Phy489 Lecture 20

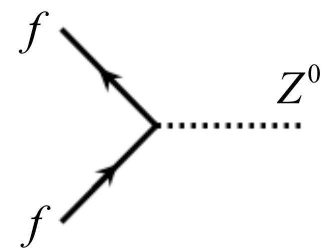
Neutral Weak Interactions

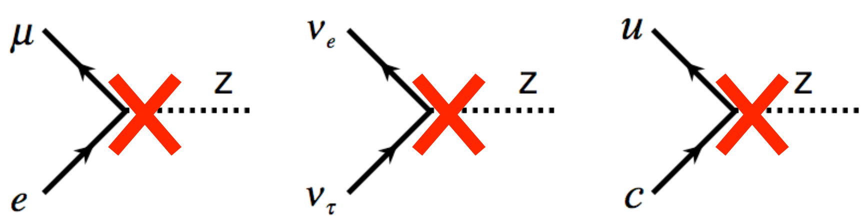
Unification of the electromagnetic and weak interactions (Glashow, Weinberg, Salam) is discussed in §9.7. We will cover some of this, but only briefly……

Discussion (to follow) of the neutral weak interaction is based on the predictions of the GWS theory in a more detailed way than for the charged weak interaction [for which we previously had the Fermi theory that worked well at low energies].

Fundamental neutral weak interaction vertex:

[Same fermion species in/out: No FCNC !!]



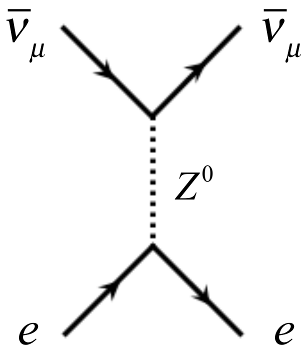


The "GIM-mechanism" suppresses FCNC due to second-order processes [see §10.5 and notes from Lecture 9].

Unification of Electromagnetic and Weak Interactions

GWS model for electroweak unification requires the existence of a "neutral current" weak interaction (which had not been previously observed). [we will see this in a future lecture]

First observed in ν-scattering experiments at CERN: $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$



This scattering process has no charged current (CC) final state contributions at lowest order, and of course has no EM contribution.

Experimentally, one sees the recoiling electron produced by a $\bar{\nu}_\mu$ beam directed onto some target.

Need neutrino scattering for investigation of weak neutral currents (NC) since other neutral processes are swamped by EM contributions.

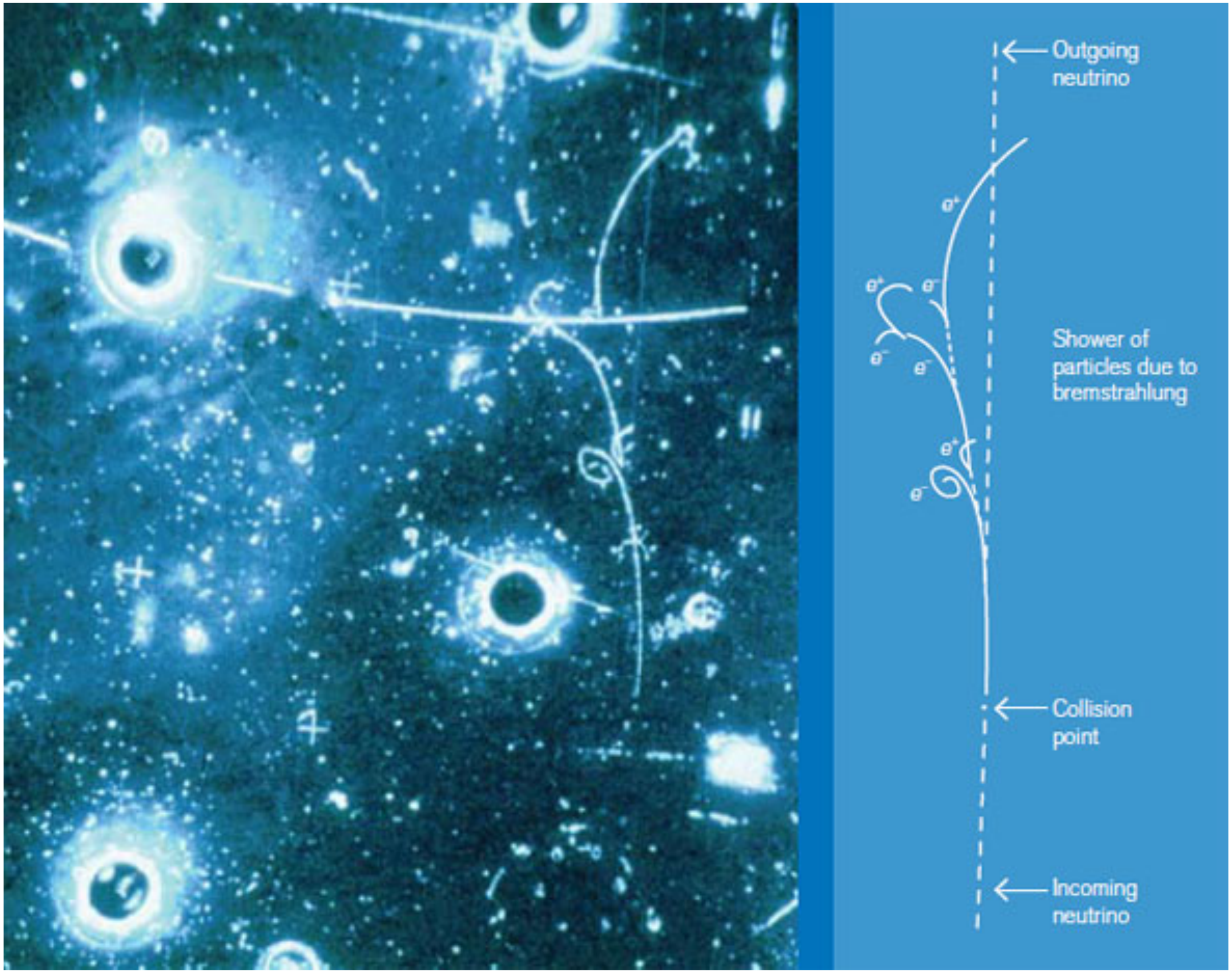
Also observe $\nu_\mu,\ \bar{\nu}_\mu$ interactions with quarks with cross-sections about 1/3 the size of those for the corresponding charged current processes:

NC
$$\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + X$$
$$\nu_\mu + N \rightarrow \nu_\mu + X$$

CC
$$\bar{\nu}_\mu + N \rightarrow \mu^+ + X$$
$$\nu_\mu + N \rightarrow \mu^- + X$$

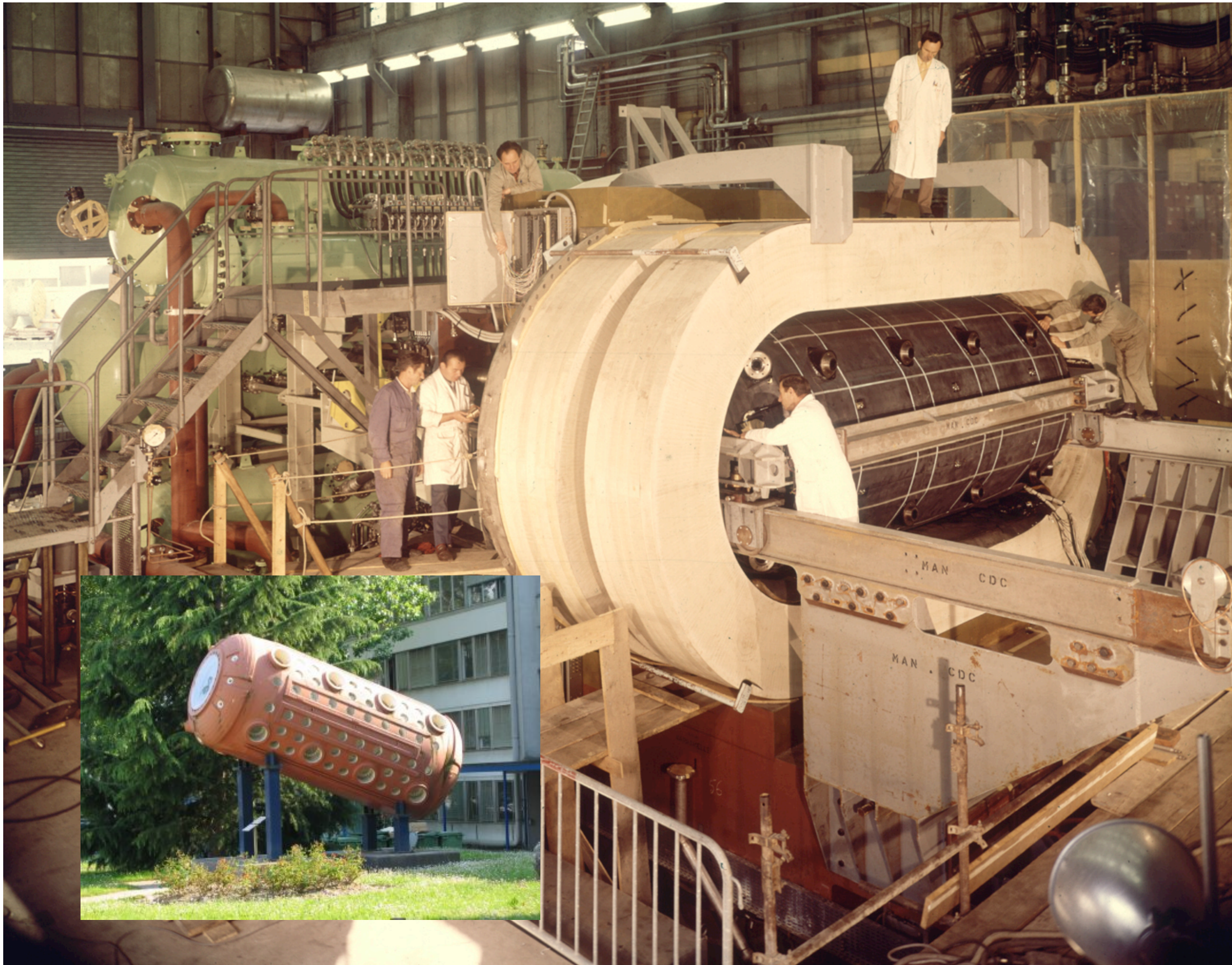
The size of the cross-section indicates a new interaction, not some higher order effect (which would be suppressed by powers of the effective weak coupling).

Neutral Current Interactions



The First Experimentally Observed NC Event (CERN, Gargamelle, 1972)

Gargamelle Experiment (CERN)



Feynman Rules for the Neutral Weak Interaction

The only difference relative to the charged weak interaction is in the form of the vertex factors:

Charged weak interaction vertex factor: $-\frac{ig_w}{2\sqrt{2}}\gamma^\mu\left(1-\gamma^5\right)$ $\left[-\frac{ig_w}{2\sqrt{2}}\gamma^\mu\left(1-\gamma^5\right)V_{qq'}\right]$

Neutral weak interaction vertex factor: $-\frac{ig_z}{2}\gamma^\mu\left(c_V^f-c_A^f\gamma^5\right)$

f	c_V	c_A
ν_e,ν_μ,ν_τ	$\frac{1}{2}$	$\frac{1}{2}$
e^-,μ^-,τ^-	$-\frac{1}{2}+2\sin^2\theta_w$	$-\frac{1}{2}$
u,c,t	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_w$	$\frac{1}{2}$
d,s,b	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_w$	$\frac{1}{2}$

Note that the couplings are different for charged and neutral leptons, and up type and down type quarks.

The neutrino couplings are derived in §9.7.3. We will derive the charged lepton couplings in an upcoming lecture (see problem 9.28).

The couplings are a function of the weak mixing angle θ_w which we will discuss later on but which satisfies the relations below:

$$\underbrace{g_w=\frac{g_e}{\sin\theta_w} \qquad g_z=\frac{g_e}{\sin\theta_w\cos\theta_w} \qquad M_W=M_Z\cos\theta_w}$$

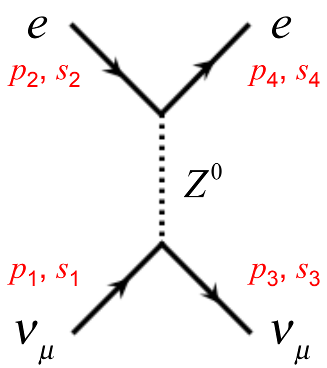
$$\left[\theta_w=28.7^o \qquad \sin^2\theta_w\approx 0.23\right]$$

measured values

Note that these are ***predictions*** of the GSW theory (which were quickly verified experimentally)

The Z^0 propagator is $-\frac{i}{q^2 - M_Z^2 c^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2 c^2}\right) \quad = \quad \frac{i g_{\mu\nu}}{M_Z^2 c^2}$ for $q^2 << M_Z^2 c^2$

Look at $\nu_\mu e^- \to \nu_\mu e^-$ (we have already looked at the charged current process $\nu_\mu e^- \to \mu^- \nu_e$)



$$-i\mathcal{M} = \left[\bar{u}(3)\left(-\frac{ig_z}{2}\right)\overbrace{\gamma^\mu\left(\frac{1}{2} - \frac{1}{2}\gamma^5\right)}^{\text{neutrino coupling}}u(1)\right]\left(\frac{ig_{\mu\nu}}{M_Z^2 c^2}\right)\left[\bar{u}(4)\left(-\frac{ig_z}{2}\right)\underbrace{\gamma^\nu\left(c_V^e - c_A^e\gamma^5\right)}_{\text{electron coupling}}u(2)\right]$$

$$\mathcal{M} = \frac{g_z^2}{8(M_z c)^2}\left[\bar{u}(3)\gamma^\mu\left(1-\gamma^5\right)u(1)\right]\left[\bar{u}(4)\gamma_\mu\left(c_V^e - c_A^e\gamma^5\right)u(2)\right]$$

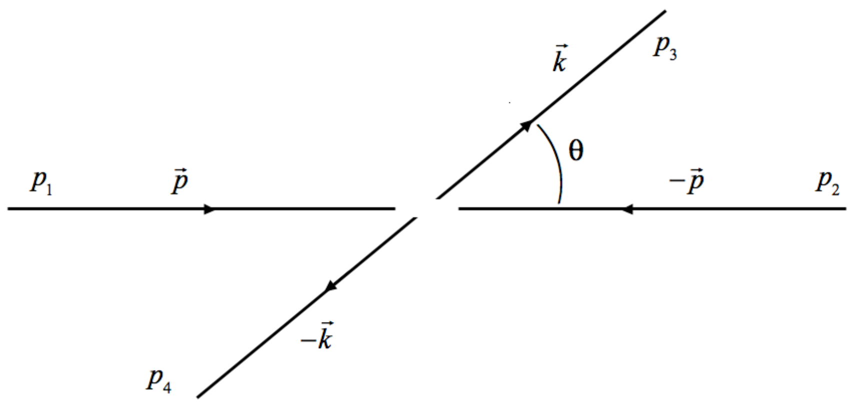
includes a factor of 1/2 from the average over initial spin states

$$\left\langle|\mathcal{M}|^2\right\rangle = 2\left(\frac{g_z}{4M_z c}\right)^4 Tr\left[\gamma^\mu\left(1-\gamma^5\right)\not{p}_1\gamma^\nu\left(1-\gamma^5\right)\not{p}_3\right]Tr\left[\gamma_\mu\left(c_V^e - c_A^e\gamma^5\right)\left(\not{p}_2 + mc\right)\gamma_\nu\left(c_V^e - c_A^e\gamma^5\right)\left(\not{p}_4 + mc\right)\right]$$

$$= \frac{1}{2}\left(\frac{g_z}{M_z c}\right)^4\left[\left(c_V^e + c_A^e\right)^2\left(p_1\cdot p_2\right)\left(p_3\cdot p_4\right) + \left(c_V^e - c_A^e\right)^2\left(p_1\cdot p_4\right)\left(p_2\cdot p_3\right) - (mc)^2\left(\left(c_V^e\right)^2 - \left(c_A^e\right)^2\right)\left(p_1\cdot p_3\right)\right]$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{2}\left(\frac{g_z}{M_z c}\right)^4 \left[\left(c_V^e + c_A^e\right)^2 (p_1 \cdot p_2)(p_3 \cdot p_4) + \left(c_V^e - c_A^e\right)^2 (p_1 \cdot p_4)(p_2 \cdot p_3) \cancel{- (mc)^2 \left(\left(c_V^e\right)^2 - \left(c_A^e\right)^2 \right)(p_1 \cdot p_3)} \right]$$

Evaluate this in the CM frame, neglecting the electron mass.



$$p_1 = \left(\frac{E}{c}, \vec{p}\right) \qquad p_2 = \left(\frac{E}{c}, -\vec{p}\right)$$

$$p_3 = \left(\frac{E}{c}, \vec{k}\right) \qquad p_4 = \left(\frac{E}{c}, -\vec{k}\right)$$

$$|p|^2 = |k|^2 \equiv p^2$$

Need $(p_1 \cdot p_2) = (p_3 \cdot p_4) = 2\frac{E^2}{c^2}$ and $(p_1 \cdot p_4) = (p_2 \cdot p_3) = \frac{E^2}{c^2} + p^2 \cos\theta = \frac{E^2}{c^2}(1+\cos\theta)$

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{2}\left(\frac{g_z}{M_z c}\right)^4 \left[\left(c_V^e + c_A^e\right)^2 \left(\frac{2E^2}{c^2}\right)^2 + \left(c_V^e - c_A^e\right)^2 \left(\frac{E^2}{c^2}(1+\cos\theta)\right)^2 \right]$$

$$= \frac{1}{2}\left(\frac{g_z}{M_z c}\right)^4 \left[\left(c_V^e + c_A^e\right)^2 \left(\frac{4E^4}{c^4}\right) + \left(c_V^e - c_A^e\right)^2 \left(\frac{E^4}{c^4}\right)(1+\cos\theta)^2 \right]$$

$$= \frac{4}{2}\left(\frac{g_z}{4M_z c}\right)^4 \left(\frac{E^4}{c^4}\right)\left[\left(c_V^e + c_A^e\right)^2 + \left(c_V^e - c_A^e\right)^2 \cos^4\left(\theta/2\right)\right]$$

[using $1+\cos\theta = 2\cos\left(\theta/2\right)$]

$$= 2\left(\frac{g_z E}{M_z c^2}\right)^4 \left[\left(c_V^e + c_A^e\right)^2 + \left(c_V^e - c_A^e\right)^2 \cos^4\left(\theta/2\right)\right]$$

[Griffiths 9.98]

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{\left\langle \left|\mathcal{M}\right|^2 \right\rangle}{\left(2E\right)^2} \boxed{\frac{\left|\vec{p}_f\right|}{\left|\vec{p}_i\right|}}$$

[Griffiths 6.47 for 2 body scattering in CM frame]

this ratio is 1 for this elastic scattering process

$$\frac{d\sigma}{d\Omega} = 2\left(\frac{\hbar c}{\pi}\right)^2 \left(\frac{g_z}{4M_z c^2}\right)^4 E^2 \left[\left(c_V^e + c_A^e\right)^2 + \left(c_V^e - c_A^e\right)^2 \cos^4\left(\theta/2\right)\right]$$

[Griffiths 9.99]

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = 2\left(\frac{\hbar c}{\pi}\right)^2 \left(\frac{g_z}{4M_z c^2}\right)^4 E^2 2\pi \int_0^\pi \left[\left(c_V^e + c_A^e\right)^2 + \left(c_V^e - c_A^e\right)^2 \cos^4\left(\theta/2\right)\right] \sin\theta d\theta$$

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = 2\left(\frac{\hbar c}{\pi}\right)^2 \left(\frac{g_z}{4M_z c^2}\right)^4 E^2 2\pi \int_0^\pi \left[\left(c_V^e + c_A^e\right)^2 + \left(c_V^e - c_A^e\right)^2 \cos^4\left(\theta/2\right)\right] \sin\theta d\theta$$

$$\int_0^\pi \left(c_V^e + c_A^e\right)^2 \sin\theta d\theta = \left(c_V^e + c_A^e\right)^2 \int_0^\pi \sin\theta d\theta = 2\left(c_V^e + c_A^e\right)^2$$

$$\int_0^\pi \left(c_V^e - c_A^e\right)^2 \cos^4\left(\theta/2\right)\sin\theta d\theta = \left(c_V^e - c_A^e\right)^2 \int_0^\pi \cos^4\left(\theta/2\right)\sin\theta d\theta = \left(c_V^e - c_A^e\right)^2 \int_0^\pi \left(\frac{1+\cos^2\theta}{2}\right)^2 \sin\theta d\theta$$

$$= \left(c_V^e - c_A^e\right)^2 \frac{1}{4}\int_0^\pi \left(1 + 2\cos\theta + \cos^2\theta\right)\sin\theta d\theta = \left(c_V^e - c_A^e\right)^2 \frac{1}{4}\int_0^\pi \left(1+\cos^2\theta\right)\sin\theta d\theta = \frac{2}{3}\left(c_V^e - c_A^e\right)^2$$

$$\sigma = 2\left(\frac{\hbar c}{\pi}\right)^2 \left(\frac{g_z}{4M_z c^2}\right)^4 E^2 2\pi \left[2\left(c_V^e + c_A^e\right)^2 + \frac{2}{3}\left(c_V^e - c_A^e\right)^2\right]$$

$$\sigma = 2\left(\frac{\hbar c}{\pi}\right)^2 \left(\frac{g_z}{4M_z c^2}\right)^4 E^2 \frac{16}{3}\pi \left[\left(c_V^e\right)^2 + \left(c_A^e\right)^2 + c_V^e c_A^e\right]$$

$$\sigma = \frac{2}{3\pi}\left(\hbar c\right)^2 \left(\frac{g_z}{2M_z c^2}\right)^4 E^2 \left[\left(c_V^e\right)^2 + \left(c_A^e\right)^2 + c_V^e c_A^e\right]$$

Compare this to the cross-section for the charged-current process $\sigma\left(\nu_\mu e^- \to \mu^- \nu_e\right)$ (which we did earlier)

Note that $g_z = \frac{g_w}{\cos\theta_w}$ $M_z = \frac{M_w}{\cos\theta_w}$ $\Rightarrow$ $\frac{g_z}{M_z} = \frac{g_w}{M_w}$

$$\sigma\left(\nu_\mu e^- \to \nu_\mu e^-\right) = \frac{2}{3\pi}\left(\hbar c\right)^2\left(\frac{g_z}{2M_z c^2}\right)^4 E^2\left[\left(c_V^e\right)^2 + \left(c_A^e\right)^2 + c_V^e c_A^e\right]$$

$$= \frac{2}{3\pi}\left(\hbar c\right)^2\left(\frac{g_z}{2M_z c^2}\right)^4 E^2\left[\left(-\frac{1}{2} + 2\sin^2\theta_w\right)^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2} + 2\sin^2\theta_w\right)\left(-\frac{1}{2}\right)\right]$$

$$= \frac{2}{3\pi}\left(\hbar c\right)^2\left(\frac{g_z}{2M_z c^2}\right)^4 E^2\left(\frac{1}{2}\right)^2\left[\left(-1 + 4\sin^2\theta_w\right)^2 + \left(-1\right)^2 + \left(-1 + 4\sin^2\theta_w\right)\left(-1\right)\right]$$

$$= \frac{2}{3\pi}\left(\hbar c\right)^2\left(\frac{g_z}{2M_z c^2}\right)^4 E^2\left(\frac{1}{2}\right)^2\left[1 - 8\sin^2\theta_w + 16\sin^4\theta_w + 1 + 1 - 4\sin^2\theta_w\right]$$

$$= \frac{2}{3\pi}\left(\hbar c\right)^2\left(\frac{g_z}{2M_z c^2}\right)^4 E^2\left(\frac{1}{2}\right)^2\left[3 - 12\sin^2\theta_w + 16\sin^4\theta_w\right]$$

$$= \frac{1}{8\pi}\left(\hbar c\right)^2\left(\frac{g_z}{M_z c^2}\right)^4 E^2\left[\frac{1}{4} - \sin^2\theta_w + \frac{4}{3}\sin^4\theta_w\right]$$

$$\sigma\left(e^-\nu_\mu \to \mu^-\nu_e\right) = \frac{1}{8\pi}\left(\left(\frac{g_w}{M_W c^2}\right)^2 \hbar c E\right)^2 = \frac{1}{8\pi}\left(\hbar c\right)^2\left(\frac{g_w}{M_W c^2}\right)^4 E^2$$

[neglecting the muon mass]

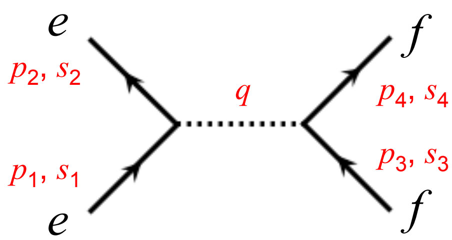
N.B. $\frac{g_w}{M_W} = \frac{g_e}{\sin\theta_W \cos\theta_W M_Z} = \frac{g_z}{M_z}$

$$\frac{\sigma\left(\nu_\mu e^- \to \nu_\mu e^-\right)}{\sigma\left(e^-\nu_\mu \to \mu^-\nu_e\right)} = \left[\frac{1}{4} - \sin^2\theta_w + \frac{4}{3}\sin^4\theta_w\right] \approx 0.09$$

Experimentally this ratio is about 0.11 with roughly 10% uncertainty.

Fermion Pair Production at the Z^0 Pole

Next look at fermion pair production in electron-positron collisions near the Z^0 pole:



Here we need the full Z^0 propagator: $-\dfrac{i}{q^2 - M_Z^2 c^2}\left(g_{\mu\nu} - \dfrac{q_\mu q_\nu}{M_Z^2 c^2}\right)$

$$\left[q = p_1 + p_2 = p_3 + p_4\right]$$

$$\mathcal{M} = -\frac{g_z^2}{4\left(q^2 - M_Z^2 c^2\right)}\left[\bar{u}(4)\gamma^\mu\left(c_V^f - c_A^f\gamma^5\right)v(3)\right]\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2 c^2}\right)\left[\bar{v}(2)\gamma^\nu\left(c_V^e - c_A^e\gamma^5\right)u(1)\right]$$

where f represents any fundamental fermion (except an electron, for the usual reason).

The second term in the propagator does not contribute (in the limit where we ignore m_f):

$$\bar{u}(4)\gamma^\mu q_\mu\left(c_V^f - c_A^f\right)v(3) = \bar{u}(4)\not{q}\left(c_V^f - c_A^f\right)v(3)$$

$\not{q} = \not{p}_3 + \not{p}_4 \quad \bar{u}(4)\not{p}_4 = 0$ This is the Dirac equation for an adjoint spinor for a massless particle (we are ignoring fermion masses): $e.g. \;\; \bar{u}\left(\gamma^\mu p_\mu - mc\right) = 0$

$\not{p}_3\left(c_V^f - c_A^f\gamma^5\right)v(3) = \left(c_V^f + c_A^f\gamma^5\right)\not{p}_3 v(3) = 0$ for the same reason.

$$\Rightarrow \quad \mathcal{M} = -\frac{g_z^2}{4\left(q^2 - M_Z^2 c^2\right)}\left[\bar{u}(4)\gamma^\mu\left(c_V^f - c_A^f\gamma^5\right)v(3)\right]\left[\bar{v}(2)\gamma_\mu\left(c_V^e - c_A^e\gamma^5\right)u(1)\right]$$

$$\mathcal{M} = -\frac{g_z^2}{4\left(q^2 - M_Z^2c^2\right)}\left[\bar{u}(4)\gamma^\mu\left(c_V^f - c_A^f\gamma^5\right)v(3)\right]\left[\bar{v}(2)\gamma_\mu\left(c_V^e - c_A^e\gamma^5\right)u(1)\right]$$

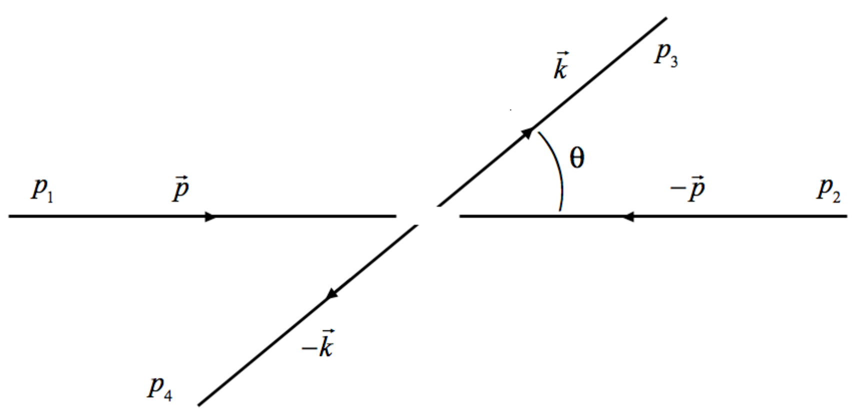
$$\left\langle|\mathcal{M}|^2\right\rangle = \left\{\frac{g_z^2}{8\left(q^2 - M_Z^2c^2\right)}\right\}^2 Tr\left[\gamma^\mu\left(c_V^f - c_A^f\gamma^5\right)\not{p}_3\gamma^\nu\left(c_V^f - c_A^f\gamma^5\right)\not{p}_4\right]Tr\left[\gamma_\mu\left(c_V^f - c_A^f\gamma^5\right)\not{p}_1\gamma_\nu\left(c_V^f - c_A^f\gamma^5\right)\not{p}_2\right]$$

(probably)

and we'll skip the trace calculation…………(it's outlined in the text and ⇧ on your assignment)

$$\left\langle|\mathcal{M}|^2\right\rangle = \frac{1}{2}\left\{\frac{g_z^2}{q^2 - M_Z^2c^2}\right\}^2\left\{\left[\left(c_V^f\right)^2 + \left(c_A^f\right)^2\right]\left[\left(c_V^e\right)^2 + \left(c_A^e\right)^2\right]\left[\left(p_1\cdot p_3\right)\left(p_2\cdot p_4\right) + \left(p_1\cdot p_4\right)\left(p_2\cdot p_3\right)\right]\right.$$
$$\left. + 4c_V^f c_A^f c_V^e c_A^e\left[\left(p_1\cdot p_3\right)\left(p_2\cdot p_4\right) - \left(p_1\cdot p_4\right)\left(p_2\cdot p_3\right)\right]\right\}$$

Look at this in the CM frame:



$$p_1 = \left(\frac{E}{c}, \vec{p}\right) \qquad p_2 = \left(\frac{E}{c}, -\vec{p}\right)$$

$$p_3 = \left(\frac{E}{c}, \vec{k}\right) \qquad p_4 = \left(\frac{E}{c}, -\vec{k}\right)$$

$$|p|^2 = |k|^2 \equiv p^2 \quad \text{for} \quad m_e, m_f \to 0$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{2}\left\{\frac{g_z^2}{q^2 - M_Z^2 c^2}\right\}^2 \left\{\left[\left(c_V^f\right)^2 + \left(c_A^f\right)^2\right]\left[\left(c_V^e\right)^2 + \left(c_A^e\right)^2\right]\left[\left(p_1 \cdot p_3\right)\left(p_2 \cdot p_4\right) + \left(p_1 \cdot p_4\right)\left(p_2 \cdot p_3\right)\right]\right.$$

$$\left. + 4 c_V^f c_A^f c_V^e c_A^e \left[\left(p_1 \cdot p_3\right)\left(p_2 \cdot p_4\right) - \left(p_1 \cdot p_4\right)\left(p_2 \cdot p_3\right)\right]\right\}$$

$$q^2 = \left(p_1 + p_2\right)^2 = \left(\frac{2E}{c}\right)^2$$

$$\left(p_1 \cdot p_4\right) = \left(p_2 \cdot p_3\right) = \frac{E^2}{c^2} + p^2 \cos\theta = \frac{E^2}{c^2}\left(1 + \cos\theta\right) \qquad \left(p_1 \cdot p_3\right) = \left(p_2 \cdot p_4\right) = \frac{E^2}{c^2} - p^2 \cos\theta = \frac{E^2}{c^2}\left(1 - \cos\theta\right)$$

$$\left(p_1 \cdot p_3\right)\left(p_2 \cdot p_4\right) + \left(p_1 \cdot p_4\right)\left(p_2 \cdot p_3\right) = \frac{E^2}{c^2}\left(1 - \cos\theta\right) \cdot \frac{E^2}{c^2}\left(1 - \cos\theta\right) + \frac{E^2}{c^2}\left(1 + \cos\theta\right) \cdot \frac{E^2}{c^2}\left(1 + \cos\theta\right)$$

$$= \frac{E^4}{c^4}\left(1 - 2\cos\theta + \cos^2\theta\right) + \frac{E^4}{c^4}\left(1 + 2\cos\theta + \cos^2\theta\right) = 2\frac{E^4}{c^4}\left(1 + \cos^2\theta\right)$$

$$\left(p_1 \cdot p_3\right)\left(p_2 \cdot p_4\right) - \left(p_1 \cdot p_4\right)\left(p_2 \cdot p_3\right) = \frac{E^2}{c^2}\left(1 - \cos\theta\right) \cdot \frac{E^2}{c^2}\left(1 - \cos\theta\right) - \frac{E^2}{c^2}\left(1 + \cos\theta\right) \cdot \frac{E^2}{c^2}\left(1 + \cos\theta\right)$$

$$= \frac{E^4}{c^4}\left(1 - 2\cos\theta + \cos^2\theta\right) - \frac{E^4}{c^4}\left(1 + 2\cos\theta + \cos^2\theta\right) = -4\frac{E^4}{c^4}\cos\theta$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = \left\{\frac{g_z^2 E^2}{\left(2E\right)^2 - \left(M_Z c^2\right)^2}\right\}^2 \left\{\left[\left(c_V^f\right)^2 + \left(c_A^f\right)^2\right]\left[\left(c_V^e\right)^2 + \left(c_A^e\right)^2\right]\left[1 + \cos^2\theta\right] - 8 c_V^f c_A^f c_V^e c_A^e \cos\theta\right\}$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = \left\{ \frac{g_z^2 E^2}{(2E)^2 - (M_Z c^2)^2} \right\}^2 \left\{ \left[(c_V^f)^2 + (c_A^f)^2 \right] \left[(c_V^e)^2 + (c_A^e)^2 \right] \left[1 + \cos^2\theta \right] - 8 c_V^f c_A^f c_V^e c_A^e \cos\theta \right\}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi} \right)^2 \frac{1}{4E^2} \left\langle |\mathcal{M}|^2 \right\rangle = \left\{ \frac{\hbar c g_z^2 E}{16\pi \left[(2E)^2 - (M_Z c^2)^2 \right]} \right\}^2 \left\{ \left[(c_V^f)^2 + (c_A^f)^2 \right] \left[(c_V^e)^2 + (c_A^e)^2 \right] \left[1 + \cos^2\theta \right] \underbrace{- 8 c_V^f c_A^f c_V^e c_A^e \cos\theta}_{\text{For } \sigma \text{ this term integrates to 0}} \right\}$$

$$\sigma = \frac{1}{3\pi} \left\{ \frac{\hbar c g_z^2 E}{4 \left[(2E)^2 - (M_Z c^2)^2 \right]} \right\}^2 \left[(c_V^f)^2 + (c_A^f)^2 \right] \left[(c_V^e)^2 + (c_A^e)^2 \right]$$

Note that this expression "blows up" at the Z^0 pole ($2E = M_Z c^2$). However, since the Z^0 is not a stable particle, its mass cannot be precisely defined. So this turns out not to be a problem.

Need to modify the propagator to account for the finite width:

$$\frac{i}{q^2 - M_Z^2 c^2} \rightarrow \frac{i}{q^2 - \left(M_Z c\right)^2 + i\hbar M_Z \Gamma_Z} \qquad\qquad \sigma \Rightarrow \frac{\left(\hbar c g_z^2 E\right)^2}{48\pi} \frac{\left[\left(c_V^f\right)^2 + \left(c_A^f\right)^2\right]\left[\left(c_V^e\right)^2 + \left(c_A^e\right)^2\right]}{\left[\left(2E\right)^2 - \left(M_Z c^2\right)^2\right]^2 + \left[\hbar M_Z c^2 \Gamma_Z\right]^2}$$

Since $\hbar\Gamma_Z << M_Z c^2$ this term is negligible except near the Z^0 pole (where it softens the spike)

In a recent lectures (Griffiths Ch. 8) we calculated the cross-section for fermion pair-production in electron-positron collisions in QED (*i.e.* interaction mediated by a virtual photon):

$$\sigma = \frac{\left(\hbar c g_e^2\right)^2}{48\pi} \frac{Q_f^2}{E^2}$$

Take the ratio of the neutral weak interaction and QED cross-sections (using $g_z = \dfrac{g_e}{\sin\theta_w \cos\theta_w}$)

$$\frac{\sigma\left(e^+e^- \rightarrow Z^0 \rightarrow \mu^+\mu^-\right)}{\sigma\left(e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-\right)} = \frac{\left[\frac{1}{2} - 2\sin^2\theta_w + 4\sin^4\theta_w\right]^2}{\left[\sin\theta_w \cos\theta_w\right]^4} \times \frac{E^4}{\left[\left(2E\right)^2 - \left(M_Z c^2\right)^2\right]^2 + \left[\hbar\Gamma_z M_z c^2\right]^2}$$

Consider this ratio at energies well below the Z^0 pole and near the Z^0 pole:

$$\frac{\sigma\left(e^+e^- \to Z^0 \to \mu^+\mu^-\right)}{\sigma\left(e^+e^- \to \gamma \to \mu^+\mu^-\right)} = \underbrace{\frac{\left[\frac{1}{2} - 2\sin^2\theta_w + 4\sin^4\theta_w\right]^2}{\left[\sin\theta_w \cos\theta_w\right]^4}}_{\approx\ 2} \times \frac{E^4}{\left[\left(2E\right)^2 - \left(M_Z c^2\right)^2\right]^2 + \left[\hbar\Gamma_z M_z c^2\right]^2}$$

Well below the Z^0 pole: $$\frac{\sigma\left(e^+e^- \to Z^0 \to \mu^+\mu^-\right)}{\sigma\left(e^+e^- \to \gamma \to \mu^+\mu^-\right)} \approx 2\left(\frac{E}{M_Z c^2}\right)^4$$

Near the Z^0 pole: $$\frac{\sigma\left(e^+e^- \to Z^0 \to \mu^+\mu^-\right)}{\sigma\left(e^+e^- \to \gamma \to \mu^+\mu^-\right)} \approx 2\frac{\left[M_z c^2/2\right]^4}{\left[\hbar\Gamma_z M_z c^2\right]^2} = \frac{1}{8}\left(\frac{M_z c^2}{\hbar\Gamma_z}\right)^2 \approx 200 \qquad \left[\hbar\Gamma_Z \sim 2.5\ \text{GeV}\right]$$

It is important to note that elsewhere, the interference effects are important:

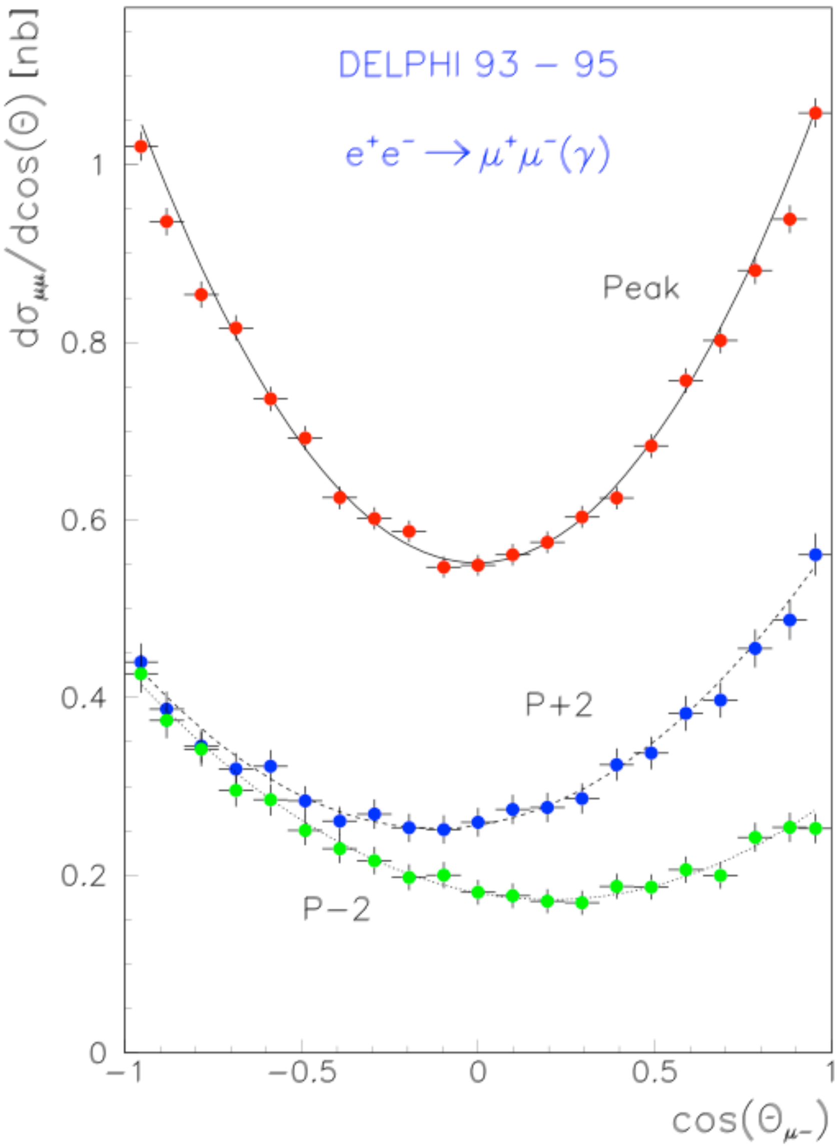
$$\mathcal{M}_{tot}^2\left(e^+e^- \to \mu^+\mu^-\right) = \mathcal{M}_Z^2 + \mathcal{M}_\gamma^2 + \underbrace{\mathcal{M}_Z^*\mathcal{M}_\gamma + \mathcal{M}_Z\mathcal{M}_\gamma^*}$$

Has Z^0 propagator squared

Has γ propagator squared

Each has a product of the photon and Z^0 propagators

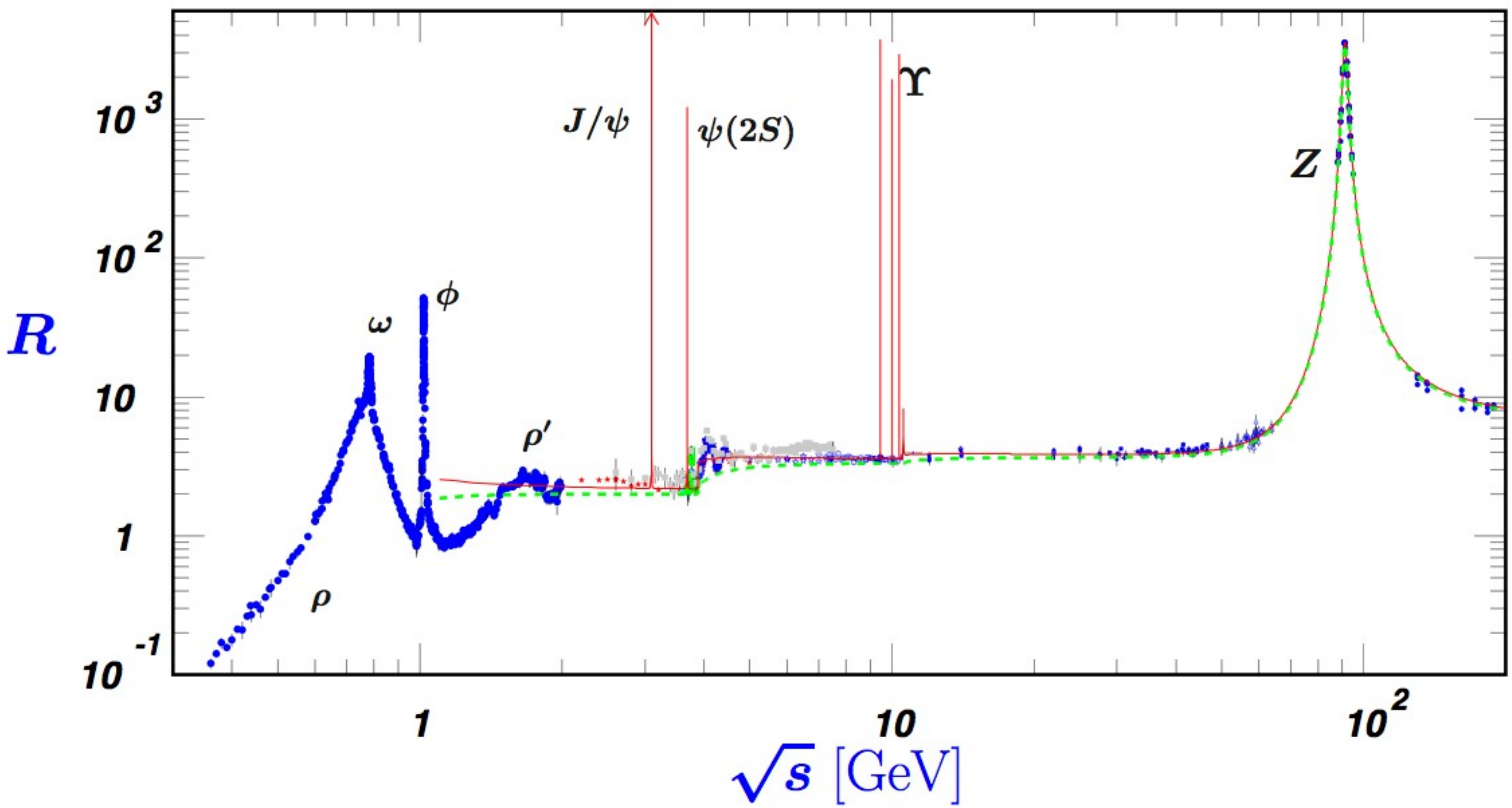
Note in particular that for energies >> $M_Z c^2$ all these terms are of similar magnitude.



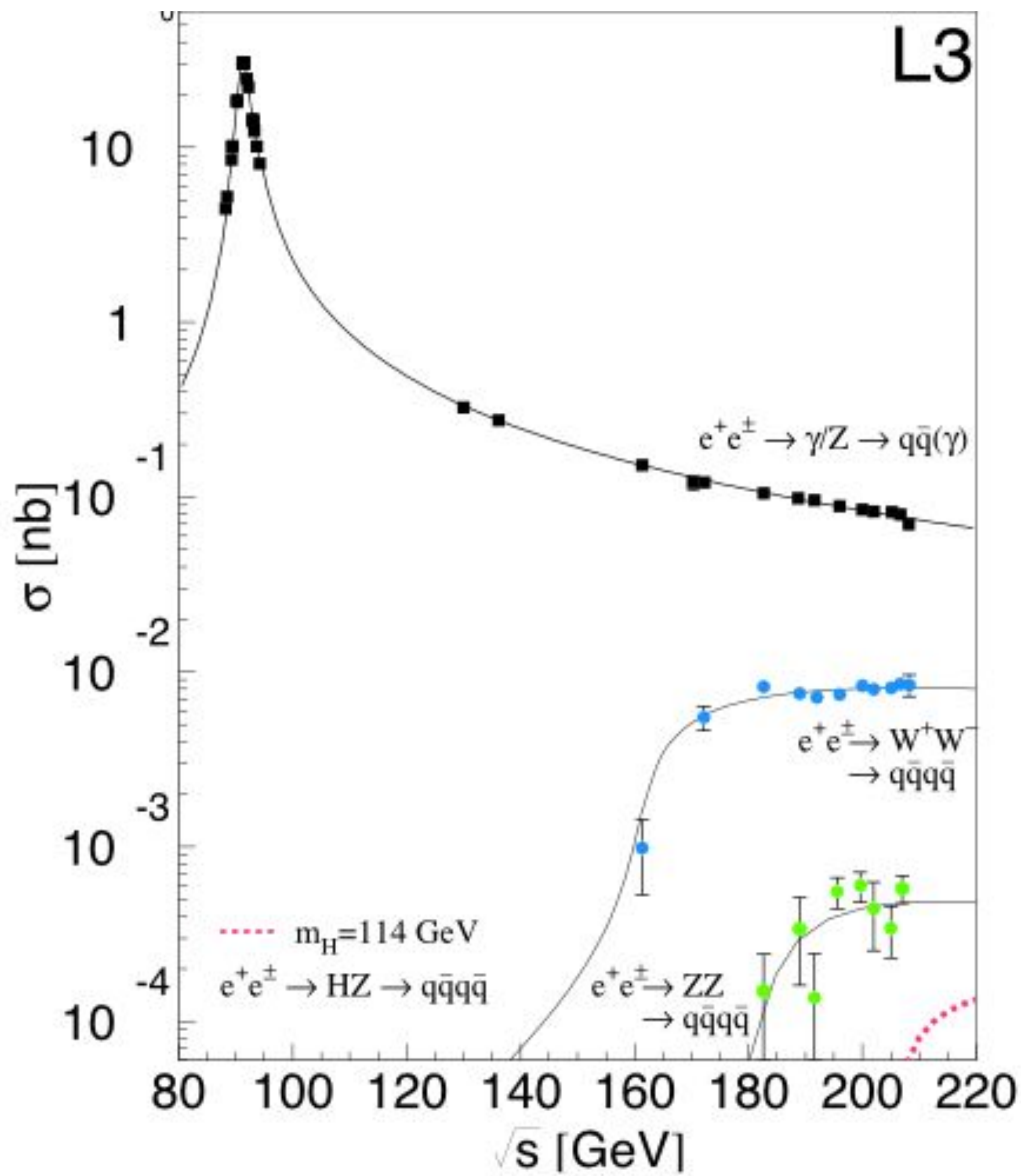
Peak means $E_{cm} = M_Z c^2$=91.2 GeV

P±2 means Peak ± 2 GeV

$$R = \frac{\sigma\left(e^+e^- \to hadrons\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} \quad \text{for} \quad \sqrt{s} \leq 209\,\mathrm{GeV}$$



Blowup of high $\sqrt{s}$ region



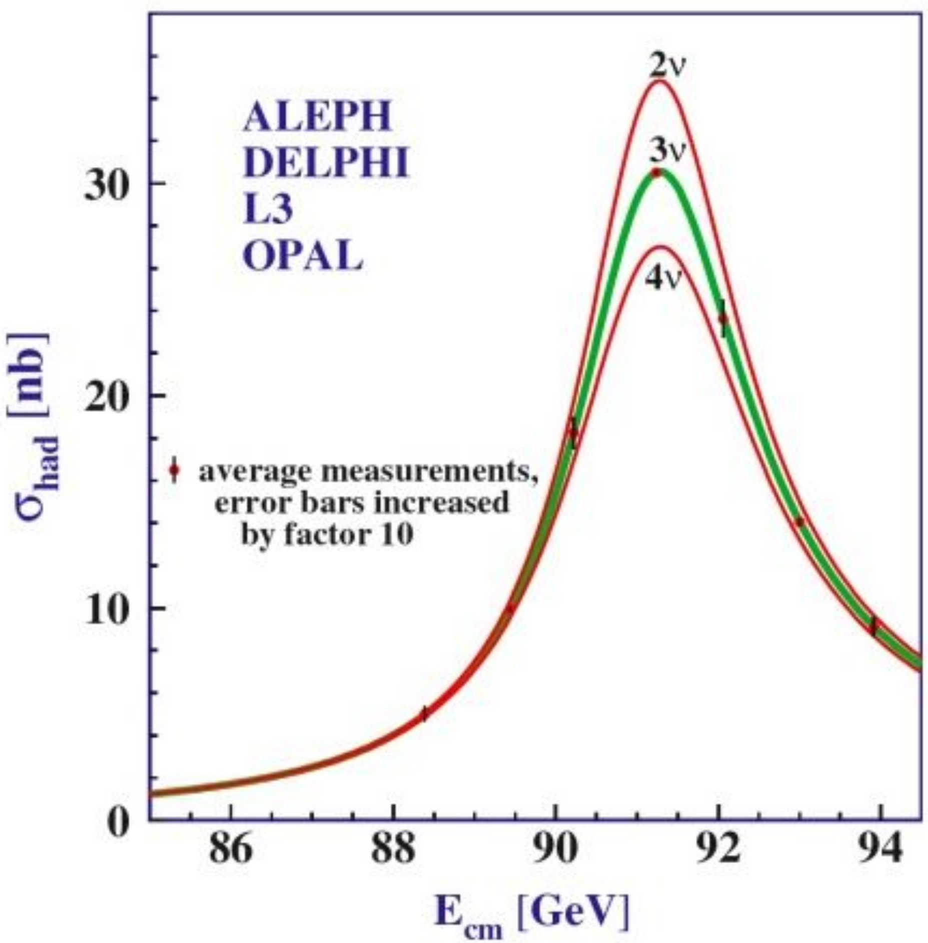
At high centre of mass energies new processes start to turn on (e.g. one reaches the threshold energy).

What causes slow turn-on? Why isn't there an abrupt increase as one reaches the threshold energy?

What does this look like if we sketch it to higher energies?

The number of light neutrino generations

Measurements of $\sigma\left(e^+e^- \to hadrons\right)$ in the region of $E_{CM} \sim M_Z$ tell us about the number of neutrino generations (at least those with masses < M_Z / 2).



Why is this the case?
How does this work?