Phy489 Lecture 21-22

Chiral Fermion States & Electroweak Unification

Question: how can we contemplate unifying two forces that appear to have couplings that are very different in form (not just in "apparent magnitudes" since it was already suspected that the "weakness" of the charged weak interaction could be attributed to the mass of the exchanged particle)?

Compare the couplings (vertex factors in Feynman rules)

QED	$ig_e\gamma^\mu$	vector
Charged Weak	$-i\frac{g_w}{2\sqrt{2}}\gamma^\mu\left(1-\gamma^5\right)$	pure vector-axial vector (V-A)
Neutral Weak	$-i\frac{g_z}{2}\gamma^\mu\left(c_V^f-c_A^f\gamma^5\right)$	mix of vector and axial vector

Note that QED is all "neutral current". We will see that the neutral weak and EM currents "mix", hence the c_V^f and c_A^f terms instead of the pure *V-A* of the charged weak interaction.

Chiral Fermions

How do we deal with the "structural differences"? (*e.g.* the different vertex factors)

Difficulty is associated with the factors of $(1-\gamma^5)$. This can be dealt with by "absorbing" a factor of $(1-\gamma^5)/2$ into the definition of the particle spinors:

$$u_L(p) = \left(\frac{1-\gamma^5}{2}\right) u(p)$$

we call this a left handed spinor, though in general it is not a helicity eigenstate

Look at the term $\gamma^5 u(p) = \begin{pmatrix} \frac{c(\vec{p}\cdot\vec{\sigma})}{E+mc^2} & 0 \\ 0 & \frac{c(\vec{p}\cdot\vec{\sigma})}{E-mc^2} \end{pmatrix} u(p)$ [see next slide]

If the particle is massless, $E = |\vec{p}|c$ and we have $\frac{c(\vec{p}\cdot\vec{\sigma})}{E+mc^2} = \frac{c(\vec{p}\cdot\vec{\sigma})}{|\vec{p}|c} = \hat{p}\cdot\vec{\sigma}$

We had (see Lecture on solutions to the Dirac equation):

$$u_A = \frac{c}{E-mc^2}\left(\vec{p}\cdot\vec{\sigma}\right)u_B \qquad\qquad u_B = \frac{c}{E+mc^2}\left(\vec{p}\cdot\vec{\sigma}\right)u_A$$

$$\gamma^5 u(p) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u_A \\ u_B \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{c}{E-mc^2}\left(\vec{p}\cdot\vec{\sigma}\right)u_B \\ \frac{c}{E+mc^2}\left(\vec{p}\cdot\vec{\sigma}\right)u_A \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{c}{E+mc^2}\left(\vec{p}\cdot\vec{\sigma}\right)u_A \\ \frac{c}{E-mc^2}\left(\vec{p}\cdot\vec{\sigma}\right)u_B \end{pmatrix}$$

$$\gamma^5 u(p) = \begin{pmatrix} \frac{c}{E+mc^2}\left(\vec{p}\cdot\vec{\sigma}\right) & 0 \\ 0 & \frac{c}{E-mc^2}\left(\vec{p}\cdot\vec{\sigma}\right) \end{pmatrix}\begin{pmatrix} u_A \\ u_B \end{pmatrix} = \begin{pmatrix} \frac{c}{E+mc^2}\left(\vec{p}\cdot\vec{\sigma}\right) & 0 \\ 0 & \frac{c}{E-mc^2}\left(\vec{p}\cdot\vec{\sigma}\right) \end{pmatrix} u(p)$$

If the particle is massless this becomes $\gamma^5 u(p) = \begin{pmatrix} \left(\hat{p}\cdot\vec{\sigma}\right) & 0 \\ 0 & \left(\hat{p}\cdot\vec{\sigma}\right) \end{pmatrix} u(p) \;=\; \left(\hat{p}\cdot\vec{\Sigma}\right)u\left(p\right)$

So, in the case of a massless particle: $\gamma^5 u(p) \quad = \quad (\hat{p} \cdot \vec{\Sigma}) u(p)$ where $\quad \vec{\Sigma} \equiv \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$

Recall that $\frac{\hbar}{2}\vec{\Sigma}$ is the spin matrix for a Dirac particle (see section 7.2), so $(\hat{p} \cdot \vec{\Sigma})$

represents the helicity, with eigenvalues of ±1.

So we have that $\frac{1}{2}(1-\gamma^5)u(p)$ = 0 if $u(p)$ carries helicity +1 (right handed)

= $u(p)$ if $u(p)$ carries helicity -1 (left handed)

Reminder: this is true in the massless limit only; for small masses it is approximate.

So $\frac{1}{2}(1-\gamma^5)$ acts as a projection operator that picks out the helicity -1 component

If the particle is NOT massless it is only in the ultra-relativistic regime ($E >> mc^2$) that

$$\gamma^5 u(p) \quad = \quad \left(\vec{p}\cdot\vec{\Sigma}\right)u(p)$$

holds approximately, and it is only in this limit that u_L carries helicity = -1. However, this is still generally referred to as a left-handed spinor.

For an anti-particle, a similar exercise yields (again, in the massless limit)

$$v_L\left(p\right)=\left(\frac{1+\gamma^5}{2}\right)v\left(p\right) \qquad \left[u_L\left(p\right)=\left(\frac{1-\gamma^5}{2}\right)u\left(p\right)\right]$$

The corresponding right handed spinors are:

$$u_R\left(p\right)=\left(\frac{1+\gamma^5}{2}\right)u\left(p\right) \qquad v_R\left(p\right)=\left(\frac{1-\gamma^5}{2}\right)v\left(p\right)$$

Adjoint Spinors (Chiral Fermions)

Need also to know the expressions for of the corresponding adjoint spinors:

$$\overline{u}_L = u_L^{\dagger}\gamma^0 = u^{\dagger}\left(\frac{1-\gamma^5}{2}\right)\gamma^0 = u^{\dagger}\gamma^0\left(\frac{1+\gamma^5}{2}\right) = \overline{u}\left(\frac{1+\gamma^5}{2}\right)$$

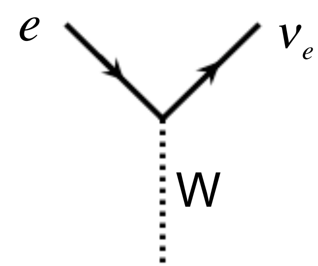
$$\overline{v}_L = \overline{v}\left(\frac{1-\gamma^5}{2}\right)$$

$$\overline{u}_R = \overline{u}\left(\frac{1-\gamma^5}{2}\right)$$

$$\overline{v}_R = \overline{v}\left(\frac{1+\gamma^5}{2}\right)$$

The Charged Current Weak Interaction

Look now at charged weak interaction vertex:



The contribution to the amplitude $\mathcal{M}$ from this vertex is $j_\mu^- = \bar{\nu}\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)e$

where, for the moment, we are using particle species (ν, e) to label the spinors

rather than u_ν, u_e (the bar however still denotes an adjoint spinor, not an antiparticle).

Note that $\left(\frac{1-\gamma^5}{2}\right)^2 = \frac{1}{4}\left(1-2\gamma^5+\left(\gamma^5\right)^2\right) = \left(\frac{1-\gamma^5}{2}\right)$ since $\left(\gamma^5\right)^2 = 1$

$\gamma_\mu\left(\frac{1-\gamma^5}{2}\right) = \left(\frac{1+\gamma^5}{2}\right)\gamma_\mu$ Multiply by $\left(\frac{1-\gamma^5}{2}\right)$ on the RHS of each side of this expression

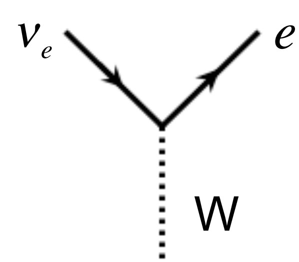
$\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)\left(\frac{1-\gamma^5}{2}\right) = \left(\frac{1+\gamma^5}{2}\right)\gamma_\mu\left(\frac{1-\gamma^5}{2}\right) \Rightarrow \gamma_\mu\left(\frac{1-\gamma^5}{2}\right) = \left(\frac{1+\gamma^5}{2}\right)\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)$ So we can write

So we can write $$j_\mu^- = \bar{\nu}\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)e = \bar{\nu}\left(\frac{1+\gamma^5}{2}\right)\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)e = \bar{\nu}_L\gamma_\mu e_L$$

Since the notation might be confusing, I remind you that this represents an adjoint particle spinor for the neutrino.

And the (charged) weak vertex factor is now purely vectorial, just as for QED, but it couples only to left-handed electrons and left-handed neutrinos. Similarly, we have

$$j_\mu^+ = \bar{e}_L\gamma_\mu\nu_L$$ for the process



We can also write the electromagnetic “current” in terms of these chiral spinors:

$$j_\mu^{em} = -\bar{e}\gamma_\mu e = -(\bar{e}_L + \bar{e}_R)\gamma_\mu(e_L + e_R) = -\bar{e}_L\gamma_\mu e_L - \bar{e}_R\gamma_\mu e_R$$

Where the factor of -1 is conventional, accounting for the charge of the electron.

I have used $u = u_L + u_R$ (which is easy to show, if it is not obvious to you)

Note that the cross-terms vanish in the expression for j_μ^{em}

$$\bar{e}_L\gamma_\mu e_R = \bar{e}\left(\frac{1+\gamma^5}{2}\right)\gamma_\mu\left(\frac{1+\gamma^5}{2}\right)e = \bar{e}\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)\left(\frac{1+\gamma^5}{2}\right)e = 0$$

Since $\left(1-\gamma^5\right)\left(1+\gamma^5\right) = 1 - \left(\gamma^5\right)^2 = 0$

This more generally implies that a vector interaction cannot couple a LH particle state to a RH particle state or a LH particle state to a LH antiparticle state etc.

Isospin in Strong Interactions (Review)

We learned earlier that the strong interactions of nucleons (protons and neutrons) do not depend on the nucleon species (so *pp*, *np*, and *nn* experience the same strong interactions).

We wrote a two component object, the nucleon as $N = \begin{pmatrix} p \\ n \end{pmatrix}$.

More formally, we can represent the isospin wavefunction of a nucleon as a linear combination of the two states:

$$\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |p\rangle \qquad \text{isospin “up”}$$

$$\chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |n\rangle \qquad \text{isospin “down”}$$

And we can introduce the isospin operator $\vec{\tau}$ which has components

$$\tau_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \tau_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \tau_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Except for factors of ½ these are the Pauli spin matrices describing spin ½ particles.

In fact, the Pauli spin matrices describe any system that has two possible states. Note that

$$\tau_3|p\rangle = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\ 0\end{pmatrix} = \frac{1}{2}|p\rangle \qquad \tau_3|n\rangle = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}0\\ 1\end{pmatrix} = -\frac{1}{2}|n\rangle$$

That is, these are eigenstates of τ_3 with eigenvalues ± 1/2.

Expect eigenvalues of $\tau^2 \equiv \vec{\tau}\cdot\vec{\tau}$ to then be ½(½+1) = ¾. Check this:

$$\tau^2\chi = \tau_1^2 + \tau_2^2 + \tau_3^2 = 3\left[\frac{1}{4}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right]\chi = \frac{3}{4}\chi \quad ✔$$

Can make isospin raising or lowering operators $\tau^{\pm} = (\tau_1 \pm i\tau_2)$

$$\tau^+ = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix} \qquad \tau^- = \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$$

Applying these operators to the two states, we see

$$\tau^+|p\rangle = 0 \qquad \tau^+|n\rangle = |p\rangle \qquad \tau^-|p\rangle = |n\rangle \qquad \tau^-|n\rangle = 0$$

Back to the weak interaction.......

We we have (so far)

$$j_\mu^- = \bar{\nu}_L \gamma_\mu e_L$$

$$j_\mu^+ = \bar{e}_L \gamma_\mu \nu_L$$

charged weak currents

$$j_\mu^{em} = -\bar{e}\gamma_\mu e = -\bar{e}_L \gamma_\mu e_L - \bar{e}_R \gamma_\mu e_R$$

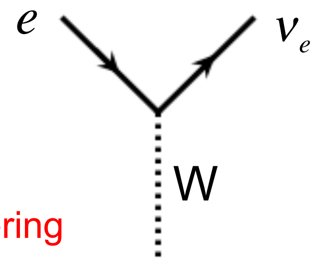
EM current (neutral)

Neutral Weak Currents

We have expressions for the "positive" and "negative" weak charged current:

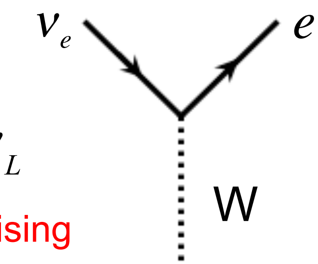
$$j_\mu^- = \bar{\nu}_L \gamma_\mu e_L$$

Weak-isospin lowering



$$j_\mu^+ = \bar{e}_L \gamma_\mu \nu_L$$

Weak-isospin raising



Can write this compactly by defining the left-handed (weak-isospin) doublet: $\chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$

Using the 2x2 matrices $\tau^+ \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $\tau^- \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ we can then write:

$$\boxed{j_\mu^\pm = \bar{\chi}_L \gamma_\mu \tau^\pm \chi_L}$$

with $\tau^\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$ **Weak-isospin raising and lowering operators, with** $\tau_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\tau_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

These are two of the Pauli spin matrices, written here as τ to avoid any confusion with ordinary spin (as we did when discussing isospin in our discussions of strong interactions). More accurately, these are two of the matrices that form a representation of the group SU(2) which describes systems in which two possible states are related by some symmetry transformation.

(e.g. the quantity in terms of which we are defining these doublets)

Refer to this as weak-isospin and anticipate full weak-isospin symmetry, which would imply the existence of a third "current" corresponding to $\frac{1}{2}\tau^3$ where τ^3 is the third Pauli spin matrix [of SU(2)]:

$$\tau^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad j_\mu^3 = \overline{\chi}_L \gamma_\mu \tfrac{1}{2}\tau^3 \chi_L = \frac{1}{2}\overline{\nu}_L \gamma_\mu \nu_L - \frac{1}{2}\overline{e}_L \gamma_\mu e_L$$

Here we have "predicted" the existence of a weak neutral current process. However, this still only couples LH particles to LH particles, while we know that the Z^0 has RH couplings as well (or the vertex factor would be just V-A like the charged weak interaction).

Consider the so-called weak-hypercharge current (which is a mixture of the two neutral currents j_μ^3 and j_μ^{em}): [here the weak hypercharge Y is defined by $Q = I_3 + \frac{1}{2}Y$]

$$j_\mu^Y = 2j_\mu^{em} - 2j_\mu^3 = 2\left(-\overline{e}_R \gamma_\mu e_R - \overline{e}_L \gamma_\mu e_L\right) - 2\left(\frac{1}{2}\overline{\nu}_L \gamma_\mu \nu_L - \frac{1}{2}\overline{e}_L \gamma_\mu e_L\right)$$

$$= -2\,\overline{e}_R \gamma_\mu e_R - \overline{e}_L \gamma_\mu e_L - \overline{\nu}_L \gamma_\mu \nu_L$$

$$j_\mu^Y = -2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu \nu_L$$ is invariant under a weak-isospin transformation

$$e_R \to e_R$$
$$e_L \to \nu_L$$
$$\nu_L \to e_L$$

$$j_\mu^Y = -2\bar{e}_R\gamma_\mu e_R - \bar{\nu}_L\gamma_\mu \nu_L - \bar{e}_L\gamma_\mu e_L$$

N.B. We have been discussing things in terms of electrons and electron-neutrinos, but the same applies to any of the weak-isospin doublets:

$$\chi_L = \begin{pmatrix}\nu_e \\ e\end{pmatrix}_L \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L \begin{pmatrix}u \\ d'\end{pmatrix}_L \begin{pmatrix}c \\ s'\end{pmatrix}_L \begin{pmatrix}t \\ b'\end{pmatrix}_L$$

where primes on the down-type quarks denote the Cabibbo-rotated states

In each case one can construct three weak isospin currents, $\vec{j}_\mu = \frac{1}{2}\bar{\chi}_L\gamma_\mu\vec{\tau}\,\chi_L$ and one weak-hypercharge current: $j_\mu^Y = 2j_\mu^{em} - 2j_\mu^3$

Weak Hypercharge

$Y = 2Q - 2I_3$ Look at values for members of weak isospin doublet:

$$\chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \begin{pmatrix} u \\ d' \end{pmatrix}_L \begin{pmatrix} c \\ s' \end{pmatrix}_L \begin{pmatrix} t \\ b' \end{pmatrix}_L \quad \begin{matrix} I_3 = +1/2 \\ I_3 = -1/2 \end{matrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad 2Q - 2I_3 = \begin{pmatrix} 2(0) - 2(1/2) \\ 2(-1) - 2(-1/2) \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad 2Q - 2I_3 = \begin{pmatrix} 2(2/3) - 2(1/2) \\ 2(-1/3) - 2(-1/2) \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1/3 \end{pmatrix}$$

e.g. value of Y is the same for the two members of a weak isospin doublet

GSW Model for Electroweak Mixing

The GWS model asserts that the three weak-isospin currents couple with strength g_w to a weak iso-triplet of vector bosons $\vec{W}$ while the weak hypercharge current couples with strength $g'/2$ to an iso-singlet vector boson B .

$$-i\left[g_w \vec{j}_\mu \cdot \vec{W}^\mu + \frac{g'}{2} j_\mu^Y B^\mu\right]$$ [where the vectors are vectors in weak-isospin space]

$$\vec{j}_\mu \cdot \vec{W}^\mu = j_\mu^1 W^{1\mu} + j_\mu^2 W^{2\mu} + j_\mu^3 W^{3\mu} = \frac{1}{\sqrt{2}} j_\mu^+ W^{+\mu} + \frac{1}{\sqrt{2}} j_\mu^- W^{-\mu} + j_\mu^3 W^{3\mu}$$

where $W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp W_\mu^2\right)$. We can now read off the couplings to the $W^\pm$:

e.g. for $e^- \to \nu_e + W^-$ $\quad j_\mu^- = \bar{\nu}_L \gamma_\mu e_L = \bar{\nu}\left[\gamma_\mu \left(\tfrac{1}{2}\right)\left(1-\gamma^5\right)\right]e$ which yields a term of

$\left\{ \text{i.e. recalling that } j_\mu^- = \bar{\nu}\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)e = \bar{\nu}\left(\frac{1+\gamma^5}{2}\right)\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)e = \bar{\nu}_L\gamma_\mu e_L \right\}$

$$-ig_w \frac{1}{\sqrt{2}} j_\mu^- W^{-\mu} = -i\frac{g_w}{2\sqrt{2}}\left[\bar{\nu}\gamma_\mu\left(1-\gamma^5\right)e\right]W^{-\mu}$$ so the vertex factor is $$-i\frac{g_w}{2\sqrt{2}}\gamma_\mu\left(1-\gamma^5\right)$$

Electroweak mixing

In the GWS model, the W^3 and B mix to produce one massless combination $\left(\gamma \equiv A^{\mu}\right)$ and one (orthogonal) massive combination, the Z^0:

$$\left.\begin{aligned} A_\mu &= B_\mu \cos\theta_w + W_\mu^3 \sin\theta_w \\ Z_\mu &= -B_\mu \sin\theta_w + W_\mu^3 \cos\theta_w \\ \\ W_\mu^3 &= Z_\mu \cos\theta_w + A_\mu \sin\theta_w \\ B_\mu &= -Z_\mu \sin\theta_w + A_\mu \cos\theta_w \end{aligned}\right\}$$

Write out neutral component of

$$-i\left[g_w \vec{j}_\mu \cdot \vec{W}^{\mu} + \frac{g'}{2} j_\mu^Y B^\mu\right]$$

In terms of the physical states A_μ and Z_μ

$$-i\left[g_w j_\mu^3 W^{3\mu} + \frac{g'}{2} j_\mu^Y B^\mu\right] = -i\left\{\left[g_w \sin\theta_w j_\mu^3 + \frac{g'}{2}\cos\theta_w j_\mu^Y\right]A^\mu + \left[g_w \cos\theta_w j_\mu^3 - \frac{g'}{2}\sin\theta_w j_\mu^Y\right]Z^\mu\right\}$$

We know that the EM coupling is $-ig_e j_\mu^{em} A^\mu$. Recall that $j_\mu^Y = 2j_\mu^{em} - 2j_\mu^3 \Rightarrow j_\mu^{em} = j_\mu^3 + \frac{1}{2} j_\mu^Y$

This implies that we need $g_w \sin\theta_w = g' \cos\theta_w = g_e$. That is, the weak and electromagnetic coupling constants are NOT independent!

We had that:

$$-i\left[g_w j_\mu^3 \cdot W^{3\mu} + \frac{g'}{2} j_\mu^Y B^\mu\right] = -i\left\{\left[g_w \sin\theta_w j_\mu^3 + \frac{g'}{2}\cos\theta_w j_\mu^Y\right]A^\mu + \underbrace{\left[g_w \cos\theta_w j_\mu^3 - \frac{g'}{2}\sin\theta_w j_\mu^Y\right]}Z^\mu\right\}$$

For the Z^0

$$g_w \cos\theta_w j_\mu^3 - \frac{g'}{2}\sin\theta_w j_\mu^Y = g_w \cos\theta_w j_\mu^3 - \frac{g'}{2}\sin\theta_w\left(2 j_\mu^{em} - 2 j_\mu^3\right)$$

$$= \left(g_w \cos\theta_w + g'\sin\theta_w\right) j_\mu^3 - g'\sin\theta_w j_\mu^{em}$$

$$= \left(\frac{g_e}{\sin\theta_w}\cos\theta_w + \frac{g_e}{\cos\theta_w}\sin\theta_w\right) j_\mu^3 - \frac{g_e}{\cos\theta_w}\sin\theta_w j_\mu^{em}$$

$$= \left(\frac{g_e \cos^2\theta_w + g_e \sin^2\theta_w}{\sin\theta_w \cos\theta_w}\right) j_\mu^3 - \left(\frac{g_e}{\sin\theta_w \cos\theta_w}\sin^2\theta_w\right) j_\mu^{em}$$

Using $g_z = \dfrac{g_e}{\cos\theta_w \sin\theta_w}$ this becomes $-ig_z\left(j_\mu^3 - \sin^2\theta_w j_\mu^{em}\right)Z^\mu$

Vertex factors for Neutral Weak Interactions

From the expression $-i\left[g_w \cos\theta_w j_\mu^3 - \frac{g'}{2}\sin\theta_w j_\mu^Y\right]Z^\mu = -ig_z\left(j_\mu^3 - \sin^2\theta_w j_\mu^{em}\right)Z^\mu$ one can

Simply read off the couplings to the Z^0. This is most straightforward for the neutrinos, e.g. for the case in which $\nu \rightarrow \nu + Z^0$ where the coupling is entirely from j_μ^3 :

$$j_\mu^3 = \frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L \cancel{- \frac{1}{2}\bar{e}_L\gamma_\mu e_L}$$ so we have $$-ig_z\left(j_\mu^3 - \sin^2\theta_w j_\mu^{em}\right)Z^\mu \Rightarrow -ig_z\left(\frac{1}{2}\bar{\nu}\frac{\gamma_\mu\left(1-\gamma^5\right)}{2}\nu\right)Z^\mu$$

And the required vertex factor is $$-i\frac{g_z}{2}\gamma_\mu\left(\frac{1}{2} - \frac{1}{2}\gamma^5\right)$$

$\frac{1}{2} \downarrow c_V^\nu \qquad \frac{1}{2} \downarrow c_A^\nu$

Z^0 Coupling to Charged Leptons

Try another: $\tau^- \to \tau^- + Z^0$

Need to again evaluate $-ig_z\left(j_\mu^3 - \sin^2\theta_w j_\mu^{em}\right)Z^\mu$. Proceed as before:

$$-ig_z\left(j_\mu^3 - \sin^2\theta_w j_\mu^{em}\right)Z^\mu = -ig_z\left[\left(-\frac{1}{2}\bar{\tau}_L\gamma_\mu\tau_L\right) + \left(\bar{\tau}_R\gamma_\mu\tau_R + \bar{\tau}_L\gamma_\mu\tau_L\right)\sin^2\theta_w\right]Z^\mu$$

$$= -ig_z\left[\left(-\frac{1}{2}\bar{\tau}\gamma_\mu\frac{\left(1-\gamma^5\right)}{2}\tau\right) + \left(\bar{\tau}\gamma_\mu\frac{\left(1+\gamma^5\right)}{2}\tau + \bar{\tau}\gamma_\mu\frac{\left(1-\gamma^5\right)}{2}\tau\right)\sin^2\theta_w\right]Z^\mu$$

$$= -ig_z\left[-\frac{1}{4}\bar{\tau}\gamma_\mu\tau + \frac{1}{4}\bar{\tau}\gamma_\mu\gamma^5\tau + \left(\frac{1}{2}\bar{\tau}\gamma_\mu\tau + \frac{1}{2}\bar{\tau}\gamma_\mu\gamma^5\tau + \frac{1}{2}\bar{\tau}\gamma_\mu\tau - \frac{1}{2}\bar{\tau}\gamma_\mu\gamma^5\tau\right)\sin^2\theta_w\right]Z^\mu$$

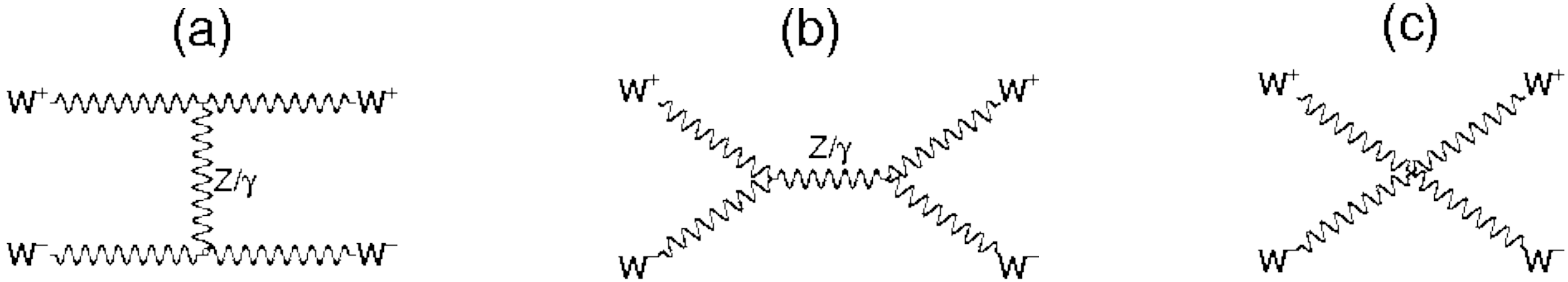
$$= -ig_z\left[-\frac{1}{4}\bar{\tau}\gamma_\mu\tau + \frac{1}{4}\bar{\tau}\gamma_\mu\gamma^5\tau + \bar{\tau}\gamma_\mu\tau\sin^2\theta_w\right]Z^\mu$$

$$= -ig_z\left[\bar{\tau}\gamma_\mu\left(-\frac{1}{4} + \sin^2\theta_w + \frac{1}{4}\gamma^5\right)\tau\right]Z^\mu = -i\frac{g_z}{2}\left[\bar{\tau}\gamma_\mu\left(\underbrace{-\frac{1}{2} + 2\sin^2\theta_w}_{c_V^\tau} + \underbrace{\frac{1}{2}}_{-c_A^\tau}\gamma^5\right)\tau\right]Z^\mu$$

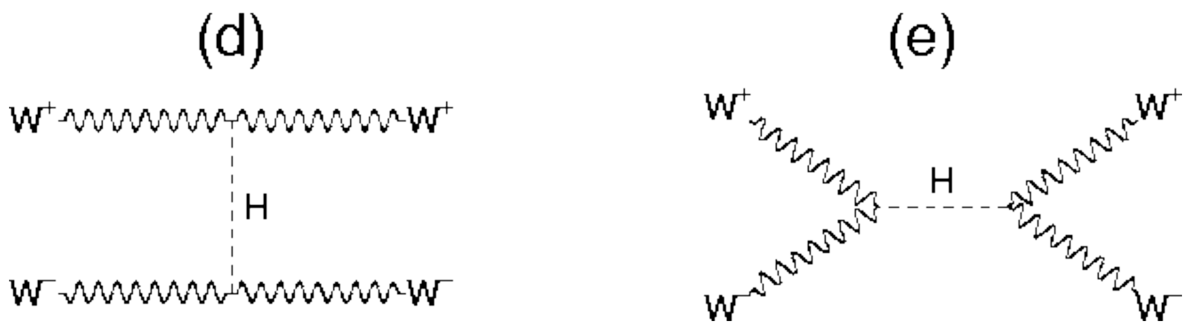
$c_V^\tau = -\frac{1}{2} + 2\sin^2\theta_w$ $\quad c_A^\tau = -\frac{1}{2}$

A few words about the Higgs boson ……

Vector Boson Scattering

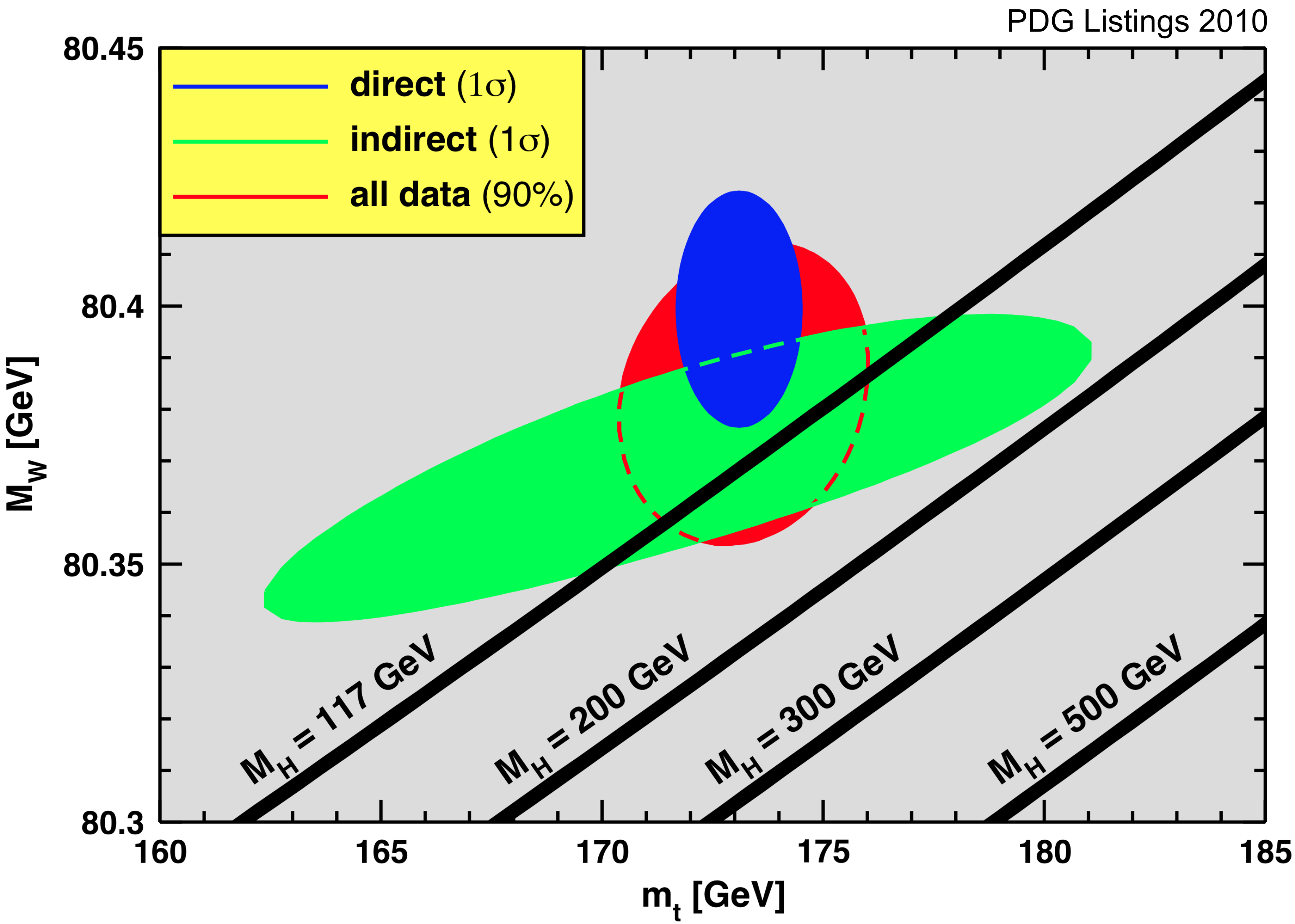


Cross-section grows with $s \equiv E_{CM}^2$. Eventually violates unitarity (probability) unless there are additional processes. Need to add



with $M_H \leq 1\ TeV$

Experimental Constraints on the Higgs Boson



Higgs Boson Decays

Once the mass of the Higgs is known (specified) we know all of it's other properties, for instance (important experimentally) what it decays into: