Phy489 Lecture 11

Question

Why are $K^{*0} - \overline{K}^{*0}$ oscillations not observed experimentally ?

(K^{*0} is the same as K^0 but with spin-1 instead of spin-0.)

$$K^0 \quad \bar{s}d \quad spin-0 \quad \uparrow\downarrow \quad M(K^0) \approx 498\, MeV/c^2$$

$$K^{*0} \quad \bar{s}d \quad spin-1 \quad \uparrow\uparrow \quad M(K^{*0}) \approx 892\, MeV/c^2$$

The mixing process is a second-order weak interaction that must take place on timescales associated with the weak interaction. However, the K^{*0} is massive enough to decay via the strong interaction so never lives long enough for this mixing process to be relevant.

Particle Lifetimes & Decay Rates

The lifetime τ of a particle is defined in it's rest frame. This lifetime represents an *average* lifetime. Quantum mechanically we cannot say anything about the lifetime of an individual particle.

The decay rate Γ represents the probability per unit time that a given particle (in a collection of particles) will decay. If we begin at t=0 with (say) N_0 charged pions, the number expected to decay in any given time interval dt is given by $\Gamma N(t)dt$, so

$$dN = -\Gamma N dt \quad \Rightarrow \quad N(t) = N_0 e^{-\Gamma t}$$

Here Γ is a probability / unit time, so has units of $(\text{time})^{-1}$ (so the exponent above is dimensionless, as it must be). The lifetime τ is defined as the reciprocal of the decay rate:

$$\tau = \frac{1}{\Gamma}$$

Branching Ratios (Branching Fractions)

Most particles decay into multiple final states, *e.g.*

$$D^0 \to K^-\pi^+,\, D^0 \to \pi^-\pi^+,\, D^0 \to K^+\pi^- \text{ + many other final states}$$

We define a decay rate for *each* available final state. The total decay rate (and thus the particle lifetime) is then given by

$$\Gamma_{TOT} = \sum_i \Gamma_i \qquad \tau = \frac{1}{\Gamma_{TOT}}$$

Sometimes refer to Γ_{TOT} as the particle "width", Γ_i as a "partial width"

The branching ratio for the i^{th} decay mode is then given by $Br(i) = \dfrac{\Gamma_i}{\Gamma_{TOT}}$

e.g. from your first assignment $Br(D^0 \to K^-\pi^+) = (3.89 \pm 0.05)\%$

[That is, 3.89% of D^0 mesons will (on average) decay into this particular final state.
Or a given D^0 meson has a 3.89% chance of decaying into this final state.]

Stable vs. Unstable Particles

Note that particles that decay only weakly or electromagnetically are typically classified as "stable" (i.e. $\tau >> 10^{-23}$ s, the timescale associated with the strong interaction).

For such particles, we often quote lifetimes: $\tau(D^0) = (410.1 \pm 1.5) \times 10^{-15}\, s$

For strongly decaying particles (or for very heavy particles that decay very quickly, such as the electroweak gauge bosons) we conventionally quote the total decay width (rate) Γ.

For instance, from PDG: $\Gamma(Z^0) = (2.4952 \pm 0.0023)\mathrm{GeV}$ (this is a weak decay)

Note the units of energy. This is for "natural" units in which $\hbar = c = 1$. $\left[\begin{array}{c} \hbar = 6.6 \times 10^{-25}\ \mathrm{GeV} \cdot \mathrm{s} \\ \text{for conversion} \end{array}\right]$

N.B. Strongly decaying particles typically have $\tau \sim 10^{-23}$ s, so

$$\Gamma \sim (6.6 \times 10^{-2})\ \mathrm{GeV} \sim 10^{-1}\ \mathrm{GeV}\ \text{ or } 100\ \mathrm{MeV}$$

For example: $Br(K^{*0} \to K\pi) \sim 100\%$ $\Gamma(K^{*0}) = (50.3 \pm 0.6)\mathrm{MeV}$

This can be thought of in terms of the uncertainty principle $\Delta E \Delta t \sim \hbar$ $E = mc^2$

D^0 $I(J^P) = \frac{1}{2}(0^-)$

Mass $m = 1864.84 \pm 0.17$ MeV (S = 1.1)
$m_{D^\pm} - m_{D^0} = 4.78 \pm 0.10$ MeV (S = 1.1)
Mean life $\tau = (410.1 \pm 1.5) \times 10^{-15}$ s
$c\tau = 122.9\ \mu$m

D^0 DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Semileptonic modes				
$K^- e^+ \nu_e$		(3.58 ±0.06) %	S=1.1	867
$K^- \mu^+ \nu_\mu$		(3.31 ±0.13) %		864
$K^*(892)^- e^+ \nu_e$		(2.18 ±0.16) %		719
$K^*(892)^- \mu^+ \nu_\mu$		(2.01 ±0.25) %		714
$K^- \pi^0 e^+ \nu_e$		(1.6 $^{+1.3}_{-0.5}$) %		861
$\overline{K}^0 \pi^- e^+ \nu_e$		(2.7 $^{+0.9}_{-0.7}$) %		860
$K^- \pi^+ \pi^- e^+ \nu_e$		(2.8 $^{+1.4}_{-1.1}$) $\times 10^{-4}$		843
$K_1(1270)^- e^+ \nu_e$		(7.6 $^{+4.2}_{-3.1}$) $\times 10^{-4}$		498
$K^- \pi^+ \pi^- \mu^+ \nu_\mu$		< 1.2 $\times 10^{-3}$	CL=90%	821
$(\overline{K}^*(892)\pi)^- \mu^+ \nu_\mu$		< 1.4 $\times 10^{-3}$	CL=90%	692
$\pi^- e^+ \nu_e$		(2.83 ±0.17) $\times 10^{-3}$		927
$\pi^- \mu^+ \nu_\mu$		(2.37 ±0.24) $\times 10^{-3}$		924
$\rho^- e^+ \nu_e$		(1.9 ±0.4) $\times 10^{-3}$		771
Hadronic modes with one $\overline{K}$				
$K^- \pi^+$		(3.89 ±0.05) %	S=1.1	861
$K^0_S \pi^0$		(1.22 ±0.06) %	S=1.2	860
$K^0_L \pi^0$		(10.0 ±0.7) $\times 10^{-3}$		860
$K^0_S \pi^+ \pi^-$	[qq]	(2.99 ±0.17) %	S=1.1	842
$K^0_S \rho^0$		(7.7 $^{+0.6}_{-0.8}$) $\times 10^{-3}$		674
$K^0_S \omega$, $\omega \to \pi^+ \pi^-$		(2.2 ±0.6) $\times 10^{-4}$		670
$K^0_S f_0(980)$, $f_0(980) \to \pi^+ \pi^-$		(1.40 $^{+0.30}_{-0.22}$) $\times 10^{-3}$		549
$K^0_S f_2(1270)$, $f_2(1270) \to \pi^+ \pi^-$		(1.3 $^{+1.2}_{-0.7}$) $\times 10^{-4}$		262
$K^0_S f_0(1370)$, $f_0(1370) \to \pi^+ \pi^-$		(2.5 $^{+0.6}_{-0.7}$) $\times 10^{-3}$		†
$K^*(892)^- \pi^+$, $K^*(892)^- \to K^0_S \pi^-$		(1.97 ±0.13) %		711
$K^*(892)^+ \pi^-$, $K^*(892)^+ \to K^0_S \pi^+$	[yy]	(1.0 $^{+1.3}_{-0.4}$) $\times 10^{-4}$		711
$K^*_0(1430)^- \pi^+$, $K^*_0(1430)^- \to K^0_S \pi^-$		(2.9 $^{+0.7}_{-0.4}$) $\times 10^{-3}$		378
$K^*_2(1430)^- \pi^+$, $K^*_2(1430)^- \to K^0_S \pi^-$		(3.3 $^{+2.2}_{-1.1}$) $\times 10^{-4}$		367
$K^*(1680)^- \pi^+$, $K^*(1680)^- \to K^0_S \pi^-$		(7 $^{+6}_{-5}$) $\times 10^{-4}$		46

isospin, spin and parity

$K^*(892)$ $\boxed{I(J^P) = \frac{1}{2}(1^-)}$

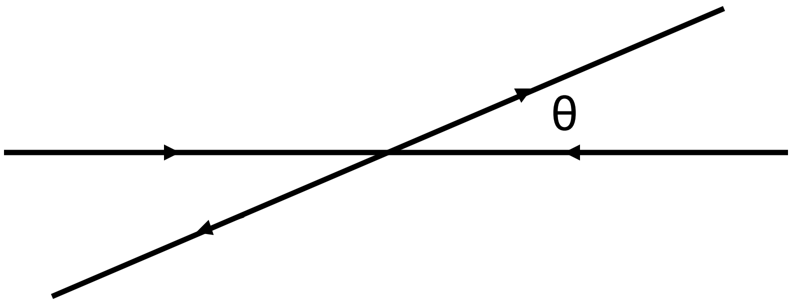
$K^*(892)^\pm$ mass $m = 891.66 \pm 0.26$ MeV
Mass $m = 895.5 \pm 0.8$ MeV
$K^*(892)^0$ mass $m = 896.00 \pm 0.25$ MeV (S = 1.4)
$K^*(892)^\pm$ full width $\Gamma = 50.8 \pm 0.9$ MeV
Full width $\Gamma = 46.2 \pm 1.3$
$K^*(892)^0$ full width $\Gamma = 50.3 \pm 0.6$ MeV (S = 1.1)

$K^*(892)$ DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$K\pi$	~ 100 %		289
$K^0\gamma$	(2.31±0.20) $\times 10^{-3}$		307
$K^\pm\gamma$	(9.9 ±0.9) $\times 10^{-4}$		309
$K\pi\pi$	< 7 $\times 10^{-4}$	95%	223

Scattering

For decays it is the decay rate that we are interested in calculating. What about for scattering processes?

Typically we are interested in the cross-section σ, or the differential cross-section with respect to some kinematic variable, *e.g.* dσ/dθ where θ is a scattering angle.



2➔2 scattering in the CM frame. Incoming particles are back to back so outgoing particles must be as well. Probability that outgoing particles will be “scattered” at an angle θ is proportional to dσ/dθ. The total cross-section is then

$$\sigma = \int_{-\pi/2}^{+\pi/2} \left(\frac{d\sigma}{d\theta} \right) d\theta$$

Note that use of the word “scattering” does NOT imply that final state particles are the same as those in the initial state.

Griffiths Example 6.1 Hard Sphere Scattering

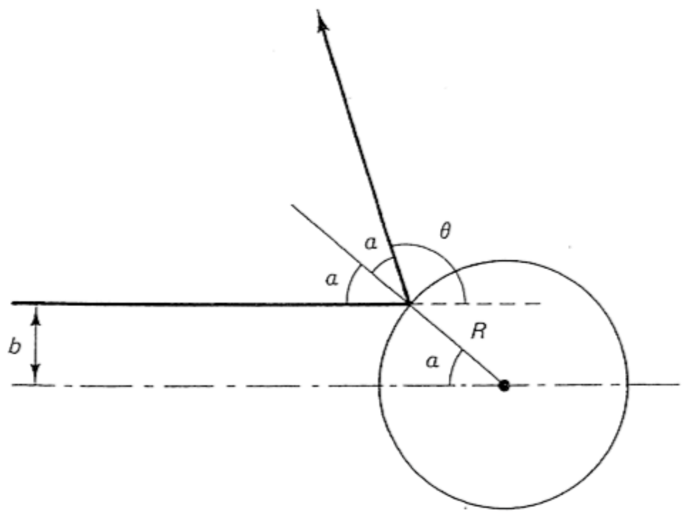


Figure 6.2 Hard-sphere scattering.

b is called the impact parameter of the incoming particle. In this classical system there is a *one-to-one correspondence* between b and the scattering angle θ, which is given by $\theta = 2\cos^{-1}(b/R)$:

$$\boxed{\begin{array}{l} b = R\sin\alpha, \quad 2\alpha + \theta = \pi \quad \Rightarrow \quad \alpha = \pi/2 - \theta/2 \\ \sin\alpha = \sin(\pi/2 - \theta/2) = \cos(\theta/2) \quad b = R\cos(\theta/2) \Rightarrow \theta = 2\cos^{-1}(b/R) \end{array}}$$

Hard Sphere Scattering in 3D

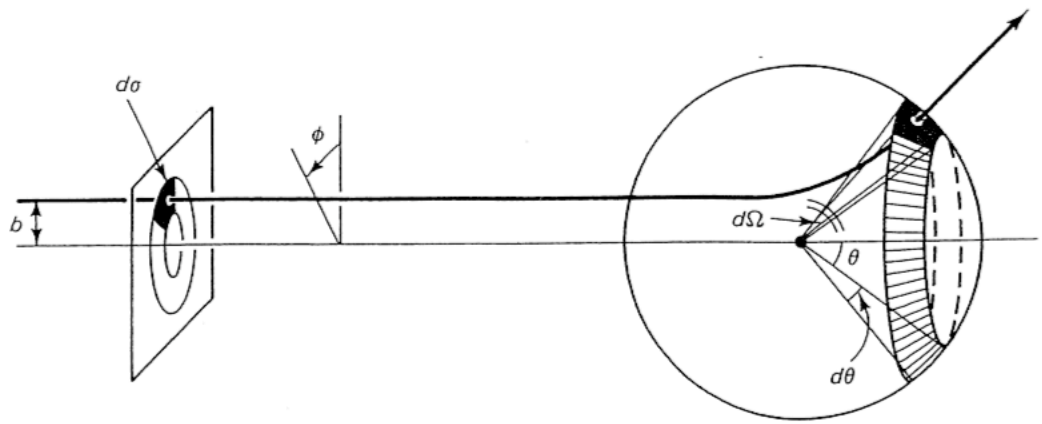


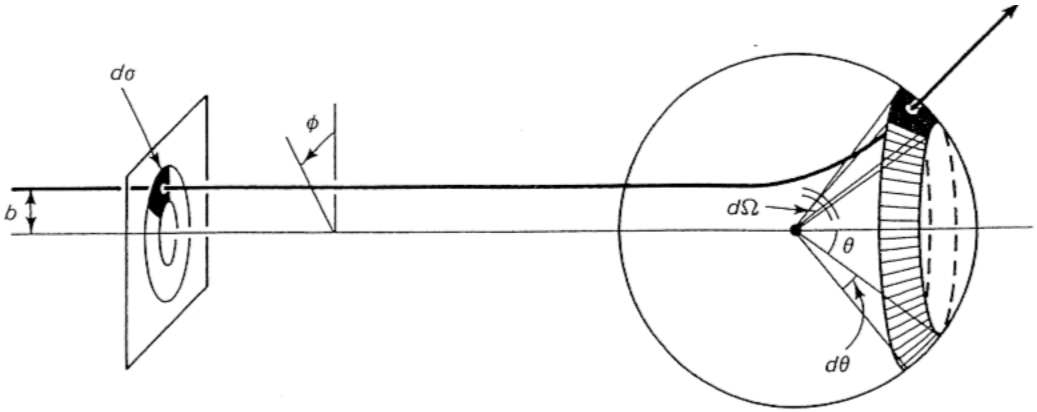
Figure 6.3 Particle incident in area $d\sigma$ scatters into solid angle $d\Omega$.

Consider and incoming particle with impact parameter in the range $b \rightarrow b+db$. Due to the 1 to 1 correspondence between b and θ, we can say that the particle will scatter into the angular region between θ and θ+$d\theta$.

Now consider a particle passing through the infinitesimal area $d\sigma$. This will scatter into a region of solid angle $d\Omega$. As $d\sigma$ increases so does $d\Omega$. We call the constant of proportionality the differential cross-section: $d\sigma = D(\theta)d\Omega$.

We will generally write this as $d\sigma/d\Omega$

In our spherically symmetric example, there is no Φ dependence.



Using the figure can show that (in this case): $d\sigma = |b\,db\,d\phi| \quad d\Omega = |\sin\theta\, d\theta\, d\phi|$

$$D(\theta) = \frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left(\frac{db}{d\theta}\right) \quad \text{and we had} \quad b = R\cos(\theta/2)$$

$$\frac{db}{d\theta} = -\frac{R}{2}\sin(\theta/2) \Rightarrow D(\theta) = \frac{R^2\cos(\theta/2)\sin(\theta/2)}{2\sin\theta} = \frac{R^2}{4}$$

so the total cross-section is given by $\sigma = \int\left(\frac{d\sigma}{d\Omega}\right)d\Omega = \int\frac{R^2}{4}d\Omega \quad = \pi R^2$

This is, of course, just the classical cross-section presented to the beam by the hard sphere. Hence the terminology “cross-section”.

Rutherford Scattering: Example 6.4

Rutherford scattering is soft scattering centre (for example some central potential).

That is, scattering of one charged particle from another (fixed) charge.

$$b = b(\theta) = \frac{q_1 q_2}{2E}\cot(\theta/2)$$

This is a non-trivial result that can commonly be found in most classical mechanics texts (see Goldstein for example).

$$D(\theta) = \frac{d\sigma}{d\Omega} = \left[\frac{q_1 q_2}{4E\sin^2(\theta/2)}\right]^2$$

N.B. this is for scattering of a particle of energy E, and charge q_1, off a stationary (fixed) charge of charge q_2.

$$\sigma_{TOT} = 2\pi\left(\frac{q_1 q_2}{4E}\right)^2 \int_0^\pi \frac{1}{\sin^4(\theta/2)}\sin\theta d\theta = \quad \infty\ !!!$$

$$y = \sin(\theta/2) \quad dy = \frac{1}{2}\cos(\theta/2)d\theta \quad \sin\theta d\theta = 2\sin(\theta/2)\cos(\theta/2)d\theta = 4y dy$$

$$\int_0^\pi \frac{1}{\sin^4(\theta/2)}\sin\theta d\theta = \int_0^1 \frac{4y}{y^4}dy = \int_0^1 \frac{4}{y^3}dy = -\frac{2}{y^2}\bigg|_0^1 = \infty$$

The infinite (total) cross-section is related to the infinite range of the EM force.

Resonances in Scattering Processes

We saw in an earlier lecture that scattering through a resonance can enhance the cross-section for a given scattering process. The example we used was $\pi^+ p$ scattering through the $\Delta^{++}(1232)$.

Specifically, we looked at the process $\pi^+ p \to \pi^+ p$ which gets a contribution from $\pi^+ p \to \Delta^{++} \to \pi^+ p$ when $\sqrt{s} \approx M\left(\Delta^{++}\right)$.

In principle, we could also get $\pi^+ p \to \Delta^{++} \to XY$ contributions to the process $\pi^+ p \to XY$ where XY is any final state to which the Δ^{++} decays.

in this particular example, there are actually no other options for XY.

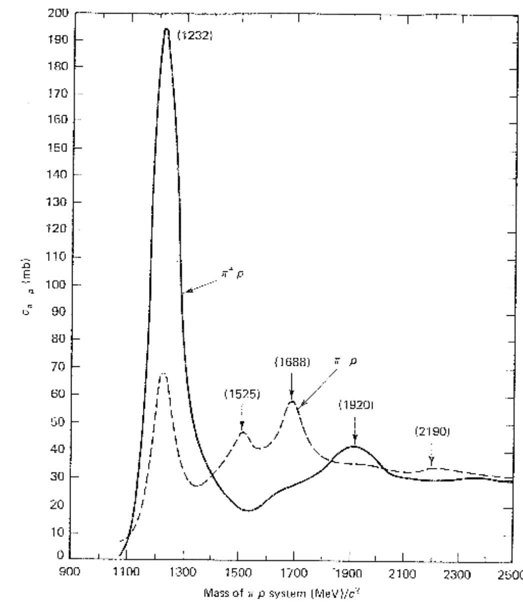


Figure 4.6 Total cross sections for π^+p (solid line) and π^-p (dashed line) scattering. (*Source:* S. Gasiorowicz, *Elementary Particle Physics* (New York: Wiley, copyright © 1966, page 294. Reprinted by permission of John Wiley and Sons, Inc.)

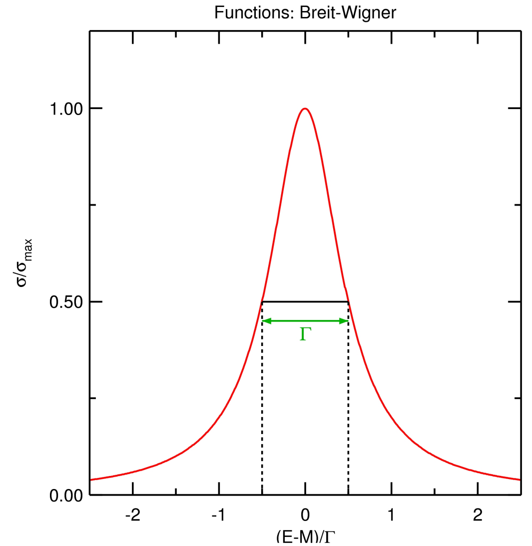
Another example (which we have also already seen) is scattering via the Z boson. This is a fundamental particle, but like the Δ^{++} it has a mass and so scattering of two particles which couple to the Z is enhanced at $\sqrt{s} \approx M(Z)$.

e.g. $e^+e^- \to Z \to q\bar{q} \to hadrons$ (more on this later in the course).

Resonant Scattering Cont'd

We had that, for an unstable state: $\Gamma = \sum_i \Gamma_i$

The energy distribution of an unstable state decaying to a final state f is given by



$$N_f(W) \propto \frac{\Gamma_f}{(W-M)^2 c^4 + \Gamma^2/4}$$

where M is the mass of the decaying particle and W is the invariant mass of the decay products (e.g. the final state). This is called a Breit-Wigner distribution.

For an unstable particle produced in a scattering process, the enhancement in the scattering cross-section follows the same Breit-Wigner form

$$\sigma_{fi} \propto \frac{\Gamma_i \Gamma_f}{\left(E - Mc^2\right)^2 + \Gamma^2/4}$$

Where E is the total energy of the system. We will see where this comes from when we calculate the cross-section for $e^+e^- \to Z \to f\bar{f}$.

Collider Physics Terminology

We won't do too much experimental stuff in the course, but it is useful to know a few more basic terms from collider physics. The LUMINOSITY of a collider

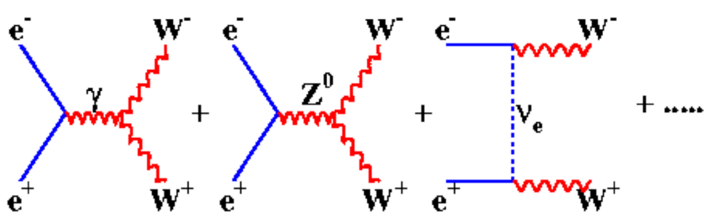
$$\mathcal{L} \approx N_1 N_2 \frac{f}{A}$$

N_1, N_2 are numbers of particles / bunch in each colliding beam
f is the bunch-crossing frequency
A is something like the cross-sectional size of the beams.

is a measure of the intensity of the colliding beams. It has units $cm^{-2}s^{-1}$.

The cross-section σ for a given process (say $e^+e^- \rightarrow W^+W^-$) is proportional to the square of the quantum mechanical amplitude, $|\mathcal{M}|^2$ and has units of $[\text{length}]^2$ (e.g. cm^2).

The amplitude $\mathcal{M}$ is the sum of all contributing processes:



[Note that this cross-section calculation will contain a lot of interference terms.]

The instantaneous rate for a given process is given by $N = \mathcal{L}\sigma$ (units of s^{-1}).

N.B. σ typically depends on the energy……and of course need to have enough energy to produce the final state of interest….e.g. $\sqrt{s} > 2M_W$ For the above process.

Sensitivity to New Particle Production

Experimental sensitivity to *rare processes* (*e.g.* low σ) determined by the luminosity $\mathcal{L}$ and how long the experiment is run:

$$N_{TOT} = \sigma \underbrace{\int \mathcal{L}\, dt}_{\text{integrated luminosity}}$$

$\mathcal{L}_{LHC}=(10^{33} - 10^{34})\ cm^{-2}s^{-1}$

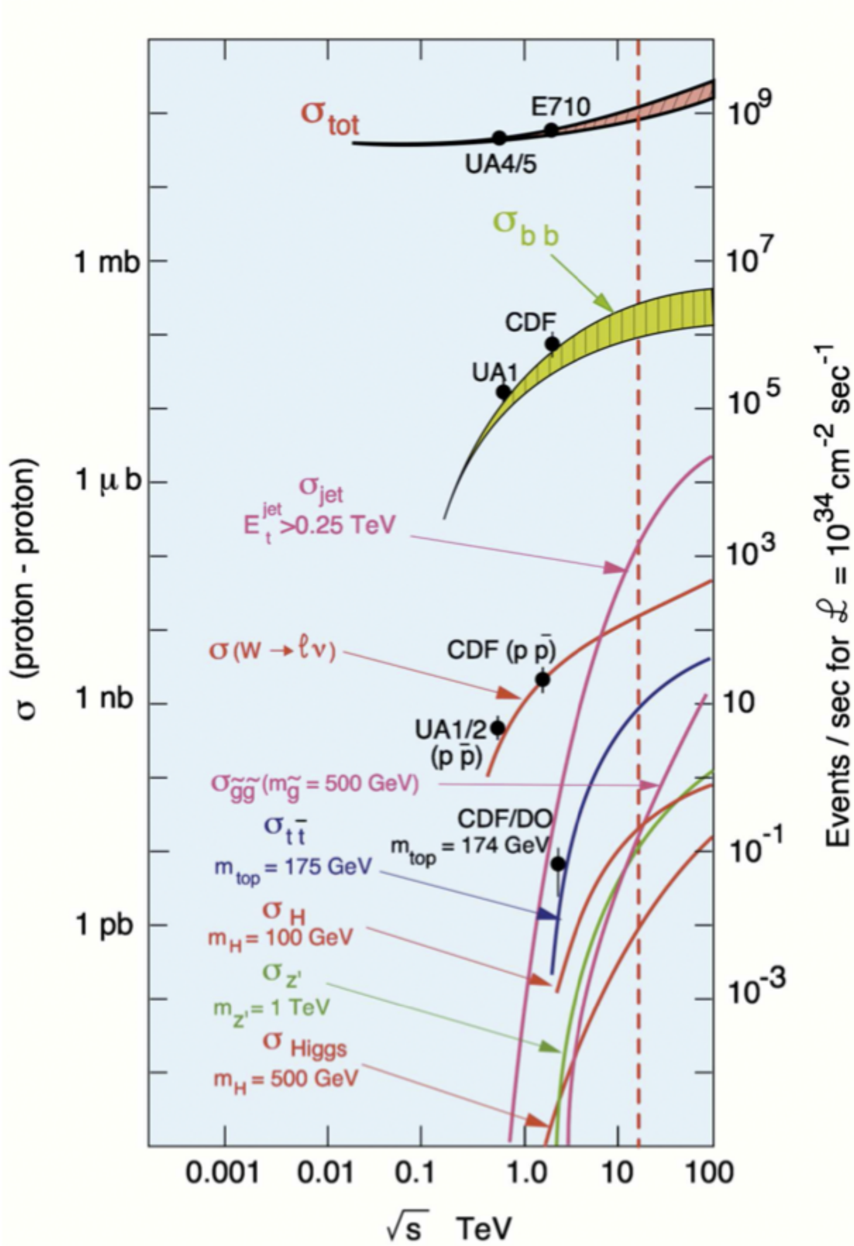
Energy or mass reach for *new particle production* is determined by the beam energies (as we have seen when discussing thresholds). For example, in the CM frame the process

$$e^{+}e^{-} \rightarrow X\bar{X}$$

proceeds only if M_X <= E_{beam} (= $E_{CM}/2$).

Situation is a bit more complicated at a hadron collider where the effective (parton-parton) CM energy varies from event to event (as we have briefly discussed).

pp and $p\bar{p}$ cross-sections as a function of $\sqrt{s}$



Units: 1 barn (b) = 10^{-28} m^2

mb = 10^{-3} barn

μb = 10^{-6} barn

nb = 10^{-9} barn $\mathcal{L}_{LHC}=(10^{33}\text{-}10^{34})\ \text{cm}^{-2}\text{s}^{-1}$

pb = 10^{-12} barn

fb = 10^{-15} barn