Phy489 Lecture 5

Relativistic Kinematics Cont'd

Last time discussed:

- Different (inertial) reference frames, Lorentz transformations
- Four-vector notation for relativistic kinematics, invariants
- Collisions and decays

This time, combined these issues to do some calculations:
I asked you to look at Ex. 3.3 in the text: $\pi(\text{at rest}) \to \mu\nu_\mu$

Initial state is $p_\pi = \left(\frac{E_\pi}{c}, \quad \vec{0}\right) = \left(m_\pi c, \quad \vec{0}\right)$

Final state is $p_f = p_\nu + p_\mu = \left(\frac{E_\nu}{c}, \quad \vec{p}\right) + \left(\frac{E_\mu}{c}, \quad -\vec{p}\right)$

Recall that (e.g.) $p_\pi \cdot p_\pi$ is an invariant = $m_\pi^2 c^2$ (and $= p_f \cdot p_f$)

Centre-of-mass (properly centre of momentum) reference frame has no net momentum (particle decay at rest, or collisions with equal and opposite momenta).

Reaction Thresholds

Early particle physics experiments were performed by colliding beams of particles with some target material (e.g. liquid hydrogen which can be thought of as a big collection of protons; the electrons are typically too light to contribute much to the interaction in the case of the scattering of a heavy particle, such as another proton).

So typically have scattering of an energetic proton with a proton at rest

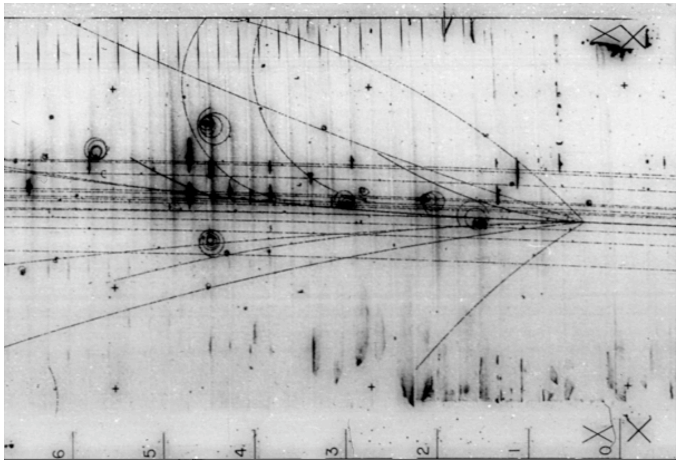
$$p + p \to X$$

Where *X* represents some (usually multi-particle) final state. Which of the fundamental forces will dominate this process?

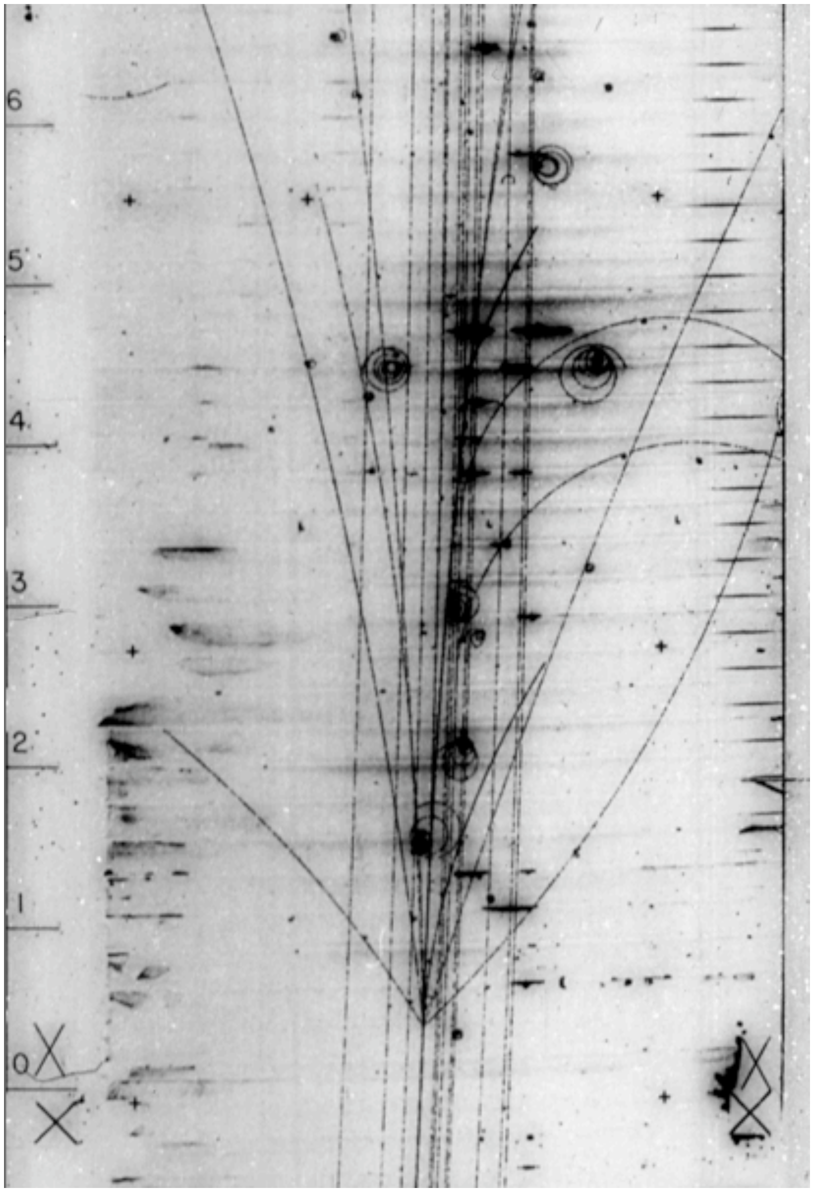
Another example, scattering of pions from protons:

$$\pi^+ + p \to X$$

Bubble chamber event from advanced undergraduate lab HEP experiment: →



Multi-particle production in fixed target collisions



Fixed target scattering:

10.3 GeV π^+ beam on liquid hydrogen target (bubble chamber).

In this event there are 8 charged particles produced in the π^+p collision (5 positive charge and 3 negative charge; charge conserved. Also need to conserve baryon number).

Some of the kinetic energy from the beam particle is converted into mass.

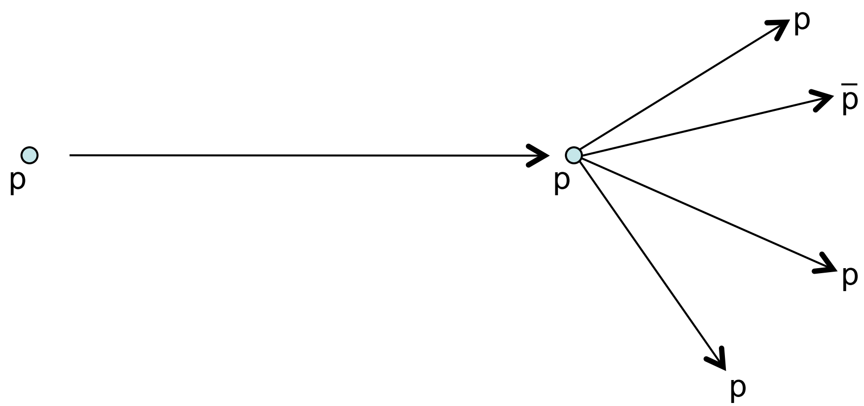
What is the maximum number of charged tracks that could emerge from one of these collisions?

What type of interaction is this?

Discovery of the antiproton ($\bar{p}$)

As previously advertised, this was the production method used for the discovery of the anti-proton (which you first need to make, and then detect):

The process $p + p \rightarrow p + p + p + \bar{p}$ is the minimal one that conserves baryon number (e.g. in pp collisions) since p is the lightest baryon:

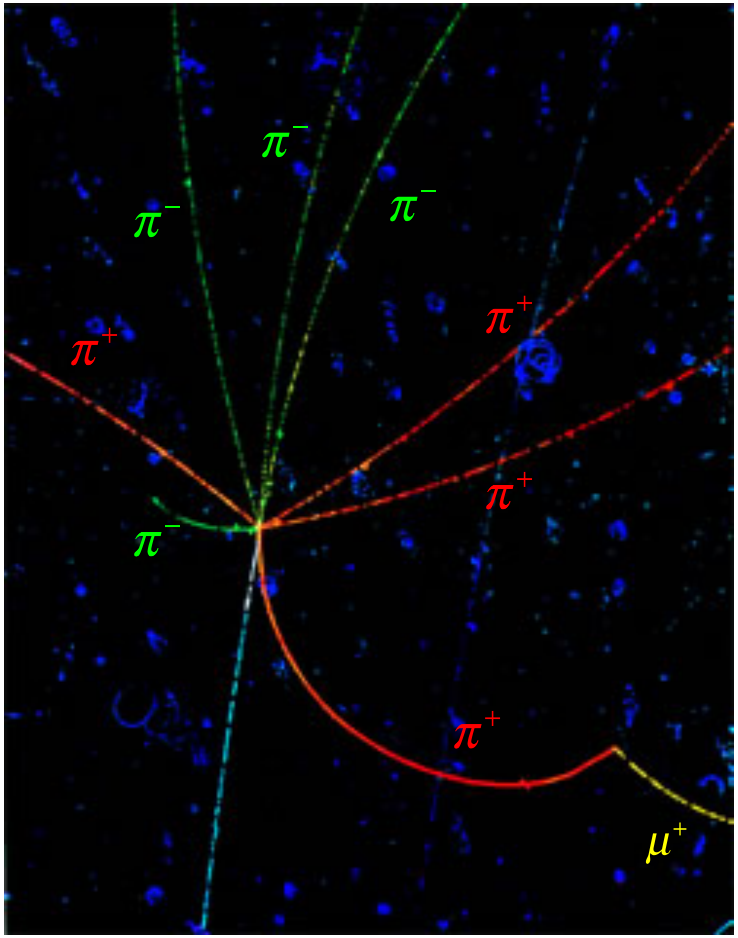


This is how the first anti-protons were made, but still need to detect them.

Question: What is the energy threshold for this process? That is, what is the minimum energy of the incoming proton such that this process is kinematically allowed (assuming the target proton is at rest)?

Discovery of the anti-proton

You don't just have to produce the anti-proton, you also need to detect it (via annihilation with one of the protons in the target material):



This is a “lab-frame” experiment. However, it is most straightforward to define the threshold requirement in the CM frame ($\vec{p}_{before} = \vec{p}_{after} = 0$). In that reference frame, the energy of the final state, at threshold, is just the mass energy of the particles in the final state, since they will be produced with no momentum. So the threshold energy in the CM frame is $4mc^2$.

We can write the total 4-momentum (p_{TOT}) in either reference frame, both before and after the collision, but $p_{TOT} \cdot p_{TOT}$ is always invariant (and always = M^2c^2, where *M* is the “invariant mass” of the system)

This is the energy we want to solve for

$$p_{TOT} = \left(\frac{E + mc^2}{c}, \quad |\vec{p}|, \quad 0, \quad 0 \right)$$ in the lab frame, *before* the collision

$$p'_{TOT} = (4mc, \quad 0, \quad 0, \quad 0)$$ in the CM frame, *after* the collision (at threshold)

$$p_{TOT}^2 = M^2c^2$$ is invariant, e.g. the same in any reference frame both before and after the interaction.

Typically we will only discuss lab frame vs. CM frame. You will seldom, if ever, need to explicitly transform between the two.

Minus sign from the metric $g_{\mu\nu}$

$$(p_{TOT})^2 = \left(\frac{E}{c} + mc\right)^2 - |\vec{p}|^2 = (p'_{TOT})^2 = 16m^2c^2$$

$$\frac{E^2}{c^2} + 2Em + m^2c^2 - |\vec{p}|^2 = 16m^2c^2 \Rightarrow \underbrace{\frac{E^2}{c^2} - |\vec{p}|^2}_{m^2c^2} + 2Em + m^2c^2 = 16m^2c^2$$

$$E = \frac{14m^2c^2}{2m} = 7mc^2$$

This is the total *relativistic* energy of the incoming beam particle, at threshold. So the kinetic energy required corresponds to 6mc^2.
(*i.e*. 1mc^2 is the rest mass).

Clearly if we could create the same process using colliding beams (of equal and opposite momentum) then the threshold beam energy would trivially be 2mc^2 (*e.g.* 1mc^2 of kinetic energy per beam). Is there a reason why doing a fixed target experiment might be better? Clearly making a lower energy beam is easier.

Colliding Beam vs. Fixed Target Scattering

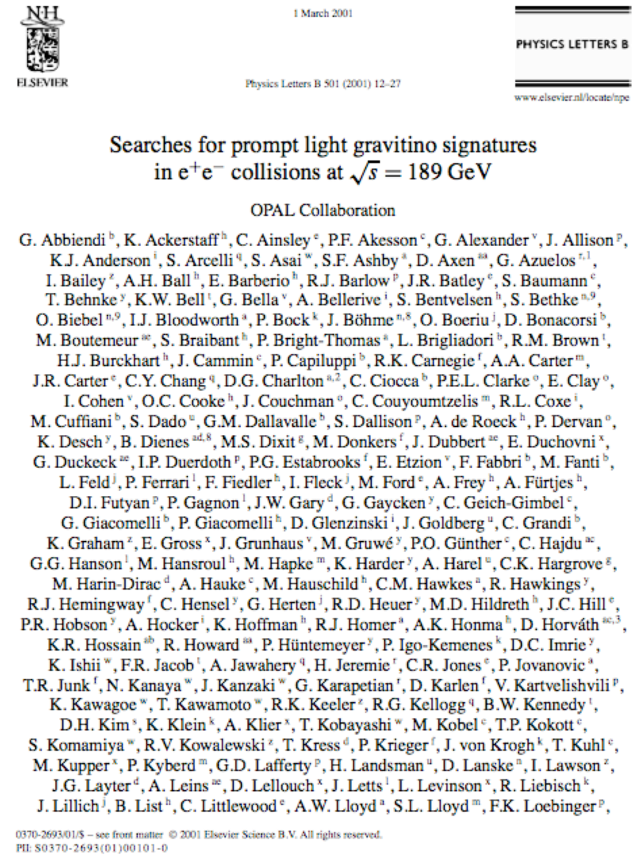
$$A + B \rightarrow C_1 + C_2 + \dots\dots\dots + C_n$$

Colliding beam machines mostly (but not always) operate in the CM frame. Some exceptions are:

- HERA (30 GeV electrons on 820 GeV protons)
- B-factory (e^+e^- with unequal energies)

The invariant $s = (p_A + p_B)^2$ is the square of the energy available for new particle production (this is again just the invariant mass of the initial state).

$\sqrt{s}$ is referred to as the CM energy of the collision



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Searches for prompt light gravitino signatures in e^+e^- collisions at $\sqrt{s} = 189$ GeV

OPAL Collaboration

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Consider the example $m_A = m_B, \quad \vec{p}_A = -\vec{p}_B \quad \Rightarrow \quad s = \left(\dfrac{E_A + E_B}{c}\right)^2 = 4\dfrac{E^2}{c^2}$

Fixed target case (again, assume $m_A = m_B = m$)

$$\left(p_A + p_B\right)^2 = \left(\frac{E}{c} + mc, \quad \vec{p} + \vec{0} \right)^2 = \left(\frac{E}{c} + mc\right)^2 - |\vec{p}|^2 = \frac{E^2}{c^2} + m^2c^2 + 2Em - |\vec{p}|^2$$

$$= \frac{E^2}{c^2} - |\vec{p}|^2 + m^2c^2 + 2Em \Rightarrow s = 2(m^2c^2 + Em) \approx 2Em \text{ for } E >> m$$

so for colliding beams we have $E_{CM} \propto E_{beam}$

and for fixed target collisions we have $E_{CM} \propto \sqrt{E_{beam}}$

Another way of looking at this is treated in Griffiths problem 3.23, which I sometime assign on the first problem set (but not this year). This looks at things in terms of the relative kinetic energy. Starts in the CM frame and asks for the relative kinetic energy in the lab frame: *e.g.* how much energy you would need to give a particle in the lab frame to get an equivalent CM energy.

Griffiths Problem 3.16 (3.14 in first edition)

Consider the collision where particle *A* (total energy *E*) hits particle *B* at rest to produce an *n*-particle final state: $A + B \to C_1 + C_2 + ...+ C_n$

Calculate the threshold energy (minimum E) for this reaction to proceed, in terms of the masses of the final-state particles.

At threshold, the minimum energy is just the masses of the RHS plus the momentum required to conserve momentum (so 0 in the CM frame)

this is the energy we want to solve for

$$p_A^2 + p_B^2 + 2 p_A \cdot p_B = (p_{RHS})^2 \quad = \quad m_A^2 c^2 + m_B^2 c^2 + 2\frac{E}{c} m_B c = (p_{RHS})^2$$

$$(p_{RHS})^2 = \left\{ \left(\frac{E_1}{c}, \quad \vec{p}_1 \right) + \left(\frac{E_2}{c}, \quad \vec{p}_2 \right) + ...+ \left(\frac{E_n}{c}, \quad \vec{p}_n \right) \right\}^2 \quad = \left(\frac{E_{TOT}^{RHS}}{c}, \quad \vec{p}_{TOT}^{RHS} \right)^2 = \frac{E^2}{c^2} - |\vec{p}|^2 = M_{TOT}^2 c^2$$

This is frame independent

$$m_A^2 c^2 + m_B^2 c^2 + 2 E m_B = c^2 \left(m_1 + m_2 + ...+ m_n \right)^2 \equiv M^2 c^2 \quad \Rightarrow \quad E = \frac{M^2 - m_A^2 - m_B^2}{2 m_B} c^2$$

in the CM frame, at threshold

Griffiths Problem 3.19 (3.16 in first edition)

Particle *A* (at rest) decays into particles *B* and *C* $(A \rightarrow B + C)$

Find the energy of the outgoing particles in terms of the three particle masses

$$p_A = p_B + p_C \qquad p_A^2 = p_B^2 + p_C^2 + 2 p_B \cdot p_C$$

$$m_A^2 c^2 = m_B^2 c^2 + m_C^2 c^2 + 2\left(\frac{E_B}{c}, \quad \vec{p}_B \right) \cdot \left(\frac{E_C}{c}, \quad \vec{p}_C \right)$$

$$m_A^2 c^2 = m_B^2 c^2 + m_C^2 c^2 + 2\left\{ \frac{E_B E_C}{c^2} - \vec{p}_B \cdot \vec{p}_C \right\} \quad = \quad m_B^2 c^2 + m_C^2 c^2 + 2\left\{ \frac{E_B (E_A - E_B)}{c^2} + \frac{E_B^2 - m_B^2 c^4}{c^2} \right\}$$

$$m_A^2 c^2 - m_B^2 c^2 - m_C^2 c^2 = 2\left\{ \frac{E_B E_A}{c^2} - \frac{E_B^2}{c^2} + \frac{E_B^2}{c^2} - \frac{m_B^2 c^4}{c^2} \right\} = 2 E_B m_A - 2 m_B^2 c^2$$

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2 m_A} c^2 \qquad E_C = \frac{m_A^2 + m_C^2 - m_B^2}{2 m_A} c^2$$

Second part of question asks for the magnitude of the final state momenta.
From the first part (previous slide) we have:

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2. \quad \text{Use} \quad \frac{E_B^2}{c^2} = |\vec{p}_B|^2 + m_B^2 c^2 \quad \Rightarrow \quad |\vec{p}_B|^2 = \frac{E_B^2}{c^2} - m_B^2 c^2$$

$$|\vec{p}_B|^2 = \frac{E_B^2}{c^2} - m_B^2 c^2 \quad \Rightarrow \quad |\vec{p}_B|^2 = \left[\frac{m_A^2 + m_B^2 - m_C^2}{2m_A}\right]^2 - m_B^2 c^2$$

$$= \frac{m_A^4 + m_B^4 + m_C^4 + 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}{4m_A^2} c^2 - m_B^2 c^2$$

$$= \left\{\frac{m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}{4m_A^2} c^2\right\} + \frac{2m_A^2 m_B^2}{4m_A^2} - m_B^2 c^2 \quad = \quad \left\{\quad\right\} + \frac{1}{2} m_B^2 c^2 - m_B^2 c^2$$

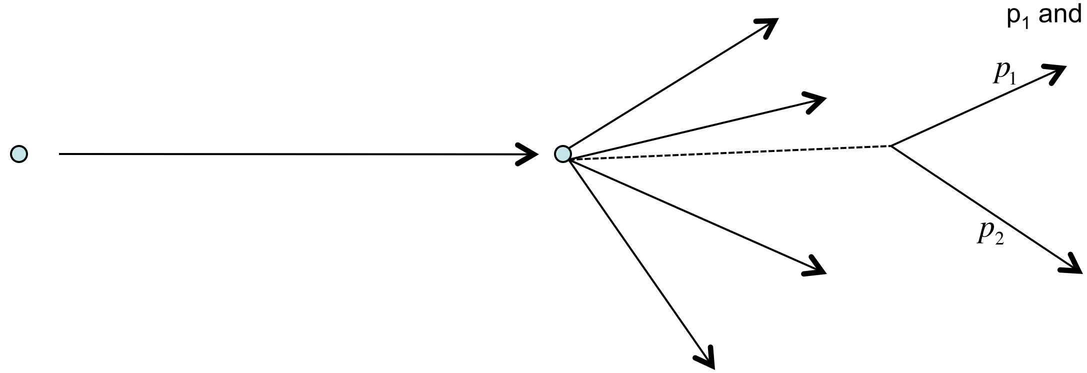
$$\left\{\quad\right\} - \frac{1}{2} m_B^2 c^2 \quad = \quad \left\{\quad\right\} + \frac{2m_A^2 m_B^2}{4m_A^2} c^2 \quad = \quad \frac{m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}{4m_A^2} c^2$$

$$|\vec{p}_B| = \frac{c}{2m_A}\sqrt{\lambda(m_A^2, m_B^2, m_C^2)} = |\vec{p}_C| \qquad \lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.$$

New Particle Searches in Collision Experiments

Consider again the process $p + p \rightarrow X$

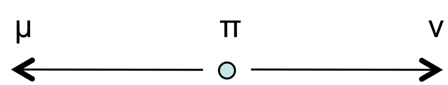
To avoid confusion, let me point out that p_1 and p_2 are four-momenta, not protons



$$\left(p_1 + p_2\right)^2 = M^2 c^2$$

where *M* is the invariant mass of the system made up of particles 1 and 2 (from the decay of a particle produced in the pp collision). This gives the mass of the decaying particle (the flight distance may or may not be observable)

Recall charged pion decay:



$$\left(p_\mu + p_\nu\right)^2 = m_\pi^2 c^2$$

In samples of many events one can search for some hypothetical (or known) particle into some particular final state (say B+C+D) by selecting all combinations of tracks consistent with being B,C and D and calculating the invariant mass. One can hope to see a signal peak atop a background from incorrect (uncorrelated) B+C+D combinations .

Invariant Mass Distributions: Examples

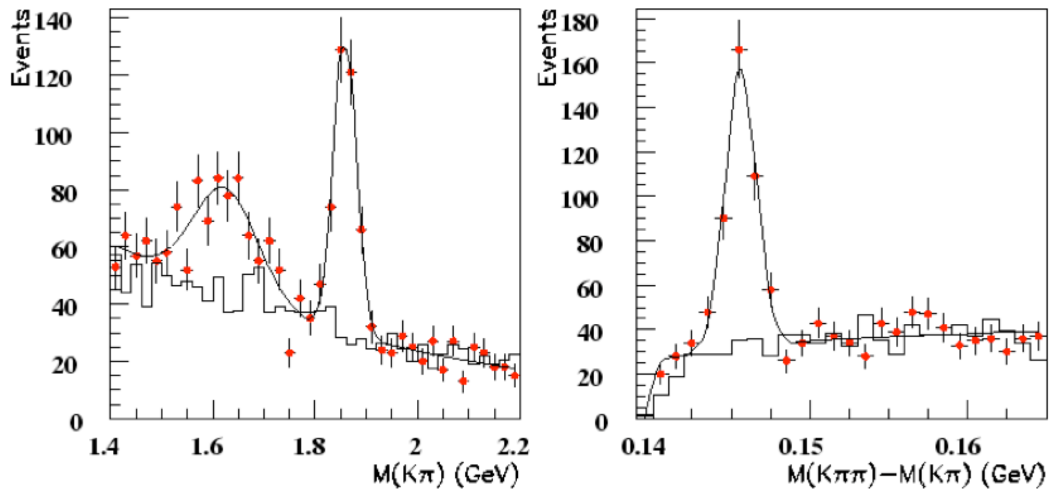
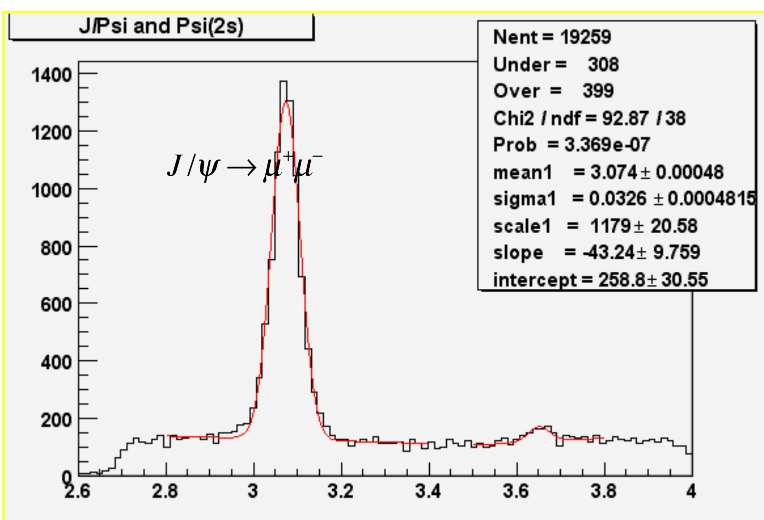
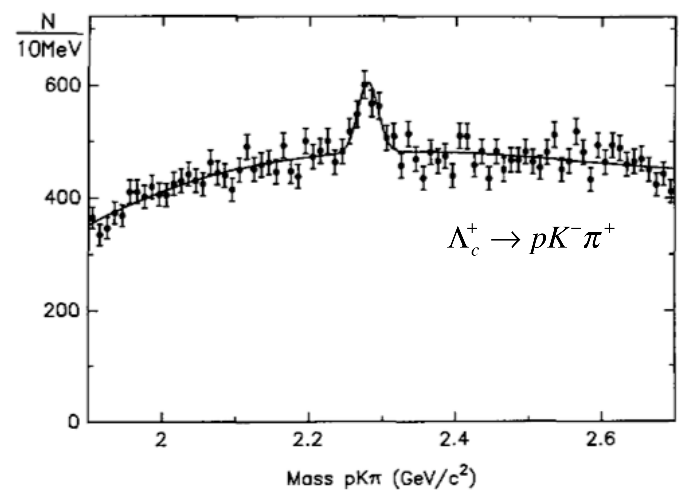


Figure 1.13: *The invariant mass signature of D^* events in the ZEUS '95 DIS data sample. The left plot shows the reconstructed D^0 mass at 1.86 GeV, the right plot shows the reconstructed difference of the D^* and D^0 mass, 2.010-1.864=0.146 GeV. The peak in the $K\pi$ mass distribution at 1.65 GeV is a reflection of the $D^0 \to K^\pm\pi^\mp\pi^0$ decay channel.*

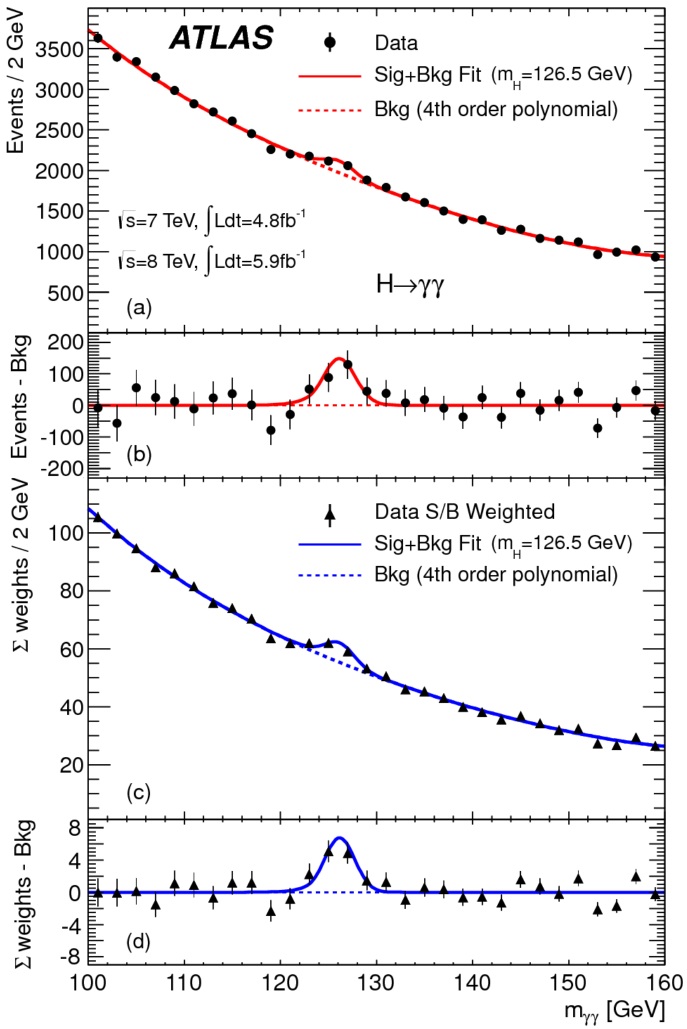


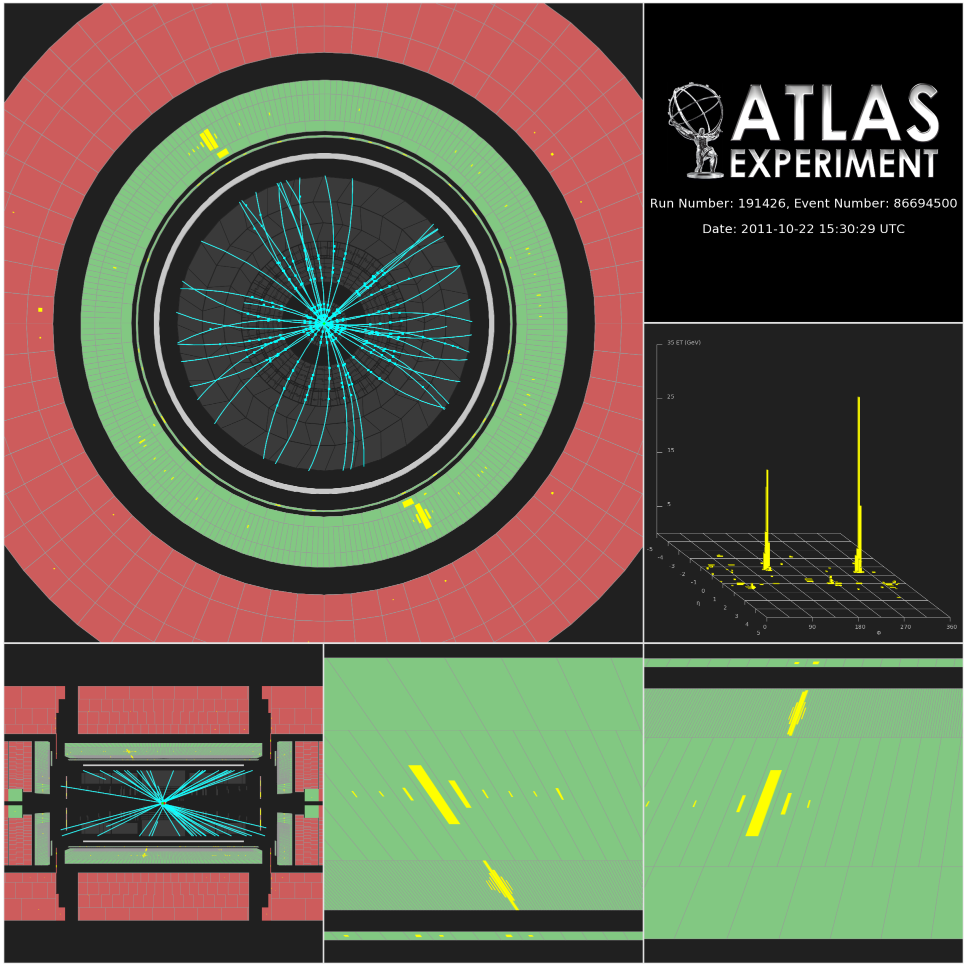
Invariant mass distribution of $\mu^+\mu^-$ combinations (CDF)



Invariant mass distribution of $pK^-\pi^+$ combinations (ARGUS)

ATLAS Higgs to γγ





ATLAS $H \to ZZ^{*} \to \mu^{+}\mu^{-}\mu^{+}\mu^{-}$