Phy489 Lecture 7

Clebsch-Gordan Coefficients

$$|\, j_1 m_1 > |\, j_2 m_2 > = \sum_{j=|j_1-j_2|}^{j_1+j_2} C^{j\ \ j_1\ \ j_2}_{m\ m_1\ m_2}\ |\, jm > \qquad m = m_1 + m_2$$

Two systems with spin j_1 and j_2 and z components m_1 and m_2 can combine to give a system which (quantum-mechanically) is a linear combination of states having spin j from $|\, j_1 - j_2 \,|$ to $j_1 + j_2$ (in integer steps) each having a z component of $m = m_1 + m_2$.

So if we make a measurement of the total angular momentum of a state made up of the two spin states as defined above, the square of the Clebsch-Gordan (CG) coefficient

$$C^{j\ \ j_1\ \ j_2}_{m\ m_1\ m_2}$$

represents the probability of obtaining a measurement of $J^2 = j(j+1)\,\hbar^2$

Tables of Clebsch-Gordan Coefficients

j_1 x j_2 Table for combining spin j_1 with spin j_2

$$|\, j_1 m_1 >|\, j_2 m_2 > = \sum_{j=|j_1-j_2|}^{j_1+j_2} C^{j\ j_1\ j_2}_{m\ m_1\ m_2}\,|\, jm > \qquad m = m_1 + m_2$$

		J	**J**	**. . .**
		m	**m**	**. . .**
m_1	**m_2**	$C^{j\ j_1\ j_2}_{m\ m_1\ m_2}$		
m_1	**m_2**			
.	.			
.	.			
.	.			
.	.			

Problem from Last Time…..

4.12

An electron in a hydrogen atom is in a state with orbital angular momentum number $\ell = 1$. If the total angular momentum quantum number is *j=3/2*, and the *z* component of total angular momentum is $\hbar / 2$ what is the probability of finding the electron with $m_s = +1/2$?

Need ½ x 1 Clebsch-Gordan Table (spin ½ + $\ell = 1$)

Clesch Gordan Coefficients from PDG Listings

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

	J	J	...
	M	M	...
m_1 m_2	Coefficients		
m_1 m_2			
. .			

1/2×1/2

	1 +1
+1/2 +1/2	1

	1 0	0 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

	1 −1
−1/2 −1/2	1

1×1/2

	3/2 +3/2
+1 +1/2	1

	3/2 +1/2	1/2 +1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3

	3/2 −1/2	1/2 −1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3

	3/2 −3/2
−1 −1/2	1

2×1

	3 +3
+2 +1	1

	3 +2	2 +2
+2 0	1/3	2/3
+1 +1	2/3	−1/3

	3 +1	2 +1	1 +1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	2/5	−1/2	1/10

	3 0	2 0	1 0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10

	3 −1	2 −1	1 −1
0 −1	2/5	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5

	3 −2	2 −2
−1 −1	2/3	1/3
−2 0	1/3	−2/3

	3 −3
−2 −1	1

1×1

	2 +2
+1 +1	1

	2 +1	1 +1
+1 0	1/2	1/2
0 +1	1/2	−1/2

	2 0	1 0	0 0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

	2 −1	1 −1
0 −1	1/2	1/2
−1 0	1/2	−1/2

	2 −2
−1 −1	1

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

2×1/2

	5/2 +5/2
+2 +1/2	1

	5/2 +3/2	3/2 +3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5

	5/2 +1/2	3/2 +1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5

	5/2 −1/2	3/2 −1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5

	5/2 −3/2	3/2 −3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5

	5/2 −5/2
−2 −1/2	1

3/2×1/2

	2 +2
+3/2 +1/2	1

	2 +1	1 +1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4

	2 0	1 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

	2 −1	1 −1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4

	2 −2
−3/2 −1/2	1

3/2×1

	5/2 +5/2
+3/2 +1	1

	5/2 +3/2	3/2 +3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	−2/5

	5/2 +1/2	3/2 +1/2	1/2 +1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6

	5/2 −1/2	3/2 −1/2	1/2 −1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2

	5/2 −3/2	3/2 −3/2
−1/2 −1	3/5	2/5
−3/2 0	2/5	−3/5

	5/2 −5/2
−3/2 −1	1

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

Combine 2 spin ½ particles as in Griffiths example 4.4

Four possible outcomes for |J,M>

$\frac{1}{2} \times \frac{1}{2}$

		1			
		+ 1			
+ 1/2	+ 1/2	1			
			1	0	
			0	0	
+ 1/2	- 1/2		1/2	1/2	1
- 1/2	+ 1/2		1/2	- 1/2	- 1
- 1/2	- 1/2				1

Combine 2 spin ½ particles as in Griffiths example 4.4

$\frac{1}{2} \times \frac{1}{2}$		1			
		+ 1	1	0	
+ 1/2	+ 1/2	1	0	0	
+ 1/2	- 1/2		1/2	1/2	1
- 1/2	+ 1/2		1/2	- 1/2	- 1
- 1/2	- 1/2				1

$$\left|\frac{1}{2}\ \boxed{\frac{1}{2}}\right\rangle \left|\frac{1}{2}\ \boxed{\frac{1}{2}}\right\rangle = \left|1\,1\right\rangle$$

Combine 2 spin ½ particles as in Griffiths example 4.4

$\frac{1}{2} \times \frac{1}{2}$

		1			
		+ 1	1	0	
+ 1/2	+ 1/2	1	0	0	
+ 1/2	- 1/2		1/2	1/2	1
- 1/2	+ 1/2		1/2	- 1/2	- 1
			- 1/2	- 1/2	1

$$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle \left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \left|1\,1\right\rangle$$

Combine 2 spin ½ particles as in Griffiths example 4.4

$$\frac{1}{2} \times \frac{1}{2}$$

		1			
		+ 1	1	0	
			0	0	
+ 1/2	+ 1/2	1			
+ 1/2	- 1/2		1/2	1/2	1
- 1/2	+ 1/2		1/2	- 1/2	- 1
			- 1/2	- 1/2	1

$$\left|\frac{1}{2}\ \boxed{\frac{1}{2}}\right\rangle \left|\frac{1}{2}\ \boxed{\frac{1}{2}}\right\rangle = \left|\boxed{1\,1}\right\rangle$$

Combine 2 spin ½ particles as in Griffiths example 4.4

$\frac{1}{2} \times \frac{1}{2}$

		1			
		+ 1	1	0	
+ 1/2	+ 1/2	1	0	0	
+ 1/2	- 1/2		1/2	1/2	1
- 1/2	+ 1/2		1/2	- 1/2	- 1
			- 1/2	- 1/2	1

$$\left|\frac{1}{2}\;\frac{1}{2}\right\rangle \left|\frac{1}{2}\;\frac{1}{2}\right\rangle = \left|1\;1\right\rangle$$

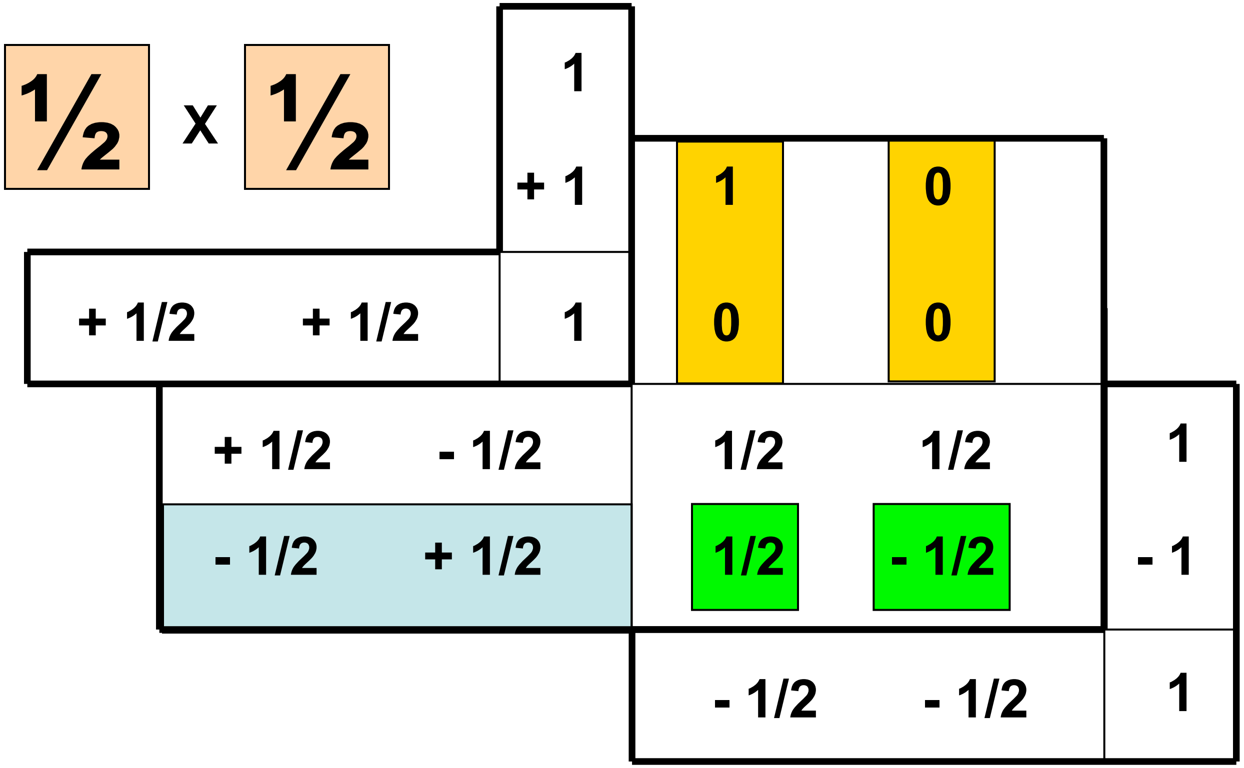
Combine 2 spin ½ particles as in Griffiths example 4.4

½ x ½

		1	1	0	
		+ 1	0	0	
+ 1/2	+ 1/2	1			
+ 1/2	- 1/2		1/2	1/2	1
- 1/2	+ 1/2		1/2	- 1/2	- 1
- 1/2	- 1/2				1

$$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle \left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left|1\,0\right\rangle + \frac{1}{\sqrt{2}}\left|0\,0\right\rangle$$

Combine 2 spin ½ particles as in Griffiths example 4.4



$$\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle=\frac{1}{\sqrt{2}}\left|1\,0\right\rangle-\frac{1}{\sqrt{2}}\left|0\,0\right\rangle$$

Combine 2 spin ½ particles as in Griffiths example 4.4

$\frac{1}{2} \times \frac{1}{2}$

		1			
		+ 1	1	0	
+ 1/2	+ 1/2	1	0	0	
+ 1/2	- 1/2		1/2	1/2	1
- 1/2	+ 1/2		1/2	- 1/2	- 1
			- 1/2	- 1/2	1

$$\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \left|\, 1\ -1\, \right\rangle$$

		1			
		+ 1	1	0	
+ 1/2	+ 1/2	1	0	0	
+ 1/2	- 1/2		1/2	1/2	1
- 1/2	+ 1/2		1/2	- 1/2	- 1
- 1/2	- 1/2				1

$$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \left|1\,1\right\rangle$$

$$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left|1\,0\right\rangle + \frac{1}{\sqrt{2}}\left|0\,0\right\rangle$$

$$\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left|1\,0\right\rangle - \frac{1}{\sqrt{2}}\left|0\,0\right\rangle$$

$$\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \left|\ 1\ -1\ \right\rangle$$

Can re-write this in terms of states of definite J,M

		1			
		+ 1	1	0	
+ 1/2	+ 1/2	1	0	0	
+ 1/2	- 1/2		1/2	1/2	1
- 1/2	+ 1/2		1/2	- 1/2	- 1
- 1/2	- 1/2				1

Spin triplet

Symmetric under $1 \longleftrightarrow 2$

$$|1\,1\rangle = \left|\frac{1}{2}\;\frac{1}{2}\right\rangle \left|\frac{1}{2}\;\frac{1}{2}\right\rangle$$

$$|1\,0\rangle = \frac{1}{\sqrt{2}}\left|\frac{1}{2}\;\frac{1}{2}\right\rangle\left|\frac{1}{2}\;-\frac{1}{2}\right\rangle + \frac{1}{\sqrt{2}}\left|\frac{1}{2}\;-\frac{1}{2}\right\rangle\left|\frac{1}{2}\;\frac{1}{2}\right\rangle$$

$$|1\;-1\rangle = \left|\frac{1}{2}\;-\frac{1}{2}\right\rangle \left|\frac{1}{2}\;-\frac{1}{2}\right\rangle$$

$$|0\,0\rangle = \frac{1}{\sqrt{2}}\left|\frac{1}{2}\;\frac{1}{2}\right\rangle\left|\frac{1}{2}\;-\frac{1}{2}\right\rangle - \frac{1}{\sqrt{2}}\left|\frac{1}{2}\;-\frac{1}{2}\right\rangle\left|\frac{1}{2}\;\frac{1}{2}\right\rangle$$

Spin singlet

Anti -symmetric under $1 \longleftrightarrow 2$

Spin ½ Formalism

Write spin up and spin down states as two-component "spinors"

"spin up" $\left|\frac{1}{2}\ \ \frac{1}{2}\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **"spin down"** $\left|\frac{1}{2}\ \ -\frac{1}{2}\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Most general state of a spin ½ system is then (a superposition of the two):

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \alpha, \beta \text{ complex}$$

Probability that measurement of S_z yields $\hbar/2$ (e.g. spin up) is α^2

Probability that measurement of S_z yields $-\hbar/2$ (e.g. spin down) is β^2

Spin Matrices

What about measurements of S_x and S_y ?

Must also be either ± ħ/2, but with what probability ?

To each component of ***S*** there is associated a 2x2 matrix [Pauli spin matrix]

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Each has eigenvalues of ± ħ/2 [*e.g.* from $\det(\hat{S}_x - \lambda I) = 0$]

Normalized eigenvectors for S_x are $\chi_\pm = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{a}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{b}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad a,b \ \textit{complex} \quad \left.\right\} \quad \begin{aligned} a &= \frac{1}{\sqrt{2}}(\alpha + \beta) \\ b &= \frac{1}{\sqrt{2}}(\alpha - \beta) \end{aligned}$$

Probability of measuring $S_x = \frac{\hbar}{2}$ is $|a|^2$

Flavour Symmetries (internal symmetries)

Note that the proton (p) and neutron (n) have very similar masses:

m_p = 938.28 MeV/c^2, m_n = 939.57 MeV/c^2

Heisenberg: try viewing this as two different states of the same particle (the nucleon, N).

$$N = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Reminiscent of spin ½ formalism.

p and n are "related" by a rotation in an abstract space referred to as isospin space (in analogy to spin). Adopt spin formalism to deal with systems of particles with different isospin (first for isospin ½ in analogy to spin ½).

For any state we can define the isospin $\mathbf{I}$ and the third component I_3 (as for J).

We label the components as I_1, I_2 and I_3 to make clear that we are not talking about components in coordinate space, but in some abstract space.

Warning….

Isospin has NOTHING to do with real spin (e.g. angular momentum). The (confusing, so perhaps inappropriate) name is historical and comes from the similarity of the mathematical formalism, to that for handling spin.

A “rotation in isopin space” does NOT have anything to do with rotations in physical (coordinate) space.

Isospin

We have seen that strong and electromagnetic interactions conserve strangeness, charm etc. Only the charged weak interaction can change quark flavour….there are no flavour changing neutral currents (FCNC) in the Standard Model.

Isospin is just a version of conservation of quark number for u, d quarks, arising from the fact that their masses are so similar.

Consider proton and neutron as members of an isospin doublet (the nucleon)

e.g. total isospin I = 1/2

$$p = \left| \frac{1}{2} \;\; \frac{1}{2} \right\rangle \qquad n = \left| \frac{1}{2} \;\; -\frac{1}{2} \right\rangle$$

Heisenberg's proposal: the strong interaction is invariant under rotations in "isospin space".

Noether's theorem then implies that isospin is conserved in all strong interactions.

Other particles fall into *different* isospin multiplets (each with 2I + 1 elements)

For example, pions have isospin I = 1 (so three charge states):

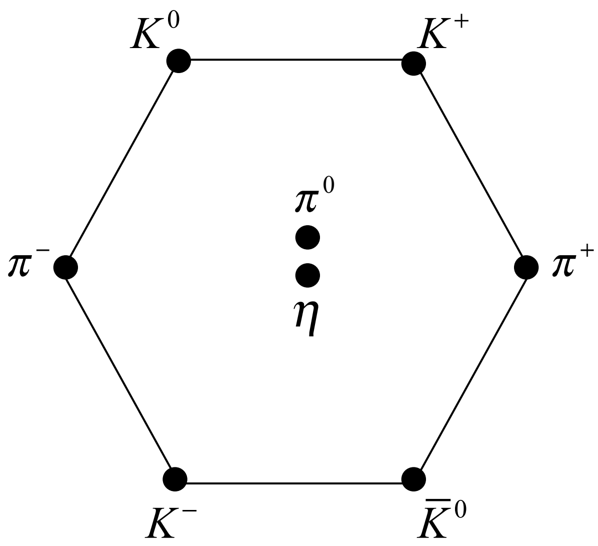
$$\pi^+ = \left|1 \;\, 1\right\rangle \qquad \pi^0 = \left|1 \;\, 0\right\rangle \qquad \pi^- = \left|1 \;\, -1\right\rangle$$

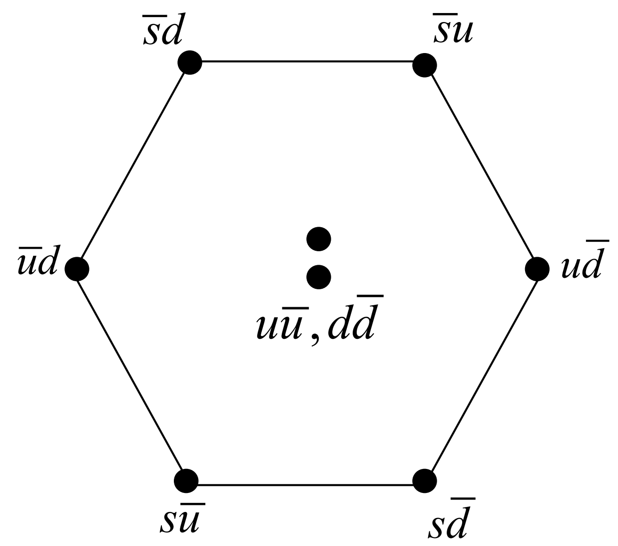
Isospin

Rules for isopin assignment are a little obscure, having to do with the positions particles occupy in the “eightfold way” multiplets discussed in Chapter 1.

Isospin relates states with different electric charges but very similar masses.

(e.g. Hadronic states differing only in their u,d quark content)





Isospin at Quark Level

$$u = \left| \frac{1}{2} \quad \frac{1}{2} \right\rangle \qquad d = \left| \frac{1}{2} \quad -\frac{1}{2} \right\rangle \qquad s = \left| 0 \;\; 0 \right\rangle$$

Other quark flavours (c,b,t) are also isospin singlets, like the strange quark

Mesons containing (only) u,d quarks can therefore have isospin 0 or 1

$$\pi^- = \bar{u}d \qquad \pi^0 = u\bar{u}, d\bar{d} \qquad \pi^+ = u\bar{d}$$

isospin triplet I = 1

$$\eta = u\bar{u}, d\bar{d}$$

isospin singlet I = 0

Mesons containing only one u,d quark can only have isospin = ½

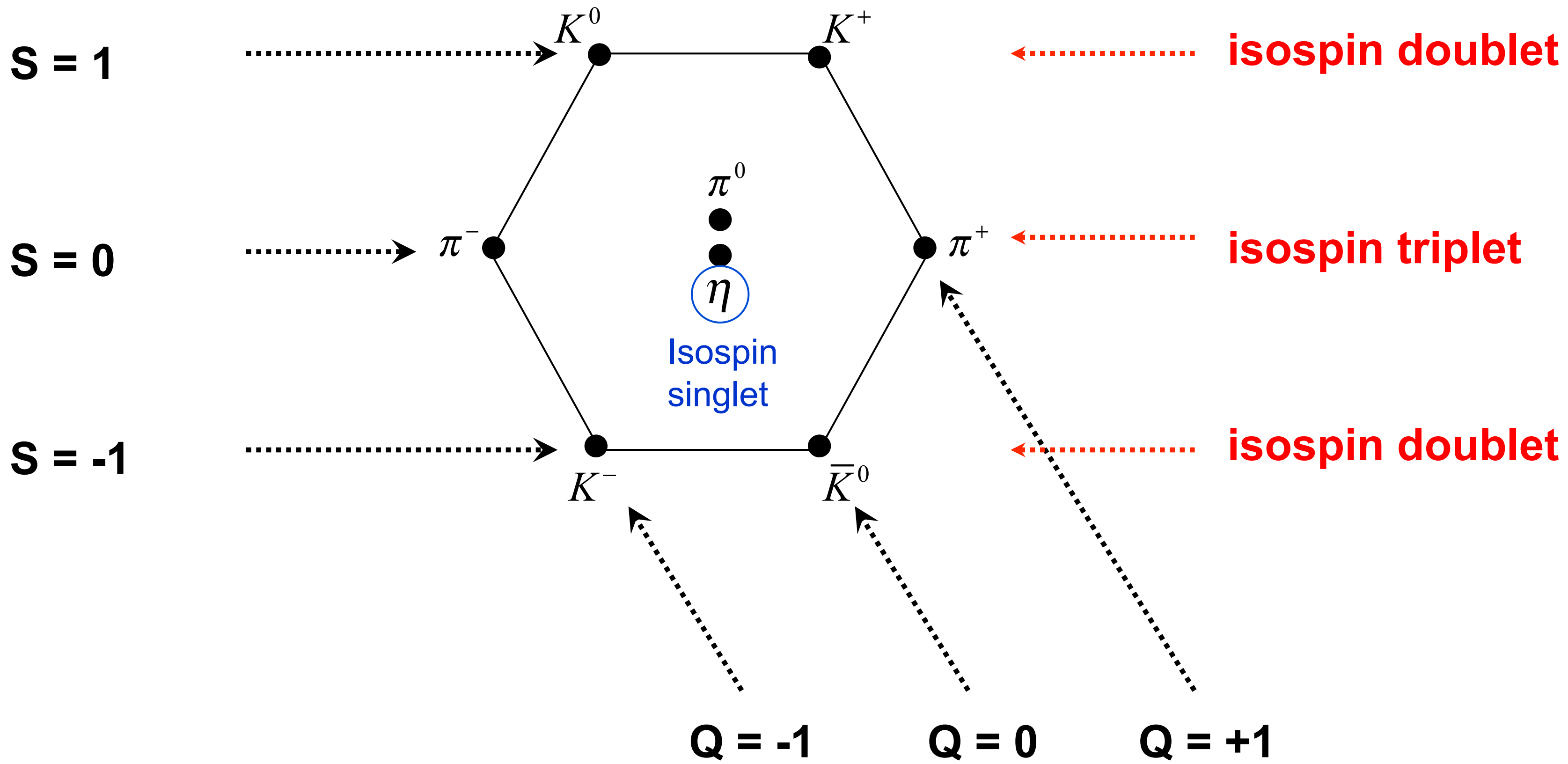
$$K^0 = u\bar{s} \qquad K^+ = d\bar{s}$$

form an isospin doublet

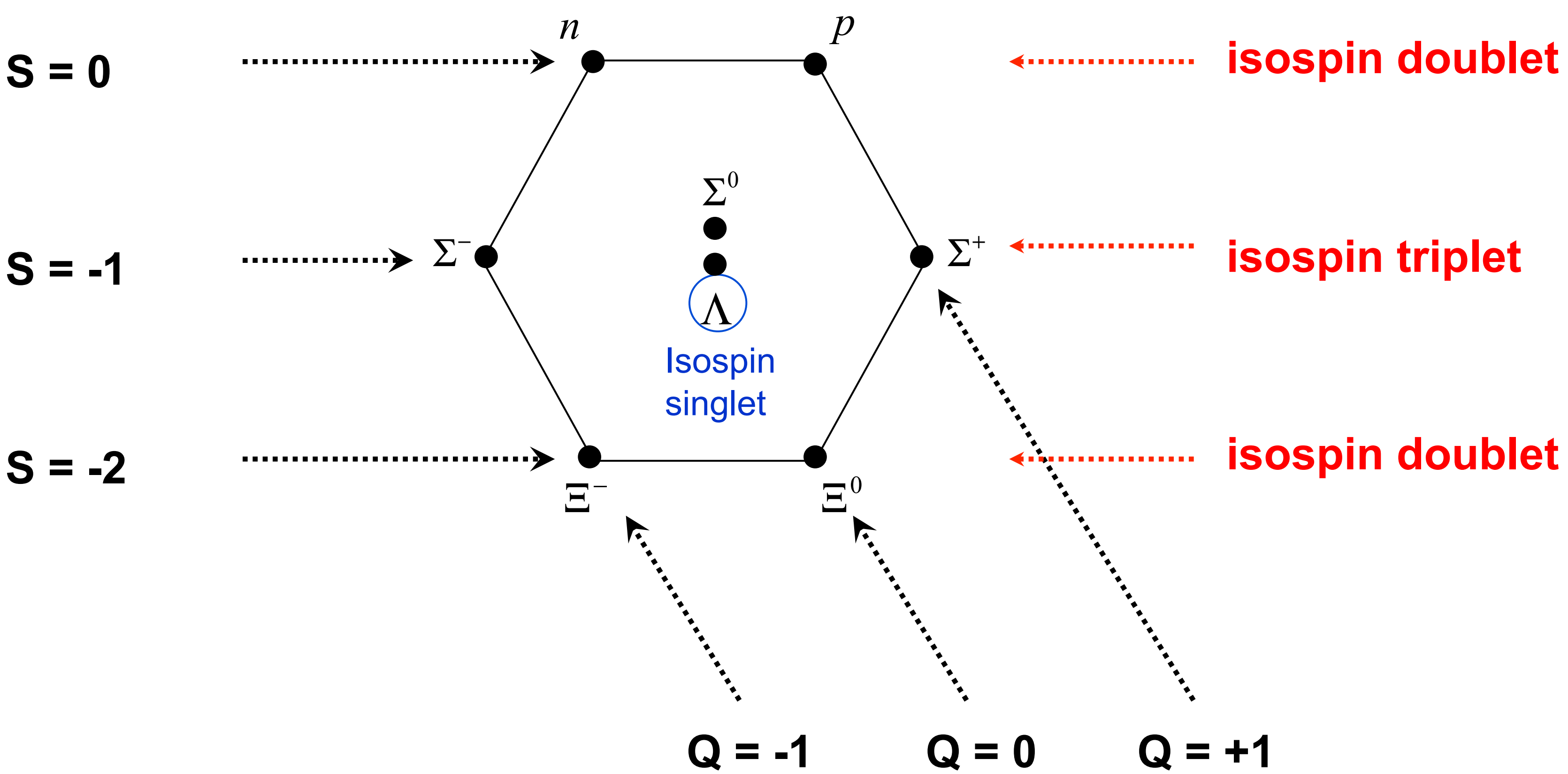
$$\bar{K}^0 = \bar{u}s \qquad K^- = \bar{d}s$$

form a *separate* isospin doublet

The “Eightfold Way” Meson Octet (u,d,s)



The “Eightfold Way” Baryon Octet (u,d,s)



Application example: pion-nucleon scattering

Six elastic $\pi N \to \pi N$ processes (e.g. same particles in and out of scattering)

$$(a)\ \pi^+ + p \to \pi^+ + p \qquad (b)\ \pi^0 + p \to \pi^0 + p$$
$$(c)\ \pi^- + p \to \pi^- + p \qquad (d)\ \pi^+ + n \to \pi^+ + n$$
$$(e)\ \pi^0 + n \to \pi^0 + n \qquad (f)\ \pi^- + n \to \pi^- + n$$

and 4 charge-exchange processes

$$(g)\ \pi^+ + n \to \pi^0 + p \qquad (h)\ \pi^0 + p \to \pi^+ + n$$
$$(i)\ \pi^0 + n \to \pi^- + p \qquad (j)\ \pi^- + p \to \pi^0 + n$$

Pions carry $\mathrm{I} = 1$ and nucleons, $\mathrm{I} = 1/2$ so the total isospin can be either 1/2 or 3/2

Since isospin is conserved (in these strong interactions) there are two distinct (quantum-mechanical) amplitudes:

$\mathcal{M}_1$ for $\mathrm{I} = 1/2$ and $\mathcal{M}_3$ for $\mathrm{I} = 3/2$

Takes an isospin ½ initial state to an isospin ½ final state.

Takes an isospin ½ initial state to an isospin ½ final state.

Pion-Nuclon Scattering Amplitudes

$$\pi^+ + p: \quad |1 \quad 1\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \left|\frac{3}{2}\ \frac{3}{2}\right\rangle$$

$$\pi^0 + p: \quad |1 \quad 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$$

$$\pi^- + p: \quad |1 \ -1\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$$

$$\pi^+ + n: \quad |1 \quad 1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$$

$$\pi^0 + n: \quad |1 \quad 0\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$$

$$\pi^- + n: \quad |1 \ -1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \left|\frac{3}{2}\ -\frac{3}{2}\right\rangle$$

See Perkins Introduction to High Energy Physics section on "Isospin in the pion nucleon system" for more detail. (Section number varies from edition to edition; it is 3.13 in the 4th edition)

$$(a)\ \pi^+ + p \rightarrow \pi^+ + p$$

$$(f)\ \pi^- + n \rightarrow \pi^- + n$$

Both are pure I = 3/2 on either side of the reaction. Both proceed purely via the I = 3/2 amplitude $\mathcal{M}_3$

Pion-Nucleon Scattering Amplitudes

$$\pi^- + p: \quad |1\ -1\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$$

$$\pi^0 + n: \quad |1\ \ 0\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$$

$(c)\ \pi^- + p \rightarrow \pi^- + p$ $\qquad$ $\mathcal{M}_c = \frac{1}{3}\mathcal{M}_3 + \frac{2}{3}\mathcal{M}_1$

$(j)\ \pi^- + p \rightarrow \pi^0 + n$ $\qquad$ $\mathcal{M}_j = \sqrt{\frac{2}{3}}\sqrt{\frac{1}{3}}\mathcal{M}_3 - \sqrt{\frac{2}{3}}\sqrt{\frac{1}{3}}\mathcal{M}_1$

Cross-section σ for some process is proportional to $|\mathcal{M}|^2$

Relative Cross-sections for πN Scattering

$$\sigma_a : \sigma_c : \sigma_j = 9\left|\mathcal{M}_3\right|^2 : \left|\mathcal{M}_3 + 2\mathcal{M}_1\right|^2 : 2\left|\mathcal{M}_3 - \mathcal{M}_1\right|^2$$

At E_{CM} = 1232 MeV there is a dramatic bump (increase) in the pion-nucleon scattering cross-section, do to a πN resonance (the Δ).

This is listed amongst the particles on the inside cover of your text

[we will discuss resonance in scattering in a future lecture]

Δ carries isospin 3/2, therefore:

Expect the scattering at this energy to be dominated by the isospin 3/2 channel

$$\sigma_a : \sigma_c : \sigma_j = 9 : 1 : 2$$

Generally, experimentally, it is easiest to measure the total cross-section, so processes (c) and (j) are combined to yield:

$$\frac{\sigma_{tot}\left(\pi^+ + p\right)}{\sigma_{tot}\left(\pi^- + p\right)} = \frac{\sigma_a}{\sigma_c + \sigma_j} = 3$$

Pion-proton Scattering Cross-sections

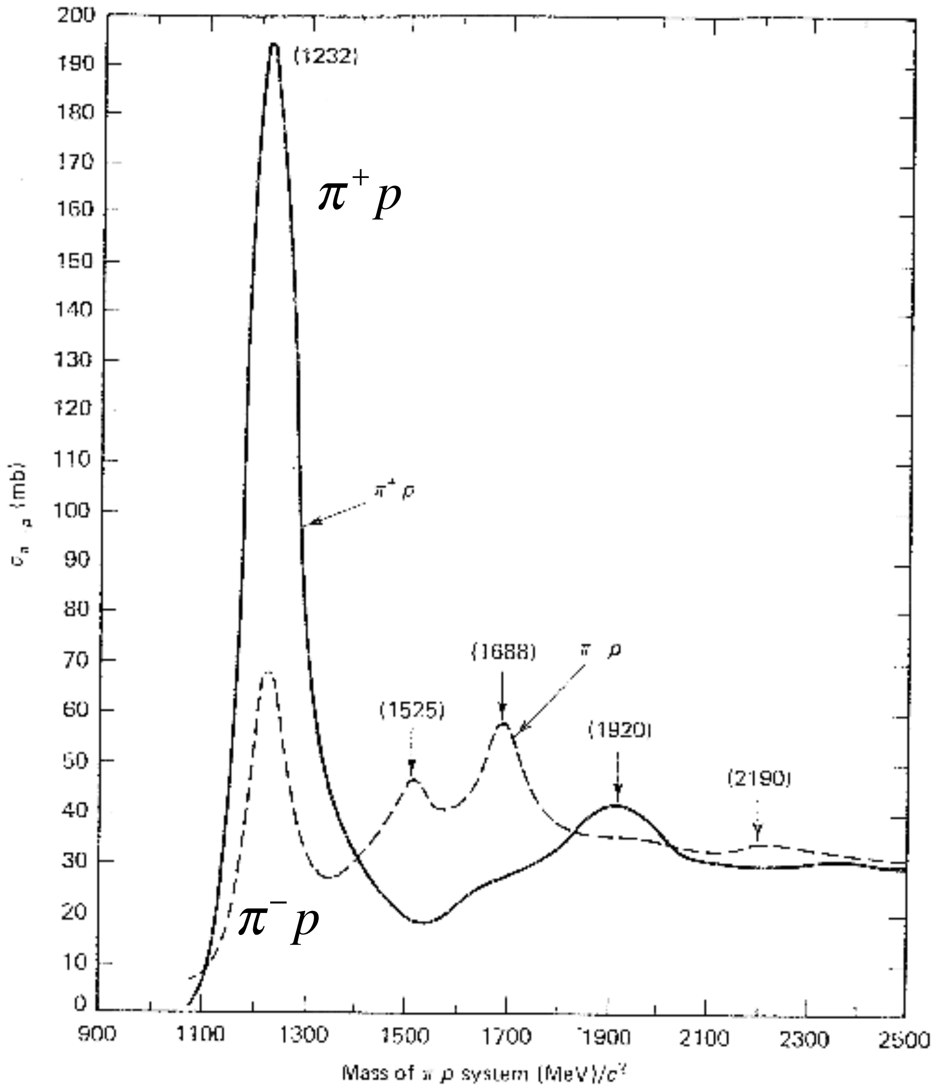


Figure 4.6 Total cross sections for $\pi^+ p$ (solid line) and $\pi^- p$ (dashed line) scattering. (*Source:* S. Gasiorowicz, *Elementary Particle Physics* (New York: Wiley, copyright © 1966, page 294. Reprinted by permission of John Wiley and Sons, Inc.)

Additional "bumps" due to other $\pi^+ p$ or $\pi^- p$ resonances.

The $\pi^+ p$ and $\pi^- p$ cross-sections become approximately equal at high energies. Is that expected?