downarrow\uparrow\right)\uparrow$$

$$\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle_{12} = \frac{1}{\sqrt{2}}\left(\uparrow\downarrow - \downarrow\uparrow\right)\downarrow$$

Anti-symmetric in 1,2

$$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_{23} = \frac{1}{\sqrt{2}}\uparrow\left(\uparrow\downarrow - \downarrow\uparrow\right)$$

$$\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle_{23} = \frac{1}{\sqrt{2}}\downarrow\left(\uparrow\downarrow - \downarrow\uparrow\right)$$

Anti-symmetric in 2,3

General spin states

All states can be written as linear combinations of these two sets of pure and mixed symmetries. We could also write mixed symmetry term asymmetric in the exchange of $1 \leftrightarrow 3$, but these would not be independent (we have 8 terms in both cases above):

$$\left|\frac{1}{2}\;\frac{1}{2}\right\rangle_{13} = \left|\frac{1}{2}\;\frac{1}{2}\right\rangle_{12} + \left|\frac{1}{2}\;\frac{1}{2}\right\rangle_{23} = \frac{1}{\sqrt{2}}\left[\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow\right] = \frac{1}{\sqrt{2}}\left[\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow\right]$$

Let's turn now to colour and flavour: there are 3^3 combinations in each case (as we wrote earlier). These can also be written in terms of combinations with pure or mixed symmetry under particle exchange:

The three combinations uuu, ddd and sss clearly have pure flavour symmetry. There are also other combinations with pure symmetry, e.g.

$$\frac{1}{\sqrt{3}}\left(uus + usu + suu\right)$$

In all there are 10 purely symmetric combinations

One can make only one purely anti-symmetric flavour combination:

$$\frac{1}{\sqrt{6}}\left(uds - usd + dsu - dus + sud - sdu\right)$$

The remaining 16 combinations are:

- 8 with mixed symmetry (anti-symmetric in $1 \leftrightarrow 2$)
- 8 with mixed symmetry (anti-symmetric in $2 \leftrightarrow 3$)

Again, we could write other mixed symmetry terms anti-symmetric in $1 \leftrightarrow 3$, but these would not be independent. So, we have (for states made of three quarks with three possible flavours [uds]):

- a decouplet of symmetric flavour states
- a singlet anti-symmetric flavour state
- two octets of mixed-symmetry states

The same arguments apply to the colour combinations (there are three flavours and there are three colours, so the combinatorics is the same). So there are again 3^3=27 combinations with the same symmetry breakdown as was just developed for the flavour combinations.

We are now in the position to make a more correct statement of the fact that quarks exist only in colourless objects (which is a common way to phrase this). That is:

<u>All free naturally-occuring particles belong to a colour singlet</u>

$$\psi(colour) = \frac{1}{\sqrt{6}}(rgb - rbg + gbr - grb + brg - bgr)$$

This is anti-symmetric under the exchange of any two quarks.

In this case, for a ground-state baryon $(\ell = \ell' = 0)$ $\psi(spin)\cdot\psi(flavour)$ must be symmetric.

For baryons that are made of three identical (same flavour) quarks, $\psi(flavour)$ can only be symmetric.

For all other cases it can be either symmetric or of mixed symmetry. In the case where the three flavours are all different (and only in this case) it can also be fully anti-symmetric.

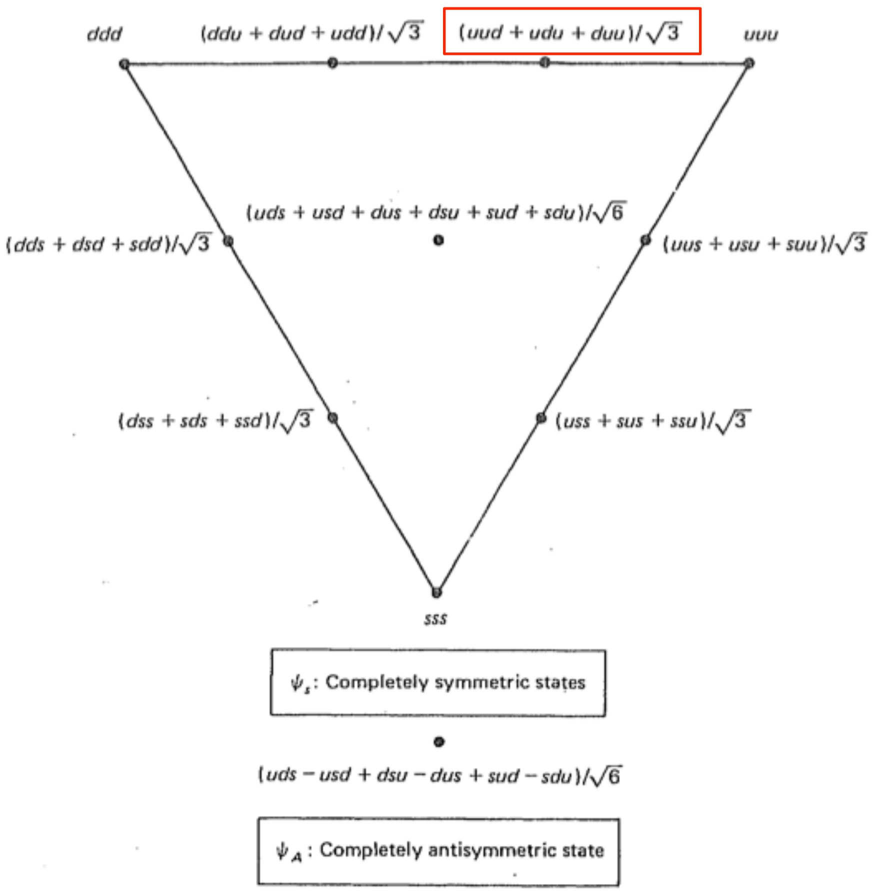
For $\psi(spin)$ one gets a fully symmetric version only for $j = \frac{3}{2}$, so the product

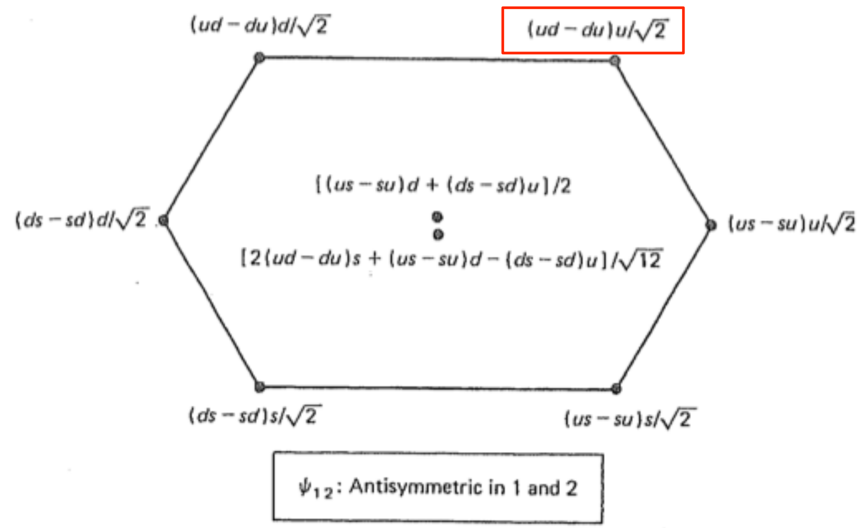
$$\psi(spin)\cdot\psi(flavour)$$

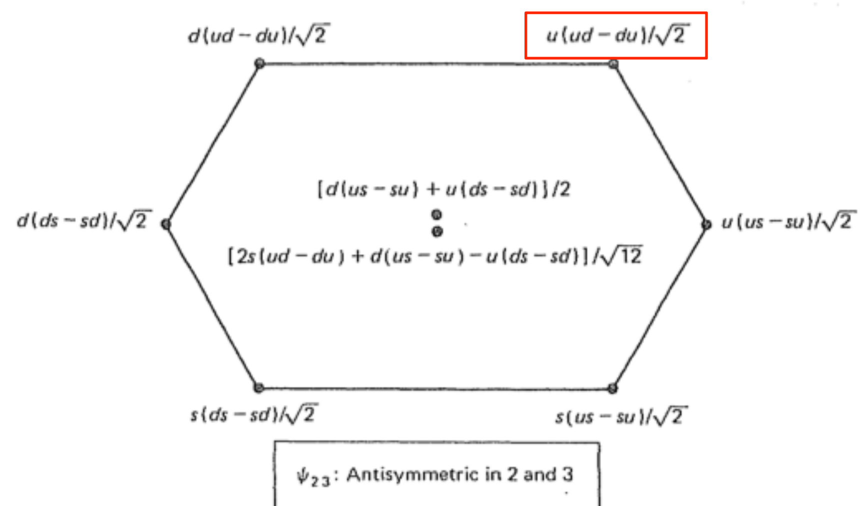
cannot be symmetric for uuu, ddd or sss baryons with spin-1/2, (e.g. $j = \frac{1}{2}$).

For combinations with two same-flavour quarks (e.g. uud) there are three possible arrangements: uud, udu and duu. One can make a symmetric combination and two of mixed symmetry (but you can’t make a fully anti-symmetric state).

Flavour multiplets (uds quark model)







For three different quarks, there are 6 arrangements:

- one fully symmetric (belongs to decouplet)
- one fully anti-symmetric (singlet)
- four of mixed symmetry (belong to the two octets)

For the total wavefunction, the (mixed symmetry) flavour octet members must be have be matched to the $\psi(spin)$ having the corresponding (anti-)symmetry:

$$\psi(baryon\ octet) =$$

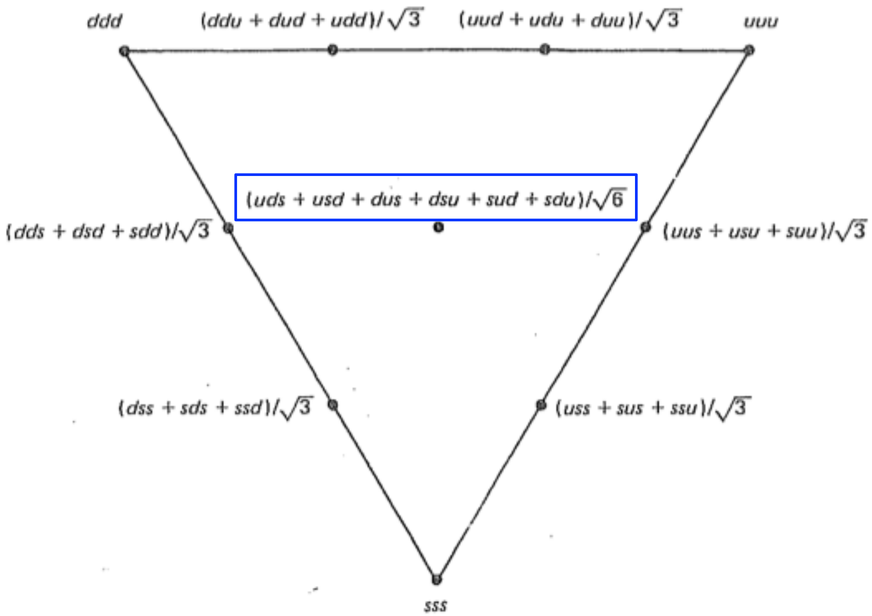
$$\frac{\sqrt{2}}{3}\left[\psi_{12}(spin)\psi_{12}(flavour) + \psi_{23}(spin)\psi_{23}(flavour) + \psi_{13}(spin)\psi_{13}(flavour)\right]$$

Look at Ω^- (sss) in the PDG. There is $\Omega(1672)^-$ $J^P = 3/2^+$

$\Omega(2250)^-$ $J^P = ?^?$ presumably $(\ell \text{ or } \ell' = 0)$

No spin1/2 version (which would presumably be lighter than the spin 3/2 version).

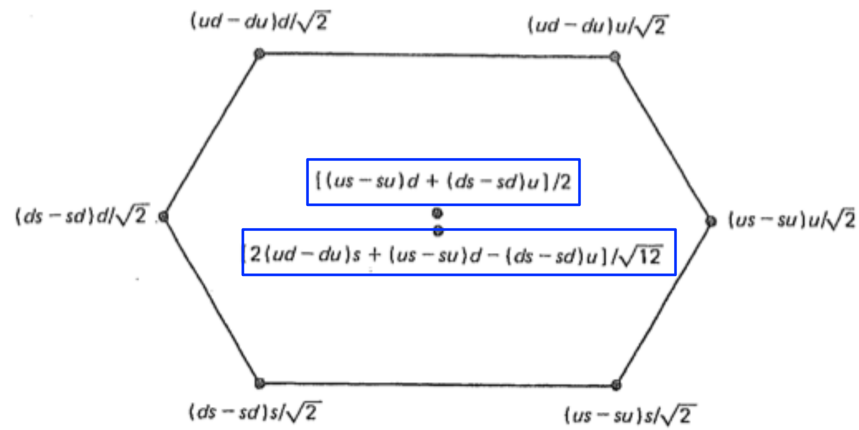
Flavour multiplets (uds quark model)



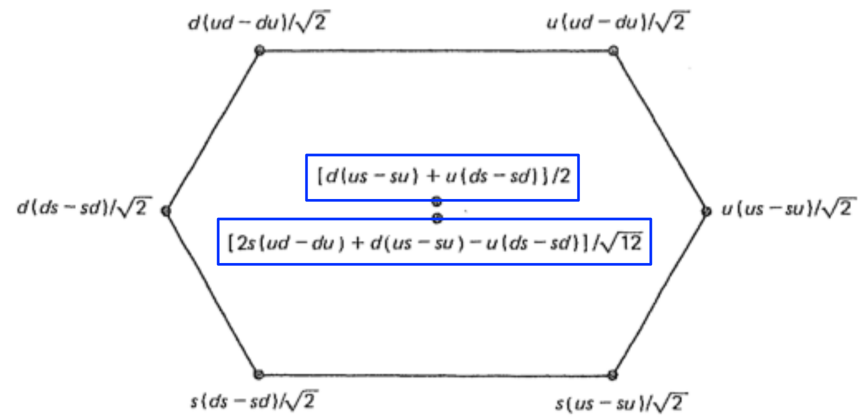
ψ_s: Completely symmetric states

$(uds - usd + dsu - dus + sud - sdu)/\sqrt{6}$

ψ_A: Completely antisymmetric state



ψ_{12}: Antisymmetric in 1 and 2



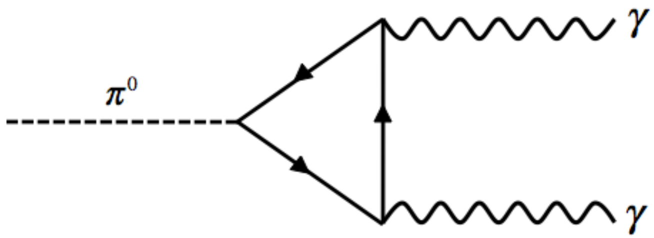
ψ_{23}: Antisymmetric in 2 and 3

Note that an understanding of the baryon wavefunction is necessary for a a discussion of (for example) baryon magnetic moments and baryon masses. These are discussed in the later sections of Griffiths Chapter 5, but we will not cover this material.

Other Evidence for Colour

The existence of the Δ^{++} led to the hypothesis of colour. There are other pieces of experimental evidence for colour. We will see one of the main ones later in the course, when we talk about hadron production in electron-positron collisions.

Another example is the decay width of the neutral pion, which decays (almost always) to two photons, $\pi^0 \to \gamma\gamma$



$$\Gamma\left(\pi^0 \to \gamma\gamma\right) = \left(\frac{N_c}{3}\right)^2 \frac{\alpha^2 m_\pi^2}{64\pi^3 f_\pi^2}$$

Here m_π is the neutral pion mass, f_π is the pion decay constant (more on this later in the course) and N_c is the number of quark colours. Value determined from measured decay width is:

$$N_c = 2.99 \pm 0.12$$

We will learn about decay widths (and their calculation) as the course progresses.

Other applications of wavefunction symmetries

Consider the decay $\rho^0 \to \pi\pi$. At first glance, it would seem that one would expect two final states (for decays into two pions – which is the case almost 100% of the time):

$$\rho^0 \to \pi^+\pi^- \qquad\qquad \rho^0 \to \pi^0\pi^0$$

However, only the final state with charged pions occurs: why?

> Note in one case we have a final state consisting of identical bosons. The wavefunction for such a system is required to be symmetric in the exchange of the two identical particles.

<u>We will discuss this in the lecture.</u>