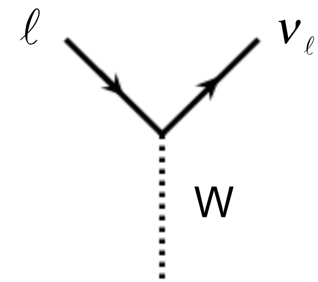
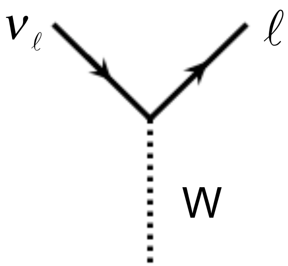
Phy489 Lecture 19

Charged Weak Interactions

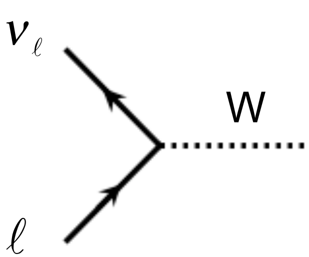
Fundamental charged weak interaction leptonic vertex:



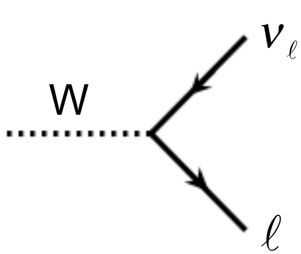
Here, a lepton emits (absorbs) a W^- (W^+) and transforms into neutrino of the same lepton species.



Here, a neutrino absorbs (emits) a W^- (W^+) and transforms into lepton of the same lepton species.



Here, a lepton and a lepton anti-neutrino annihilate to produce a W^-.



Here, a W^- decays into lepton and a lepton anti-neutrino.

As discussed earlier, these vertices represent the same fundamental interaction (vertex).

The charged weak interaction propagator is: $$-\frac{i}{q^2 - M_W^2 c^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2 c^2}\right) = \frac{i g_{\mu\nu}}{M_W^2 c^2} \quad \text{for } q^2 << M_W^2 c^2$$

This is known as the reduced propagator

What about the vertex factor? In QED: $i g_e \gamma^\mu$

Weak interaction (GWS): $$-\frac{i g_w}{\boxed{2\sqrt{2}}}\gamma^\mu\left(1-\gamma^5\right)$$

$\sqrt{4\pi\alpha_w}$

convention

Recall that γ^μ gives a vector coupling. $\gamma^\mu\gamma^5$ gives an axial vector coupling.

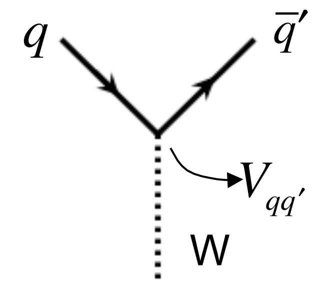
See Griffiths 7.68: $\bar{\psi}\gamma^\mu\psi$ is a vector, $\bar{\psi}\gamma^\mu\gamma^5\psi$ is an axial vector.

Parity is thus MAXIMALLY violated: i.e. the two couplings are of equal strength, rather than there being (for example) just a small contribution from the parity violating part, *e.g.* $\gamma^\mu\left(1-\varepsilon\gamma^5\right)$.

This is sometimes referred to as V-A coupling. Since only the charged weak interaction is responsible for weak decays of hadrons, this is the relevant interaction. (We will see that the coupling to the Z^0 is different).

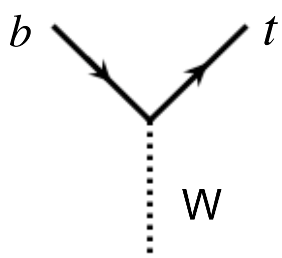
Charged Weak Interactions (Quarks)

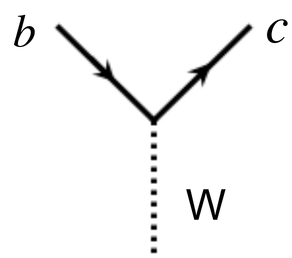
Fundamental charged weak interaction vertex for quarks:

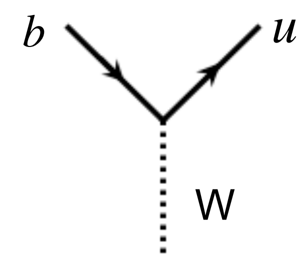


Same discussion of time orderings (as for leptonic vertices) applies.

Here, a (up or down-type) quark (flavour q) emits (absorbs) a W^- (W^+) and transforms into down or up-type quark, q' (not necessarily of the same generation – quark mixing Is allowed – probabilities determined by the CKM matrix elements):







$V_{qq'} = V_{tb}$ $\quad$ $V_{qq'} = V_{cb}$ $\quad$ $V_{qq'} = V_{ub}$

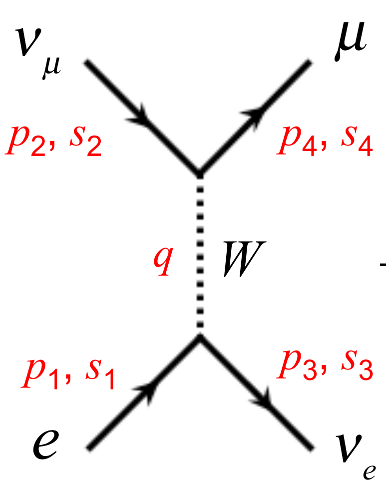
vertex factor: $-\frac{ig_w}{2\sqrt{2}} V_{tb}\gamma^\mu\left(1-\gamma^5\right)$ $\quad$ $-\frac{ig_w}{2\sqrt{2}} V_{cb}\gamma^\mu\left(1-\gamma^5\right)$ $\quad$ $-\frac{ig_w}{2\sqrt{2}} V_{ub}\gamma^\mu\left(1-\gamma^5\right)$

Inverse Muon Decay

Look at so-called “inverse muon decay process”: $e^- \nu_\mu \to \mu^- \nu_e$

[instead of $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ for which we we would need three body kinematics]

Assume that $q^2 \ll M_W^2 c^2$ so that we can use the so-called “reduced propagator”:



[don’t need to specify W+ or W- until we pick the direction of q. The result is independent of the choice]

For the time being set $g = g_w$

$$-i\mathcal{M} = \underbrace{\left[\bar{u}(3)\left(-\frac{ig}{2\sqrt{2}}\right)\gamma^\mu\left(1-\gamma^5\right)u(1)\right]}_{\text{electron line}}\left(\frac{ig_{\mu\nu}}{M_W^2 c^2}\right)\underbrace{\left[\bar{u}(4)\left(-\frac{ig}{2\sqrt{2}}\right)\gamma^\nu\left(1-\gamma^5\right)u(2)\right]}_{\text{muon line}}$$

$$\mathcal{M} = \frac{g^2}{8M_W^2c^2}\left[\bar{u}(3)\gamma^\mu\left(1-\gamma^5\right)u(1)\right]\left[\bar{u}(4)\gamma_\mu\left(1-\gamma^5\right)u(2)\right]$$

Procedure for summing over initial and final state spins is the same as before:

Recall that we had $\sum_{all\ spins} \left[\bar{u}(a)\Gamma_1 u(b)\right]\left[\bar{u}(a)\Gamma_2 u(b)\right]^* = Tr\left(\Gamma_1\left(\not{p}_b + m_b c\right)\bar{\Gamma}_2\left(\not{p}_a + m_a c\right)\right)$ with $\bar{\Gamma}_2 \equiv \gamma^0\Gamma_2^\dagger\gamma^0$

So (using m_ν=0) we get

$$\sum_{spins}|\mathcal{M}|^2 = \left(\frac{g^2}{8M_W^2c^2}\right)^2 Tr\left[\gamma^\mu\left(1-\gamma^5\right)\left(\not{p}_1 + m_e c\right)\gamma^\nu\left(1-\gamma^5\right)\left(\not{p}_3\right)\right] Tr\left[\gamma_\mu\left(1-\gamma^5\right)\left(\not{p}_2\right)\gamma_\nu\left(1-\gamma^5\right)\left(\not{p}_4 + m_\mu c\right)\right]$$

Evaluating these traces is about 1 page of algebra each, requiring traces rules 13 and 16 as well as knowledge of the commutation properties of γ^5:

$$\left\{\gamma^5,\gamma^\mu\right\} = 0 \qquad \text{i.e. } \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5 \qquad \text{for all } \mu = 0,1,2,3 \qquad \left(\gamma^5\right)^2 = 1 \ \left(\text{e.g. } I_{4x4}\right)$$

$$Tr\left[\gamma^\mu\left(1-\gamma^5\right)\left(\not{p}_1 + m_e c\right)\gamma^\nu\left(1-\gamma^5\right)\left(\not{p}_3\right)\right] = 8\left[p_1^\mu p_3^\nu + p_1^\nu p_3^\mu - g^{\mu\nu}\left(p_1\cdot p_3\right) - i\varepsilon^{\mu\nu\lambda\sigma}p_{1\lambda}p_{3\sigma}\right]$$

$$Tr\left[\gamma_\mu\left(1-\gamma^5\right)\left(\not{p}_2\right)\gamma_\nu\left(1-\gamma^5\right)\left(\not{p}_4 + m_\mu c\right)\right] = 8\left[p_{2\mu}p_{4\nu} + p_{2\nu}p_{4\mu} - g_{\mu\nu}\left(p_2\cdot p_4\right) - i\varepsilon_{\mu\nu\kappa\tau}p_2^\kappa p_4^\sigma\right]$$

$$\varepsilon^{\mu\nu\lambda\sigma} = \begin{cases} -1 \text{ if } \mu\nu\lambda\sigma \text{ is an even permutation of } 0,1,2,3 \\ +1 \text{ if } \mu\nu\lambda\sigma \text{ is an odd permutation of } 0,1,2,3 \\ \ 0 \text{ if any of the two indices are the same} \end{cases}$$

You should attempt this calculation (it will probably appear on an assignment).

It follows (after a few pages of algebra of the type that we have already seen,and which I recommend that you attempt if you need practice with this type of calculation) that:

$$\sum_{spins} \left|\mathcal{M}\right|^2 = 4\left(\frac{g}{M_W c}\right)^4 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right)$$

Since we average over the initial spin states we have

$$\left\langle \left|\mathcal{M}\right|^2 \right\rangle = 2\left(\frac{g}{M_W c}\right)^4 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right)$$

Where, in this case, there are only two possible spin configurations in the initial state because the neutrino only has 1 spin (helicity) state [or, at least, only one helicity state that couples to the weak interaction].

Look at this process in the CM frame, neglecting the electron mass….

$$\left\langle |\mathcal{M}|^2 \right\rangle = 2\left(\frac{g}{M_W c}\right)^4 (p_1 \cdot p_2)(p_3 \cdot p_4)$$ In the CM frame we have:

$$(p_1 \cdot p_2) = \left(\frac{E}{c}, \vec{p}_e\right) \cdot \left(\frac{E}{c}, -\vec{p}_e\right) = \frac{E^2}{c^2} + |\vec{p}_e|^2 = \frac{E^2}{c^2} + \frac{E^2}{c^2} - \cancel{m_e^2 c^2} = 2\frac{E^2}{c^2}$$

$$(p_3 \cdot p_4) = \left(\frac{E_\mu}{c}, \vec{p}_\mu\right) \cdot \left(\frac{E_\nu}{c}, -\vec{p}_\mu\right) = \frac{E_\mu E_\nu}{c^2} + |\vec{p}_\mu|^2$$

$$2E = E_\nu + E_\mu \Rightarrow 4E^2 = E_\nu^2 + E_\mu^2 + 2E_\nu E_\mu \Rightarrow \boxed{E_\nu E_\mu = \frac{4E^2 - E_\nu^2 - E_\mu^2}{2}}$$

$$(p_3 \cdot p_4) = \frac{4E^2 - E_\nu^2 - E_\mu^2}{2c^2} + |\vec{p}_\mu|^2 \quad = \quad 2\frac{E^2}{c^2} - \frac{|\vec{p}_\mu|^2}{2} - \frac{|\vec{p}_\mu|^2 c^2 + m_\mu^2 c^4}{2c^2} + |\vec{p}_\mu|^2$$

$$= 2\frac{E^2}{c^2} - \frac{\cancel{|\vec{p}_\mu|^2}}{2} - \frac{\cancel{|\vec{p}_\mu|^2}}{2} - \frac{m_\mu^2 c^2}{2} + \cancel{|\vec{p}_\mu|^2} = 2\frac{E^2}{c^2} - \frac{m_\mu^2 c^2}{2} = 2\frac{E^2}{c^2}\left(1 - \frac{m_\mu^2 c^4}{4E^2}\right)$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = 2\left(\frac{g}{M_W c}\right)^4 \left(\frac{2E^2}{c^2}\right)\frac{2E^2}{c^2}\left(1 - \frac{m_\mu^2 c^4}{4E^2}\right) \quad \Rightarrow \quad \left\langle |\mathcal{M}|^2 \right\rangle = 8\left(\frac{gE}{M_W c^2}\right)^4 \left[1 - \left(\frac{m_\mu c^2}{2E}\right)^2\right]$$

So we have $\left\langle |\mathcal{M}|^2 \right\rangle = 8\left(\frac{gE}{M_W c^2}\right)^4 \left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]$.

Using Griffiths 6.47 for the kinematics, we get, for the differential cross-section in the CM frame:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{\left\langle |\mathcal{M}|^2 \right\rangle}{(2E)^2} \frac{|\vec{p}_\mu|}{|\vec{p}_e|} \qquad \frac{|\vec{p}_\mu|}{|\vec{p}_e|} = \left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right] \quad \text{in the limit as } m_e \to 0$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{1}{4E^2}\left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]\left\langle |\mathcal{M}|^2 \right\rangle = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{1}{4E^2}\left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right] 8\left(\frac{gE}{M_W c^2}\right)^4 \left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]$$

$$= \left(\frac{\hbar c}{8\pi}\right)^2 \frac{8}{4E^2}\left(\frac{gE}{M_W c^2}\right)^4 \left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]^2 = 2\left(\frac{\hbar c g^2 E}{\left(M_W c^2\right)^2}\right)^2 \left(\frac{1}{8\pi}\right)^2 \left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]^2$$

$$= \frac{1}{2}\left(\frac{\hbar c g^2 E}{4\pi\left(M_W c^2\right)^2}\right)^2 \left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]^2$$

For the total cross-section one gets an additional factor of 4π.

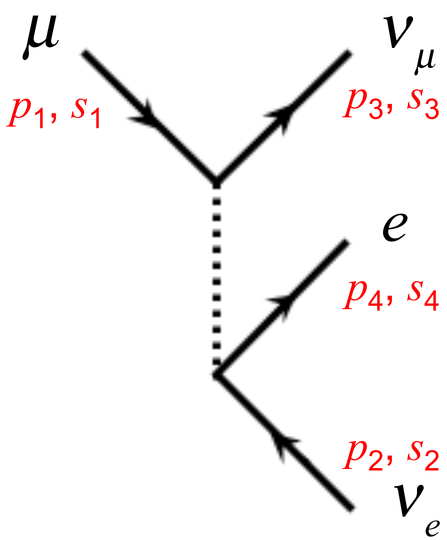
Since there is no angular dependence

[Griffiths 9.13]

$$\Rightarrow \sigma\left(e^- \nu_\mu \to \mu^- \nu_e\right) = \frac{1}{8\pi}\left(\left(\frac{g}{M_W c^2}\right)^2 \hbar c E\right)^2 \left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]^2$$

[Griffiths 9.14]

Muon decay: amplitude is similar, but need three-body kinematics:



now need outgoing anti-particle spinor

$$\mathcal{M} = \frac{g^2}{8 M_W^2 c^2}\left[\bar{u}(3)\gamma^\mu\left(1-\gamma^5\right)u(1)\right]\left[\bar{u}(4)\gamma_\mu\left(1-\gamma^5\right)v(2)\right]$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = 2\left(\frac{g}{M_W c}\right)^4 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right)$$ as before.

Calculate the decay rate in the muon rest frame: $p_\mu = \left(m_\mu c, \vec{0}\right)$

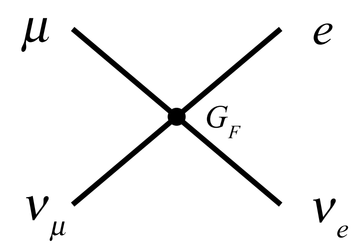
Muon decay rate calculation described in detail in §9.2 (please have a look at this). Need to go back to the Golden Rule for decays and do the three-body kinematics from scratch. Momenta of individual final-state particles and not “fixed” like in two body decays, so integrations are no longer trivial.

Results of muon decay rate calculation:

$$\Gamma = \left(\frac{m_\mu g_w}{M_W}\right)^4 \frac{m_\mu c^2}{12\hbar(8\pi)^3} \quad \Rightarrow \quad \tau_\mu = \frac{1}{\Gamma} = \left(\frac{M_W}{m_\mu g_w}\right)^4 \frac{12\hbar(8\pi)^3}{m_\mu c^2}$$

Note that g_w and M_W do not appear separately, only as a ratio. The original β-decay theory of Fermi expressed this process as a "contact" interaction with strength:

$$G_F = \frac{\sqrt{2}}{8}\left(\frac{g_w}{M_W c^2}\right)^2 (\hbar c)^3 \qquad \tau_\mu = \frac{192\pi^3\hbar^7}{G_F^2 m_\mu^5 c^4}$$



Putting in the observed muon mass and lifetime ➔ G_F = 1.166 x 10^{-5} GeV^{-2}.

$$\Rightarrow g_w = 0.653 \quad \alpha_w = \frac{g_w^2}{4\pi} \sim 1/29.5 \quad >> \quad 1/137$$

Weakness of the weak interaction is due to the mass of the exchanged particles, not to a small coupling constant.

Form Factors

Neutron decay width calculation is similar to the case of muon decay that we just discussed. Need the same three-body kinematics.

$$n \rightarrow p e^- \bar{\nu}_e$$

A big difference is that the decaying muon is a fundamental particle, while the weak decay of a neutron (or any other hadron) arises from the the decay of one of it's constituent quarks.

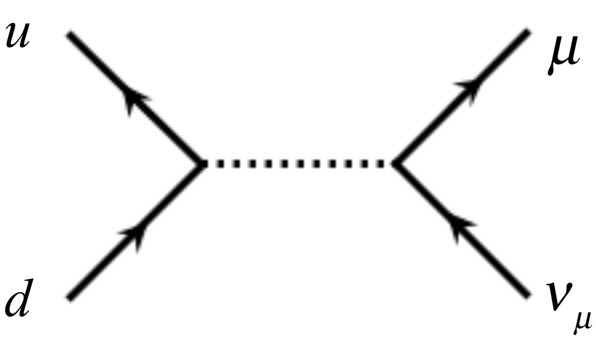
Please read §9.3 of the text, where this is discussed. That discussion introduces the concept of the "form factor" which we can use to parametrize our ignorance of the true (effective) form of the coupling.

Need this also to discuss the case of pion decay, which is a simpler system with which to illustrate this.

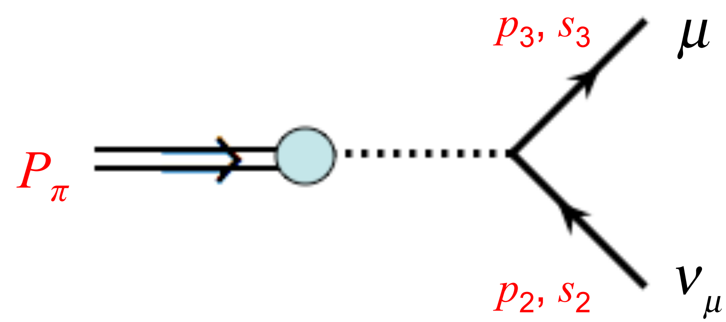
$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

Charged Pion Decay $\pi^- \to \mu^- \bar{\nu}_\mu$

At some level this looks like $\bar{u}d$ scattering, but actual decay depends upon the quark wavefunctions (and their overlap).



$\Rightarrow$



For π^- decay, the incident quarks are in a hadronic bound state (see also Ex. 7.8 for $e^+e^- \to \gamma\gamma$ in positronium). How this decay proceeds depends on the quark wavefunctions which we don't know (and could calculate only from QCD).

We parametrize this ignorance with a "form factor" F. Think about what form this can take based on the structure of the amplitude: it must be a four-vector since it must couple to the leptonic part of the amplitude.

$$\mathcal{M} = \overbrace{\bar{u}(3)\frac{ig_w}{2\sqrt{2}}\gamma^\mu\left(1-\gamma^5\right)v(2)}^{\text{leptonic part}}\frac{ig_{\mu\nu}}{\left(M_W c\right)^2}\boxed{\frac{ig_w}{2\sqrt{2}}}F^\nu$$

here $q^2 = m_\pi^2 c^2$ so we can use the reduced propagator

By convention we do not include this factor in F.

$$\mathcal{M} = \frac{g_w^2}{8(M_W c)^2}\left[\bar{u}(3)\gamma^\mu(1-\gamma^5)v(2)\right]F_\mu$$

What quantities are available from which to form a four vector?

The pion has 0 spin so the only available four-vector is the pion four-momentum p^μ. Thus $F^\mu = f_\pi p^\mu$ where f_π could in principle depend on the scalar quantity p^2, but the pion is real ("on it's mass shell") so $p^2 = m_\pi^2 c^2$, so f_π is just a constant (called the pion decay constant).

$$\left\langle |\mathcal{M}|^2 \right\rangle = \left[\frac{f_\pi}{8}\left(\frac{g_w}{M_W c}\right)^2\right]^2 \sum_{s2,s3}\left[\bar{u}(3)\gamma^\mu(1-\gamma^5)v(2)\right]\left[\bar{u}(3)\gamma^\nu(1-\gamma^5)v(2)\right]^* p_\mu p_\nu$$

$$= \left[\frac{f_\pi}{8}\left(\frac{g_w}{M_W c}\right)^2\right]^2 Tr\left(\gamma^\mu(1-\gamma^5)\not{p}_2\gamma^\nu(1-\gamma^5)(\not{p}_3+mc)\right)p_\mu p_\nu \qquad \boxed{\text{N.B. } m \equiv m_\mu,\ m_\nu = 0}$$

$$= \left[\frac{f_\pi}{8}\left(\frac{g_w}{M_W c}\right)^2\right]^2 p_\mu p_\nu\left\{8\left(p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - g^{\mu\nu}(p_2\cdot p_3) - i\varepsilon^{\mu\nu\kappa\tau}p_{2\kappa}p_{3\tau}\right)\right\}$$

$\varepsilon^{\mu\nu\kappa\tau}p_{2\kappa}p_{3\tau}$: each term appears twice, but with opposite sign, so this term yields 0.

[convince yourself]

$$= \left[\frac{f_\pi}{8}\left(\frac{g_w}{M_W c}\right)^2\right]^2 8\left[(p\cdot p_2)(p\cdot p_3) + (p\cdot p_2)(p\cdot p_3) - p^2(p_2\cdot p_3)\right]$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{8}\left[f_\pi \left(\frac{g_w}{M_W c} \right)^2 \right]^2 \left[2(p \cdot p_2)(p \cdot p_3) - p^2 (p_2 \cdot p_3) \right]$$

Need to evaluate: $p \cdot p_2$, $p \cdot p_3$, p^2 and $p_2 \cdot p_3$ (use $p = p_2 + p_3$)

$$(p \cdot p_2) = (p_2 + p_3) \cdot p_2 = p_2^2 + p_2 \cdot p_3 = p_2 \cdot p_3 \qquad (\text{since } p_2^2 = m_\nu^2 c^2)$$

$$(p \cdot p_3) = (p_2 + p_3) \cdot p_3 = p_3^2 + p_2 \cdot p_3 = m_\mu^2 c^2 + p_2 \cdot p_3$$

$$p^2 = (p_2 + p_3)^2 = m_\pi^2 c^2 = \cancel{p_2^2} + p_3^2 + 2 p_2 \cdot p_3 \Rightarrow 2 p_2 \cdot p_3 = \left(m_\pi^2 - m_\mu^2 \right) c^2$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{8}\left[f_\pi \left(\frac{g_w}{M_W c} \right)^2 \right]^2 \left[2(p \cdot p_2)(p \cdot p_3) - p^2 (p_2 \cdot p_3) \right]$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{8}\left[f_\pi \left(\frac{g_w}{M_W c} \right)^2 \right]^2 \left[2(p_2 \cdot p_3)\left(p_2 \cdot p_3 + m_\mu^2 c^2\right) - \left(p_3^2 + 2 p_2 \cdot p_3\right)(p_2 \cdot p_3) \right]$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{8}\left[f_\pi \left(\frac{g_w}{M_W c} \right)^2 \right]^2 \left[\left(m_\pi^2 - m_\mu^2\right) c^2 \left(m_\mu^2 c^2 + \frac{m_\pi^2 - m_\mu^2}{2} c^2 \right) - \left(m_\mu^2 c^2 + \left(m_\pi^2 - m_\mu^2\right) c^2 \right) \left(\frac{\left(m_\pi^2 - m_\mu^2\right) c^2}{2} \right) \right]$$

$$\left\langle \left|\mathcal{M}\right|^2 \right\rangle = \frac{1}{8}\left[f_\pi \left(\frac{g_w}{M_W c} \right)^2 \right]^2 \left[m_\mu^2 \left(m_\pi^2 - m_\mu^2 \right) c^4 + \frac{\left(m_\pi^2 - m_\mu^2 \right)^2 c^4}{2} - \frac{m_\mu^2 \left(m_\pi^2 - m_\mu^2 \right)^2 c^4}{2} - \frac{\left(m_\pi^2 - m_\mu^2 \right)^2 c^4}{2} \right]$$

$$= \frac{1}{8}\left[f_\pi \left(\frac{g_w}{M_W c} \right)^2 \right]^2 \left[\frac{m_\mu^2 \left(m_\pi^2 - m_\mu^2 \right)^2 c^4}{2} \right] \qquad \Rightarrow \quad \left\langle \left|\mathcal{M}\right|^2 \right\rangle = \left(\frac{g_w}{2M_W} \right)^4 f_\pi^2 m_\mu^2 \left(m_\pi^2 - m_\mu^2 \right)$$

From Griffiths 6.35 (two-body decay kinematics): $\Gamma\left(1 \to 2+3\right) = \frac{\left|\vec{p}_2\right|}{8\pi\hbar m_\pi^2 c} \left\langle \left|\mathcal{M}\right|^2 \right\rangle$

Simple relativistic kinematics (see Ex. 3.3) gives: $\left|\vec{p}_2\right| = \frac{c}{2m_\pi}\left(m_\pi^2 - m_\mu^2 \right)$

$$\Gamma\left(\pi^- \to \mu^- \bar{\nu}_\mu\right) = \frac{f_\pi^2}{\pi\hbar m_\pi^3} \left(\frac{g_w}{4M_W} \right)^4 m_\mu^2 \left(m_\pi^2 - m_\mu^2 \right)^2$$

Similarly, $$\Gamma\left(\pi^- \to e^- \bar{\nu}_e\right) = \frac{f_\pi^2}{\pi\hbar m_\pi^3} \left(\frac{g_w}{4M_W} \right)^4 m_e^2 \left(m_\pi^2 - m_e^2 \right)^2$$

Ratio of branching ratios for the two kinematically accessible final states for charged pion decay:

$$\frac{\Gamma\left(\pi^- \to e^- \bar{\nu}_e\right)}{\Gamma\left(\pi^- \to \mu^- \bar{\nu}_\mu\right)} = \frac{m_e^2\left(m_\pi^2 - m_e^2\right)^2}{m_\mu^2\left(m_\pi^2 - m_\mu^2\right)^2} \approx 1.3\times 10^{-4}$$

Note that, if electrons were massless, instead of just very light, the decay into the electron, electron-antineutrino final state would be forbidden. Why is this?

The pion has spin 0, so the electron and antineutrino must emerge with opposite spins (so the same helicity).

The anti-neutrino is always (only) right-handed, so the electron must be right-handed as well. If the electron were massless then, like the neutrino it would exist only as a left-handed particle. More precisely, the factor of $\left(1-\gamma^5\right)$ in the weak interaction vertex factor would couple only to LH electrons (see Prob 9.15).

Charged Weak Interactions of Quarks

In the leptonic case, the W coupling stays within generations.

In the quark sector, we have already seen that inter-generational couplings exist, with strengths determined by the elements of the CKM matrix (which are determined empirically).

Consider the leptonic decay of hadrons (we just looked at pions). Compare the two decays $\pi^- \to \ell^- \bar{\nu}_\ell$ and $K^- \to \ell^- \bar{\nu}_\ell$.

$$\Gamma\left(K^- \to \ell^- \bar{\nu}_\ell\right) = \frac{f_K^2}{\pi \hbar m_K^3} \left(\frac{g_w}{4 M_W}\right)^4 m_\ell^2 \left(m_K^2 - m_\ell^2\right)^2$$

The weak vertex factor in $\pi^- \to \ell^- \bar{\nu}_\ell$ contains a factor of V_{ud} so $\Gamma \propto \left|V_{ud}\right|^2$: this is absorbed into the pion decay constant f_π . Can write this as $\cos\theta_c$.

$\Gamma\left(K^- \to \ell^- \bar{\nu}_\ell\right) \propto \left|V_{us}\right|^2$ which we can write as $\sin^2\theta_c$.

$$\frac{\Gamma\left(K^- \to \ell^- \bar{\nu}_\ell\right)}{\Gamma\left(\pi^- \to \ell^- \bar{\nu}_\ell\right)} = \left(\frac{m_\pi}{m_K}\right)^3 \left(\frac{m_K^2 - m_\ell^2}{m_\pi^2 - m_\ell^2}\right)^2 \left|\frac{V_{us}}{V_{ud}}\right|^2 \qquad \left[= \left(\frac{m_\pi}{m_K}\right)^3 \left(\frac{m_K^2 - m_\ell^2}{m_\pi^2 - m_\ell^2}\right)^2 \tan^2\theta_c \right]$$