Phy489 Lecture 3

So far……

- Fundamental interactions in the Standard Model (SM)
  - Mediated by particle exchange
  - Three fundamental forces
    - have different effective strengths
    - act over different timescales

} These two statements are correlated

- Particle content of the Standard Model
  - Matter particles (fundamental spin-1/2 fermions)
  - Force carriers (spin-1 gauge bosons)
  - Higgs boson
  - Particle masses
  - Hadrons (made up of quarks, which do not exist freely)
- Feynman diagrams for fundamental processes
- Particle Decays
- Today continue this discussion and introduce first simple conservation laws……

Note that not all conservation laws apply to all three fundamental interactions (the ones we will discuss today do).

Kinematic effects in Particle Decays

If multiple final states are available (for the same interaction) the dominant one (kinematically) will often be the one with the largest mass difference between the initial and final states. We will see that for two-body decays 1→ 2 + 3:

$$\frac{1}{\tau} \equiv \underset{\uparrow \atop \text{decay rate (width)}}{\Gamma} = \underset{\text{kinematics}}{\boxed{\frac{|\vec{p}|}{8\pi\hbar m_1^2 c}}} \overset{\text{statistical factor} \atop \downarrow}{S} \underset{\uparrow \atop \text{dynamics}}{|\mathcal{M}|^2}$$

$|\vec{p}|$ is the magnitude of the momentum of the final state products (2,3). This is a measure of the energy release in the decay. We will derive this expression later on.

However, this rule of thumb is seldom of much practical use: charged pion decay occurs dominantly (99.988%) via $\pi^+ \to \mu^+ \nu_\mu$ although $\pi^+ \to e^+ \nu_e$ is kinematically favoured. The difference is from the dynamics (as we will see explicitly, later in the course).

Or look at it the other way around

- This principle is perhaps better illustrated in reverse: so consider the decay of a free neutron (why do I say “free”?): this occurs via $n \rightarrow p\ e^{-}\ \overline{\nu}_e$ with a very long lifetime of about 15 minutes. Why?
- Note that this is a three-body final state, so the kinematics are slightly different, but the principle is similar.
- Look at the energy release in the decay:
  - m_p = 938.280 MeV/c^2
  - m_n = 939.573 MeV/c^2
  - m_e = 0.511 MeV/c^2

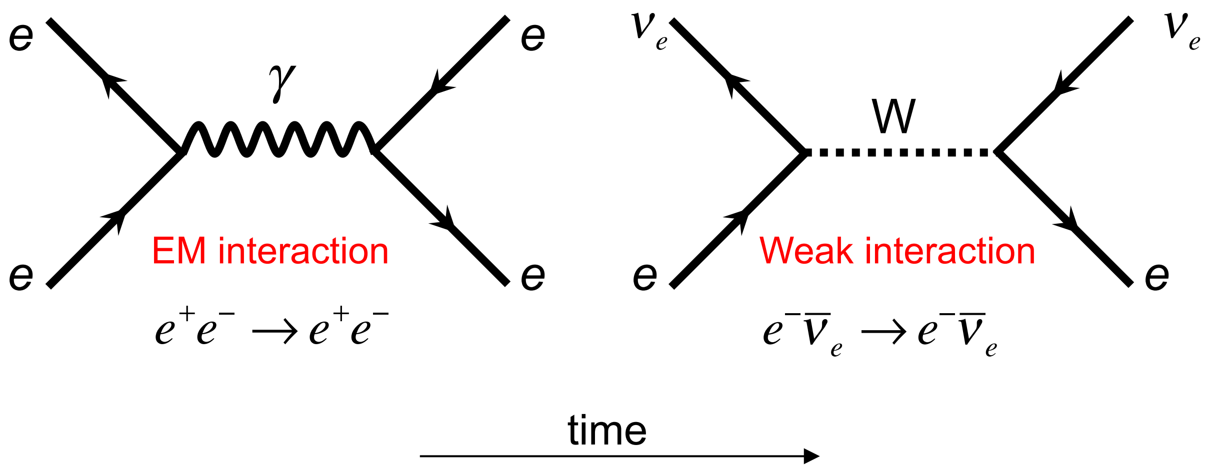
$m_n - m_p - m_e = (1.293 - 0.511)$ MeV/c^2

- The energy release is very low. There is very little “phase space” for the decay and thus it proceeds only very slowly. (More on phase-space later in the course).

Conservation Laws

e.g. conservation of energy and momentum

- So far we have mostly discussed kinematic issues. Need to consider other conservation laws as well.
- Start with a familiar one: conservation of electric charge.



$e^+e^- \rightarrow e^+e^-$

$e^-\bar{\nu}_e \rightarrow e^-\bar{\nu}_e$

Electric charge is conserved

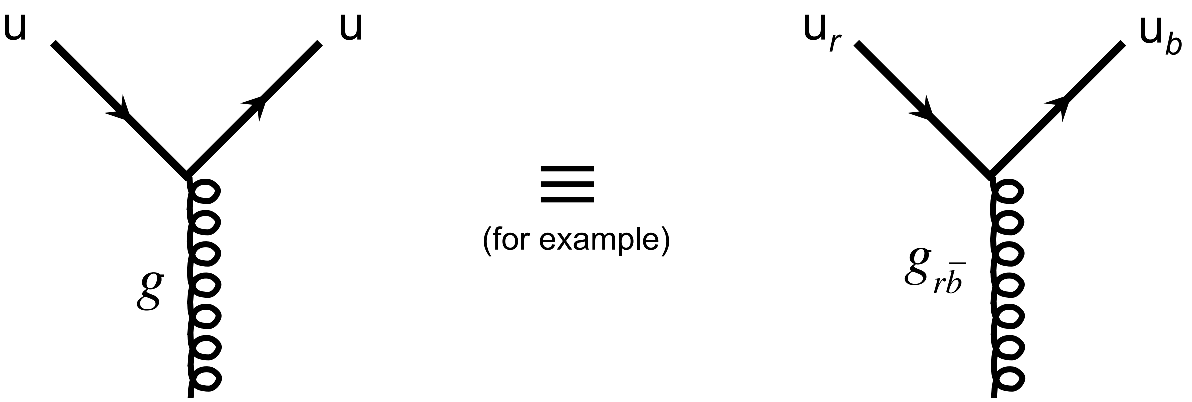
Weak charge is also conserved

Strong charge is trivially conserved

Note that charge is conserved at each fundamental vertex, not just in the process as a whole. Conservation of charge is “built into” the theory (in terms of the allowed vertices).

Charge Conservation

- As noted on the last slide, strong charge (colour) is trivially conserved at weak interaction and EM interaction vertices.
- Colour is conserved at all strong interaction vertices:



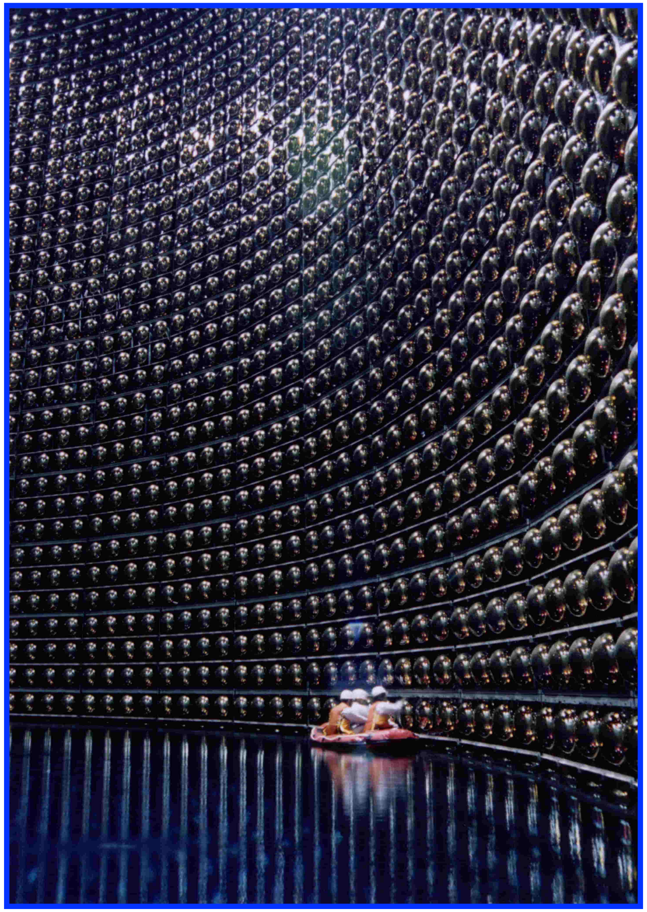
In this case the gluon is being emitted rather than absorbed

Conservation laws dealing with sources (charges) are considered inviolable.

Conservation of electric charge is a result of local gauge invariance in QED. Both the electroweak theory and QCD are locally invariant gauge theories (we will not discuss this).

Conservation of Baryon Number

SuperKamiokande Detector, Japan



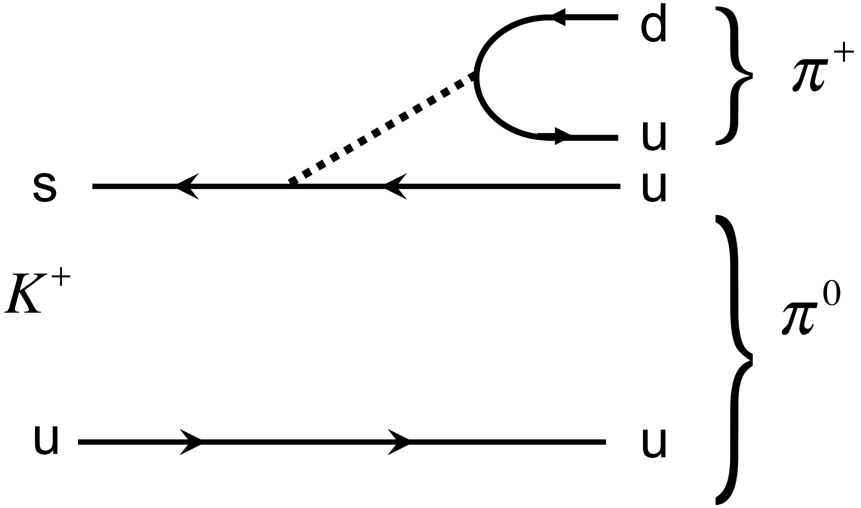
- Why does the proton not decay? (Experimentally, at least, we know that $\tau_p > 10^{35}$ years). →
- Assign "baryon number" B
  - ➢ B= +1 for all baryons
  - ➢ B = -1 for all anti-baryons
  - ➢ B = 0 for all other particles (incl. mesons)
- Can think of this as 1/3 for each quark, -1/3 for each anti-quark (also yields the correct value B=0 for mesons).
- Baryon number is conserved in the SM.
- There is no such thing as meson number….
  the number of mesons in not a conserved quantity

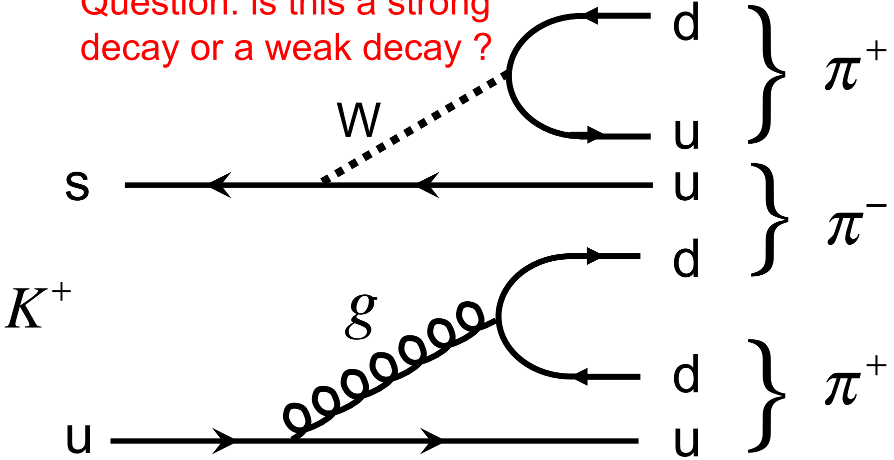
Discovery of the anti-proton

- Discovery process was $p + p \rightarrow p + p + p + \bar{p}$.
- The minimum kinetic energy required in the initial state was that required for the production of two additional baryons, because baryon number has to be conserved.

“threshold energy”: more on this later on.

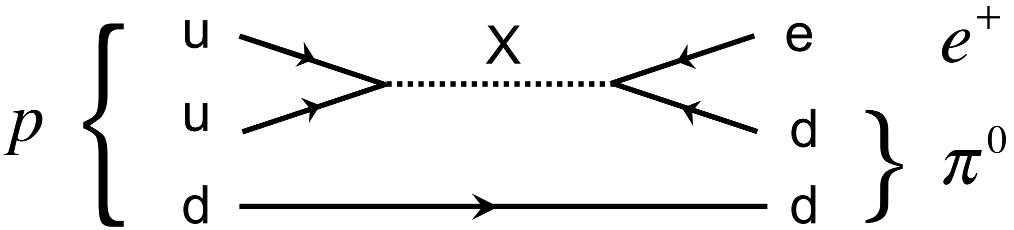
- The proton is the lightest baryon. It is absolutely stable in the Standard Model. In some extensions to the SM it can decay, but with a long lifetime (*i.e.* consistent with current experimental constraints).
- Contrast with the meson decay (below): 1 meson in initial state, *n* in final state. Where *n* can be as many as allowed by kinematics. Just need to create a q$\bar{\text{q}}$ pair for each one….




Question: is this a strong decay or a weak decay ?

Proton Decay

In some “Beyond the Standard Model” (BSM) theories, there can be new interactions that allow the proton to decay. Of course, the predictions of such a theory must not contradict known experimental constraints.



Requires a new force carrying particle X with electric charge +4/3 that couples leptons to quarks (lepto-quarks). These form part of some Grand Unified Theories (GUTs) but do not exist in the Standard Model.

Clearly this new interaction does not conserve baryon number.

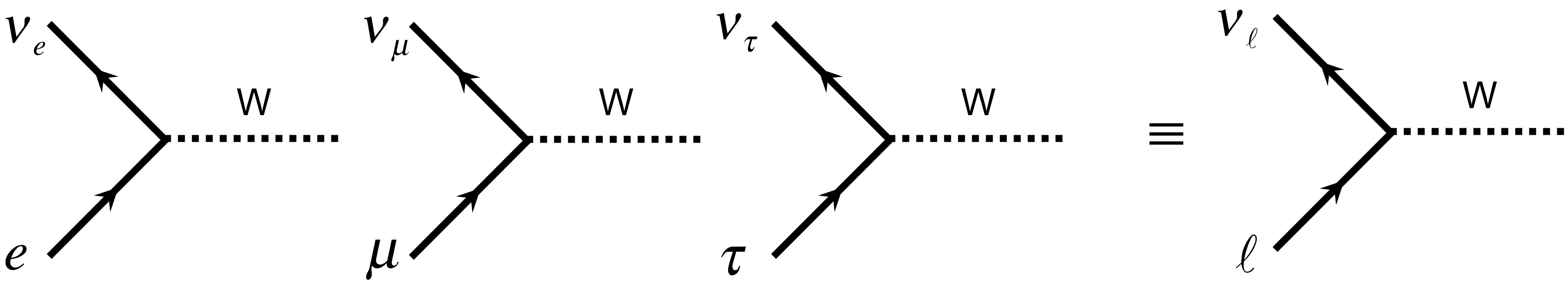
Often models of physics “Beyond the Standard Model” (BSM) will predict the existence of new particles. One of the things experimental particle physicists do is to search for these.

Conservation of Lepton Number

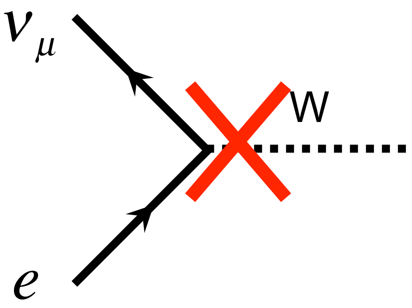
- See Griffiths (section 1.5) for the history.
- Separate conservation of electron, muon and tau numbers

$$L_e = \begin{cases} +1 & e^-, \nu_e \\ -1 & e^+, \overline{\nu}_e \end{cases} \qquad L_\mu = \begin{cases} +1 & \mu^-, \nu_\mu \\ -1 & \mu^+, \overline{\nu}_\mu \end{cases} \qquad L_\tau = \begin{cases} +1 & \tau^-, \nu_\tau \\ -1 & \tau^+, \overline{\nu}_\tau \end{cases}$$

- So the charged (leptonic) weak interaction has the allowed vertices:

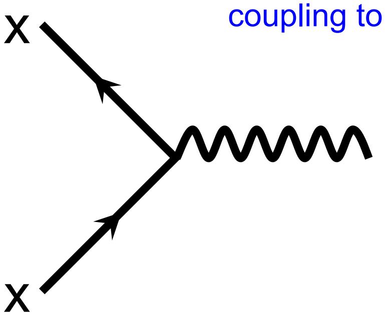


- But never (for example):



Lepton number conservation in EM Interactions

The photon couples to electric charge. Allowed vertices are of the form

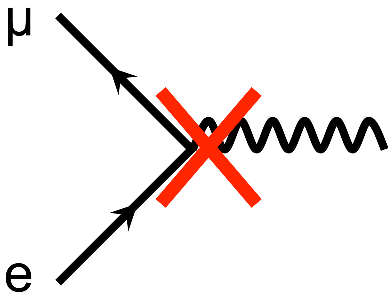


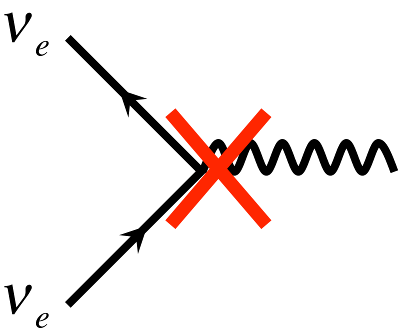
In fact, a particle's electric charge is just a measure of its coupling to the photon, e.g. the mediator of the EM interaction

x = e, μ, τ , u, d, c, s, b, t, W (all have charge)

Can read this as emission or absorption of a photon, or particle anti-particle annihilation or creation, depending on the time ordering

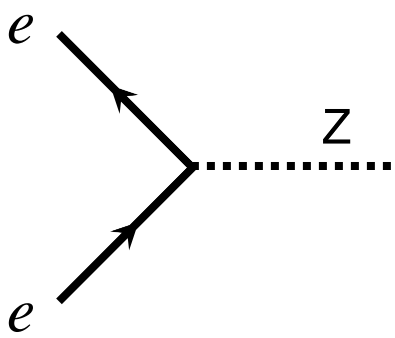
So for photon coupling to leptons, there is no coupling to neutrinos and there is no coupling of one type of lepton to another even though this does conserve electric charge:





Similarly for the Neutral Weak Interaction

So for instance



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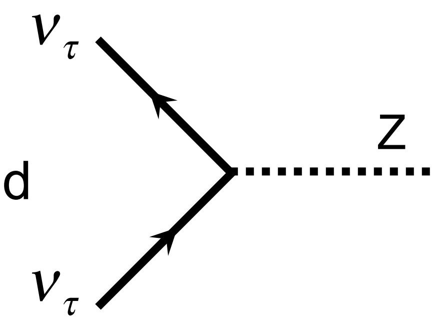
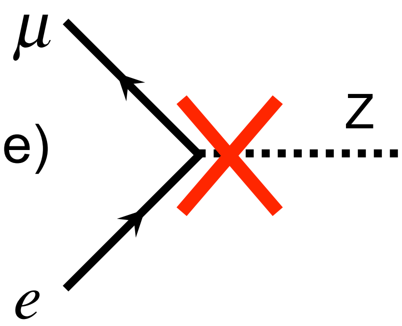


are allowed

But (for example)



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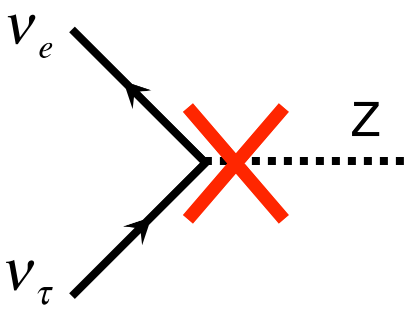


are not.

In contrast to the photon, the Z also couples to neutrinos (which carry weak charge).

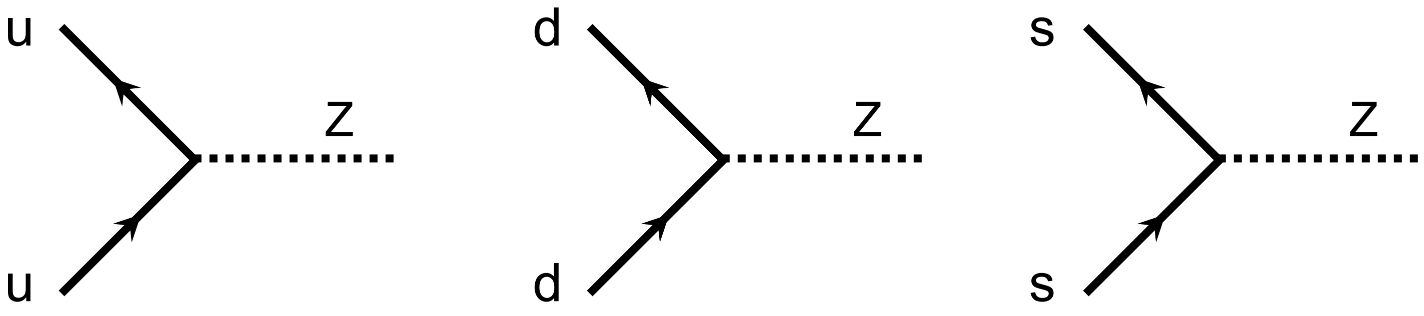
There is no “generational” mixing in the leptonic sector.

Also say there is no lepton “flavour mixing” (this is the same statement as the previous one).

There is one special exception to this: neutrino flavour oscillations. These have been shown to exist but are not part of the SM. We will not discuss this (but this explained the long-standing solar neutrino problem: see the SNO experiment). [see chapter 11 if you are interested]

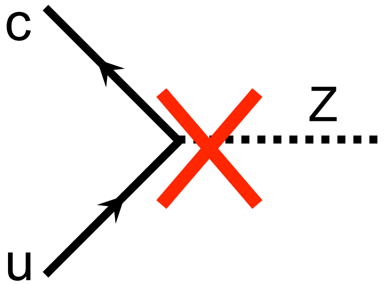
What about for Quarks ?

For the neutral weak interaction the same rule applies: no intergenerational (i.e. flavour mixing). For example



are all allowed (and equivalent diagrams for other quark flavours) as are the diagrams in which the Z is replaced by a photon (as we have seen).

But diagrams such as this one are not:



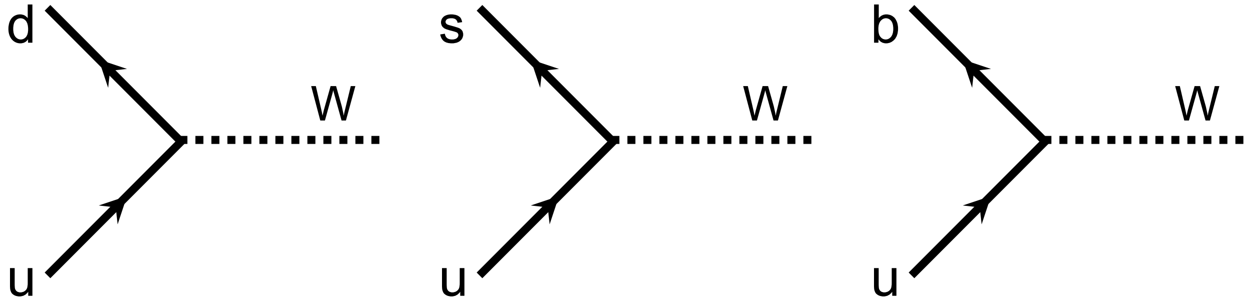
despite the fact that it conserves (for instance) electric charge.

There are no "*Flavour Changing Neutral Currents*" (FCNC) in the SM !

EM couplings to quarks are as on slide 11

Quark Mixing

For the charged weak interaction, however, things are different:



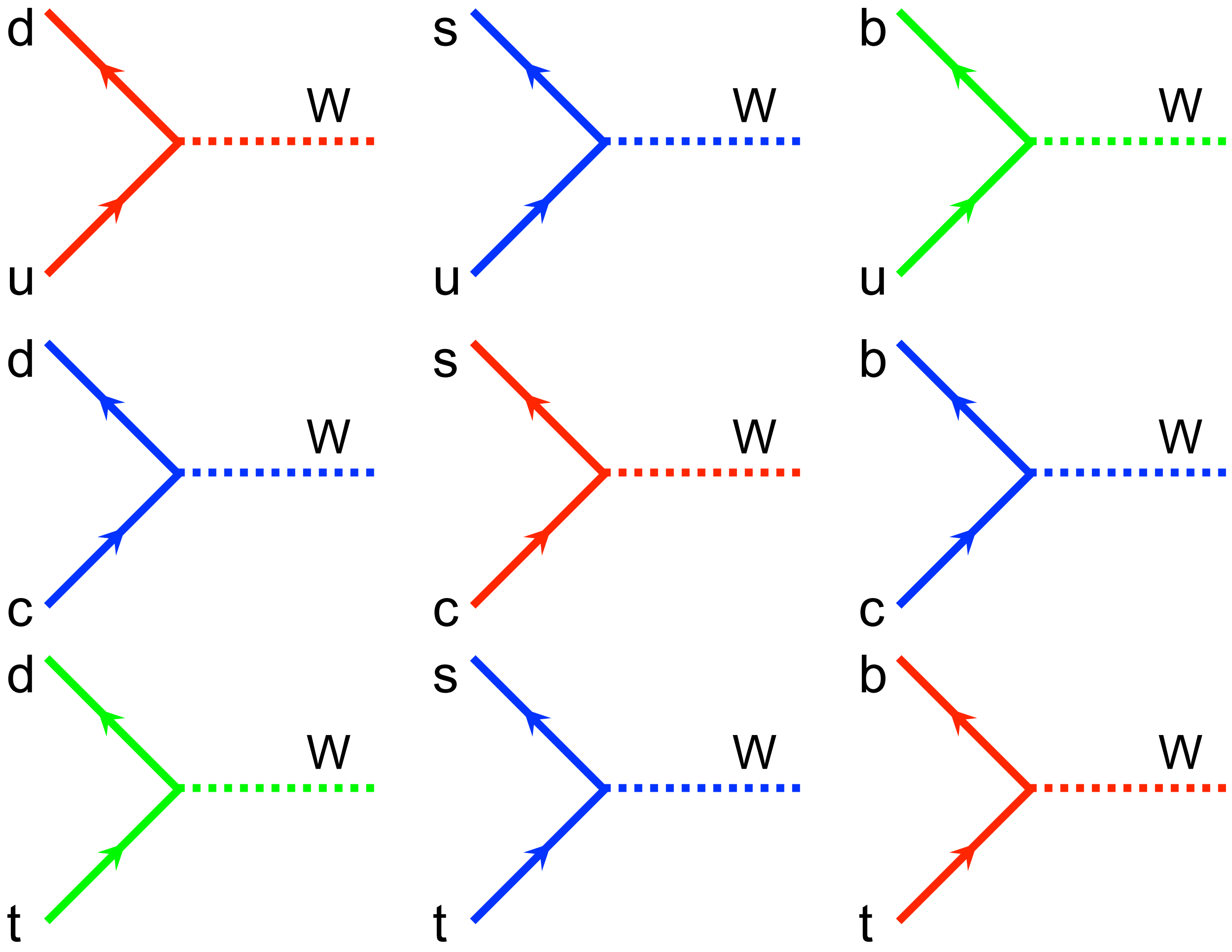
are all allowed. That is, each “up-type” quark (|Q|=2/3) couples to all three “down-type” quarks (|Q|=1/3) via the charged weak interaction (*i.e.* not just the one in the same generation).

(similarly for other up-type quarks - see next slide)

The charged weak interaction is the ONLY mechanism in the SM for coupling one flavour of quark to another. This is responsible for ALL weak decays of hadrons.

Quark Flavour Mixing

Strongest coupling is between up and down type quarks in the same generation. **Next strongest** is between neighbouring generations. **Weakest** between first and third generations (here strongest to weakest refers to the coupling strength at the vertex).



Quark Mixing Cont'd

Charged weak interaction eigenstates are not the same as the physical states (i.e. of definite mass). The charge weak interaction couples the pair (for example) $\begin{pmatrix} u \\ d' \end{pmatrix}$ where the primed states represent the linear combinations:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{td} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

CKM Matrix

Wolfenstein parametrization of the CKM matrix.

$$\lambda = \sin\theta_c \approx 0.23$$

$$1-\lambda^2/2 \approx 0.97$$

Elements are known rather precisely.
For example V_{ud} = 0.97383 ± 0.00024

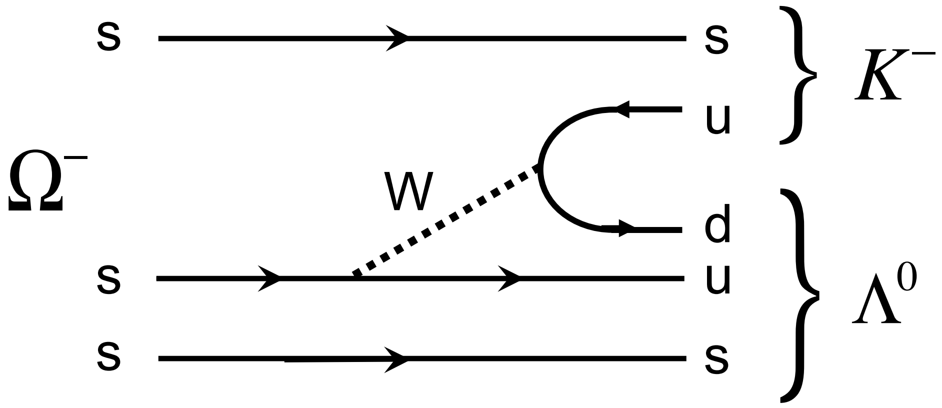
We will return to this later in the course.

Weak Decays of Hadrons (more examples)

The two main decay modes of the Ω^- are:

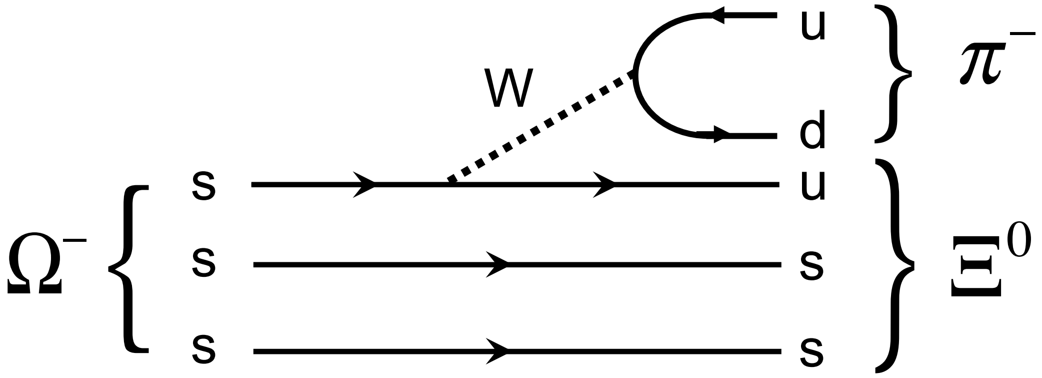
$$\Omega^- \to \Lambda^0 K^-$$

$$sss \to sud\;\bar{u}s$$



$$\Omega^- \to \Xi^0 \pi^-$$

$$sss \to ssu\;\bar{u}d$$

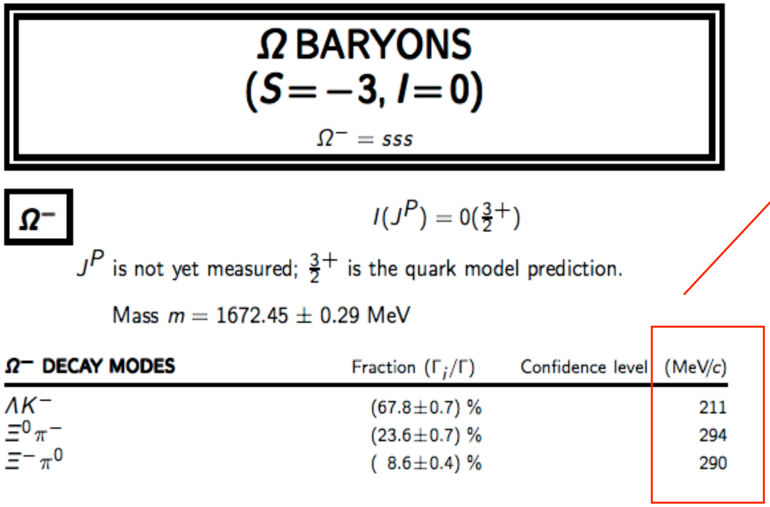


Final state quark content is the same in each case.

However, arrangement of these quarks into hadrons is different.

Weak Decays of the Ω^- cont'd

From particle data book listings



Ω BARYONS
(S = −3, I = 0)
$\Omega^- = sss$

Ω^- $I(J^P) = 0(\frac{3}{2}^+)$

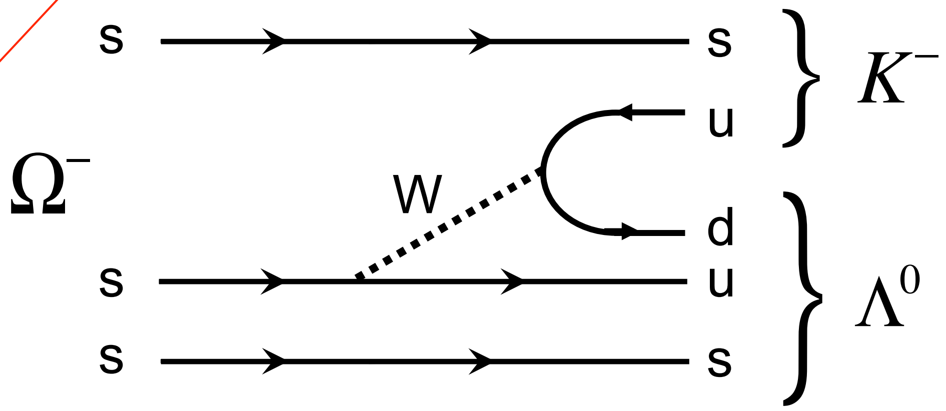
J^P is not yet measured; $\frac{3}{2}^+$ is the quark model prediction.

Mass $m = 1672.45 \pm 0.29$ MeV

Ω^- DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	(MeV/c)
ΛK^-	(67.8±0.7) %		211
$\Xi^0\pi^-$	(23.6±0.7) %		294
$\Xi^-\pi^0$	(8.6±0.4) %		290

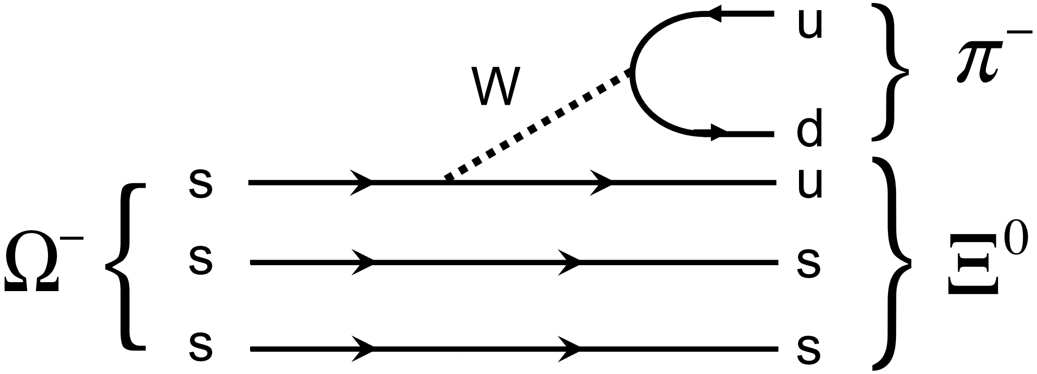
Momentum $|\vec{p}|$ of final state particles.

See kinematics on slide 3.



If relative rates are due only to kinematics expect ratio of branching ratios

$$\frac{Br(\Omega^- \to \Lambda K^-)}{Br(\Omega^- \to \Xi\pi)} \approx \frac{211}{294} \approx 0.7$$



Instead of the observed ratio of more than 2. Clearly there are issues here other than kinematics, despite the similarities of the final states.

Let’s try some typical problems

Similar to problem 1 on first assignment (will be posted by this Wednesday).

For the following reactions, say whether it is possible in the Standard Model, at lowest order -- this means two vertices in the reaction that mediates the interaction, but note that one can always add strong interaction vertices to produce quark-anti-pairs, if these are needed in the final state, since the strong coupling constant is of order 1 in these cases.

$$\mu^-\bar{\nu}_e \to \tau^-\bar{\nu}_\tau$$

$$e^+e^- \to Z^0Z^0$$

$$pp \to \pi^+\pi^+\pi^0\pi^0\pi^0$$

$$B^+ \to \pi^+\pi^+\pi^-$$

$$e^+e^- \to K^-\pi^+\pi^0$$