Phy489 Lecture 9

CP Violation

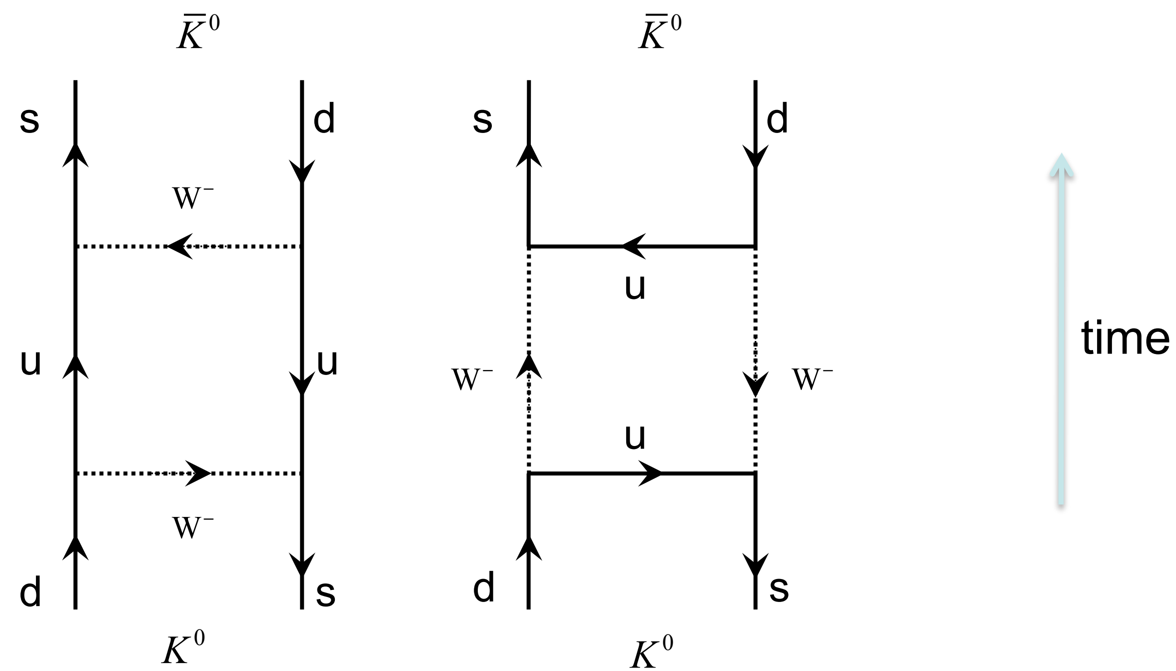
Standard Model contains only left-handed neutrinos and right-handed anti-neutrinos

$$C\left|\nu_L\right\rangle = \left|\bar{\nu}_L\right\rangle$$ charge conjugation not a symmetry of the weak interaction

$$P\left|\nu_L\right\rangle = \left|\nu_R\right\rangle$$ parity also not conserved in weak interactions

$$CP\left|\nu_L\right\rangle = \left|\bar{\nu}_R\right\rangle$$ what about the combined CP transformation…..looks OK ?

Kaon Oscillations (K^0-$\overline{K}^0$ Mixing)



Neutral mesons can “oscillate” back and forth between particle and antiparticle states.

[equivalent diagrams mix D^0 with $\overline{D}^0$, B^0 with $\overline{B}^0$]

This (second-order) process mixes states of strangeness = 1, strangeness = -1.

Neutral kaons are produced in states of definite strangeness (i.e. via the strong interaction)

CP Operations on Neutral Kaons

Neutral kaons are ground state spin-0 $\bar{s}d$ mesons, so are pseudoscalars [$P = (-1)^{\ell+1}$]

$$P|K^0\rangle = -|K^0\rangle \qquad P|\overline{K}^0\rangle = -|\overline{K}^0\rangle \qquad CP|K^0\rangle = -|\overline{K}^0\rangle$$

$$C|K^0\rangle = \ |\overline{K}^0\rangle \qquad C|\overline{K}^0\rangle = \ |K^0\rangle \qquad CP|\overline{K}^0\rangle = -|K^0\rangle$$

Normalized eigenstates of *CP* are therefore:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right) \qquad CP|K_1\rangle = +|K_1\rangle$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right) \qquad CP|K_2\rangle = -|K_2\rangle$$

Decays of Neutral Kaons

If *CP* is conserved by the weak interaction:

K_1 decays only to *CP* = +1 final states (*CP* even)

K_2 decays only to *CP* = -1 final states (*CP* odd)

Neutral kaons are the lightest neutral mesons containing a strange quark.

Cannot decay via strong interaction: $M(K^0) - M(K^-) \sim 4$ MeV/c^2 << M_π

Cannot decay electromagnetically (why not?)

Dominant decays are weak decays (s → u transition) into two or three pions

System of n pions (without orbital angular momentum) has $P = (-1)^n$

$$K_1 \to \pi\pi \qquad M_{K^0} - 2M_\pi \approx 220\ \mathrm{MeV/c^2}$$

$$K_2 \to \pi\pi\pi \qquad M_{K^0} - 3M_\pi \approx 85\ \mathrm{MeV/c^2}$$

Decays of Neutral Kaons

Decay of K_1 takes place 500 times faster than K_2 (phase space considerations)

Imagine a beam of neutral kaons (produced via the strong interaction)

$$\left|K^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K_1\right\rangle + \left|K_2\right\rangle\right)$$

Mean decay length (in lab frame) is $\gamma\beta c\tau$ (γ the same for K_1 and K_2) so mean decay length of K_2 is ~500 times longer that for K_1

K_1 component of beam will decay very quickly after production.

Expect to see many $K^0 \rightarrow \pi\pi$ decays near the production point

Far downstream expect an almost pure beam of K_2

CP Violation in Neutral Kaon Decays

By going very far downstream from the K^0 beam production point, expect an arbitrarily pure beam of K_2; if *CP* is indeed conserved by the weak interaction will observe only the three π final state very far from the production point.

Cronin and Fitch, 1964 (Nobel Prize 1980) performed this experiment.

Found small admixture (1/500) of 2π decays very far from the production point (no matter how far you go……this is the asymptotic value).

Long-lived neutral kaon state (K^0 Long) is NOT a CP eigenstate:

$$\left|K_L^0\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\left|K_2\right\rangle + \varepsilon\left|K_1\right\rangle\right)$$

CP is therefore NOT a good symmetry of the weak interaction, although the level at which it is "*broken*" (or violated) is rather small ($\varepsilon = 2.3 \times 10^{-3}$).

Other Manifestations of *CP* Violation in Kaon System

If K_L^0 were a CP eigenstate (eg ε = 0) then the two decays

$$K_L^0 \rightarrow \pi^- e^+ \nu_e \qquad\qquad K_L^0 \rightarrow \pi^+ e^- \overline{\nu}_e$$

Would occur with equal probability: instead

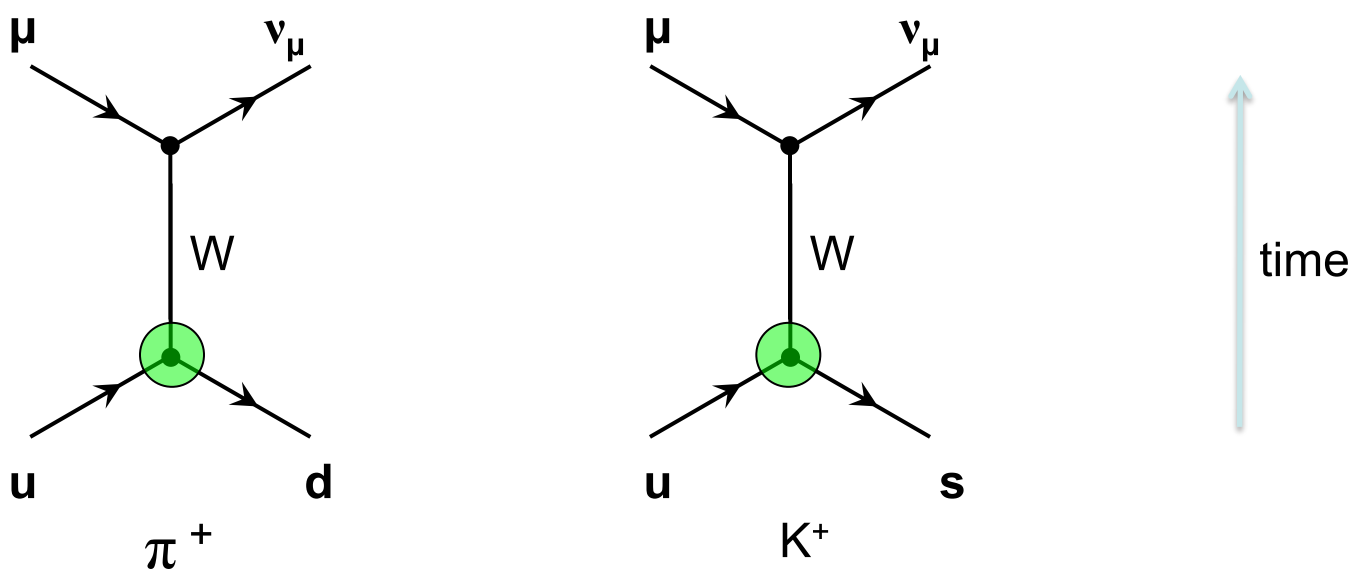
$$\frac{BR(K_L^0 \rightarrow \pi^- e^+ \nu_e) - BR(K_L^0 \rightarrow \pi^+ e^- \overline{\nu}_e)}{BR(K_L^0 \rightarrow \pi^- e^+ \nu_e) + BR(K_L^0 \rightarrow \pi^+ e^- \overline{\nu}_e)} = .0033$$

Quark Mixing and the Cabibbo Angle

Weak decays of s quarks requires su coupling (i.e. generational transition) unlike in pion decays (for example).

Cabibbo introduced quark mixing to explain relative branching ratios weak decays of mesons into leptons:

Will see later that we need "form factors" to describe how this takes place in a bound state



Relative strengths imply smaller coupling at **suW** vertex than at **udW**

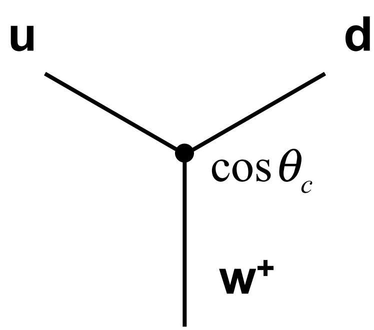
Quark Mixing and the Cabibbo Angle

Cabibbo's hypothesis (only u,d, and s quarks were "known" at the time).

$$d' = d\cos\theta_c + s\sin\theta_c$$
$$s' = -d\sin\theta_c + s\cos\theta_c$$

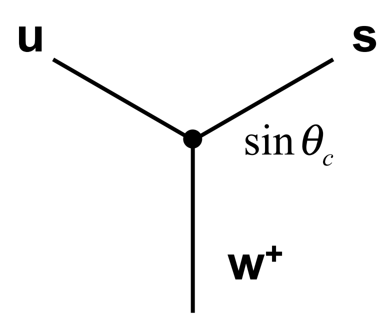
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

θ_c is the Cabibbo angle (~ 13º)



Cabibbo-favoured

$\pi^+ \to \mu^+ \nu_\mu$



Cabibbo-suppressed

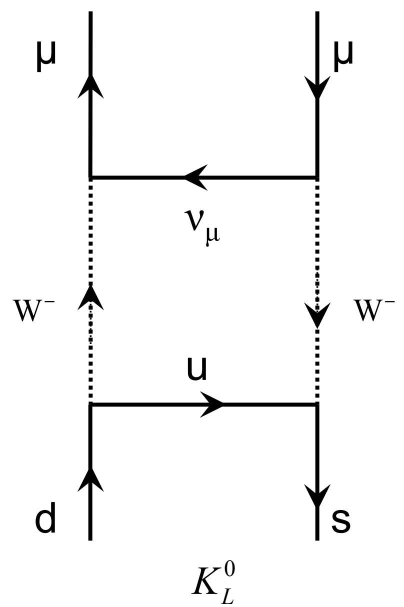
$K^+ \to \mu^+ \nu_\mu$

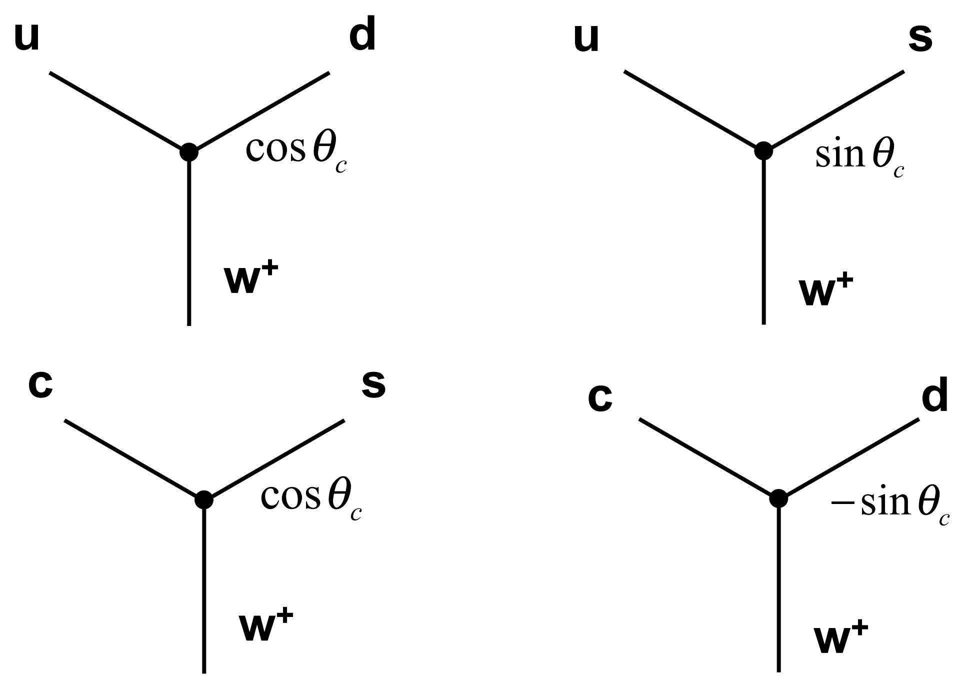
The GIM Mechanism (1970)

Glashow, Iliopoulos and Maini postulate the existence of a fourth quark (charm) on the basis of the measured branching fraction: $BR(K_L^0 \rightarrow \mu^+\mu^-) = 7 \text{ x } 10^{-9}$

This is very small….

Should occur at a much higher rate via the process:

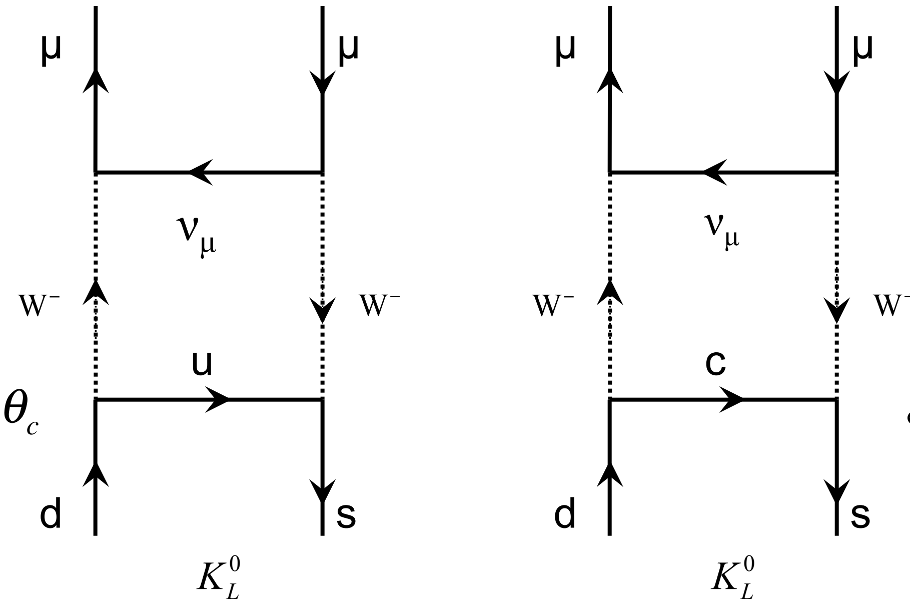




The GIM Mechanism (1970)

Contribution from process with internal u quark exchange partially cancelled by same diagram but with internal c quark exchange

$$\propto V_{ud} V_{us} \sim \sin\theta_c \cos\theta_c$$



$$\propto V_{cd} V_{cs} \sim -\sin\theta_c \cos\theta_c$$

If the masses of the u and c quarks were identical, these contributions would exactly cancel. Since they are not, this merely leads to the (observed) suppression of such processes.

The Kobayashi Maskawa Prediction (1973)

Later we will see, in the weak interaction Lagrangian:

$$(\overline{u},\overline{c},\overline{t})\gamma^{\mu}(1-\gamma^{5})V\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

→ quark mixing matrix

complex N x N matrix has $2N^2$ real free parameters

Unitarity eliminates N^2

2N (number of quarks) can be absorbed into the phases in the quark wavefunctions. Not sensitive to an overall phase, so this eliminates 2N-1

This means $N^2-2N+1 = (N-1)^2$ are required to parametrize the mixing matrix

N(N-1)/2 real mixing angles + (N-1)(N-2)/2 complex phases

A complex phase in the quark mixing matrix allows for CP violation in the SM

However, need N>2 for this.

Based on this, Kobayashi and Maskawa predicted a third generation in 1973

Cabibbo Kobayashi Maskawa Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM mixing matrix for three generation of quarks

1,2,3 label mixing angles, δ is the phase

s_1 = sin(angle 1), c_1 = cos(angle 1) etc

$$\begin{pmatrix} c_1 & s_1 c_1 & s_1 c_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}$$

Three rotations and a phase

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ \lambda & 1-\lambda^2/2 & A\lambda^3 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^3 & 1 \end{pmatrix} + O(\lambda^4)$$

Wolfenstein parametrization emphasizes the magnitudes of the off-diagonal elements

$$\lambda = \sin\theta_c \sim 0.225$$

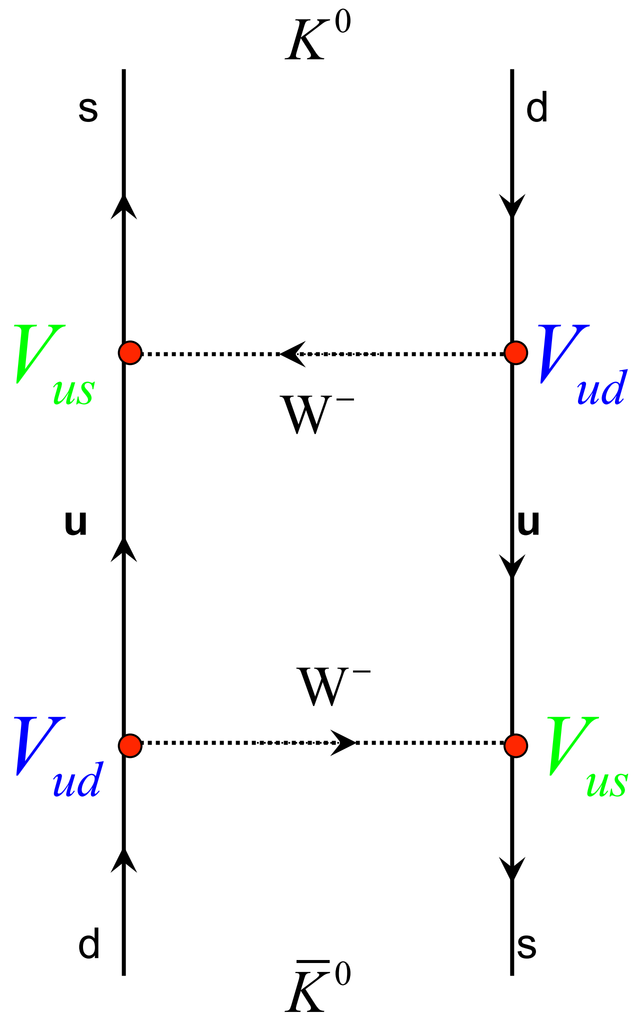
Experimental Determination of CKM Matrix Elements

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9739-0.9751 & 0.221-0.227 & 0.0029-0.0045 \\ 0.221-0.227 & 0.9730-0.9744 & 0.039-0.044 \\ 0.0048-0.014 & 0.037-0.043 & 0.9990-0.9992 \end{pmatrix}$$

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ \lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad \lambda = \sin\theta_c$$

~ 0.225

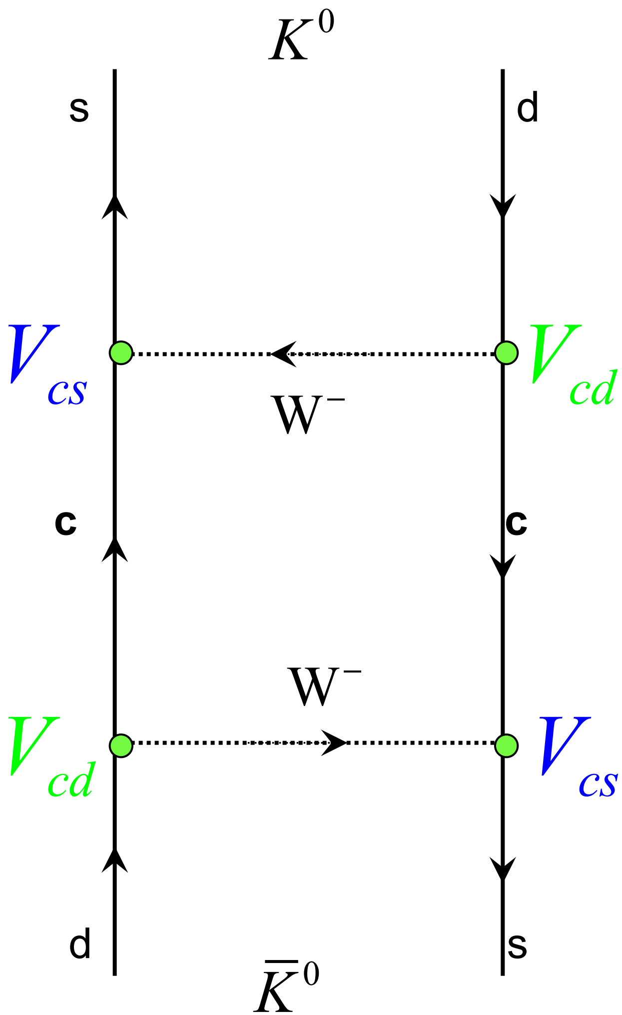
CP Violation in the K System



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

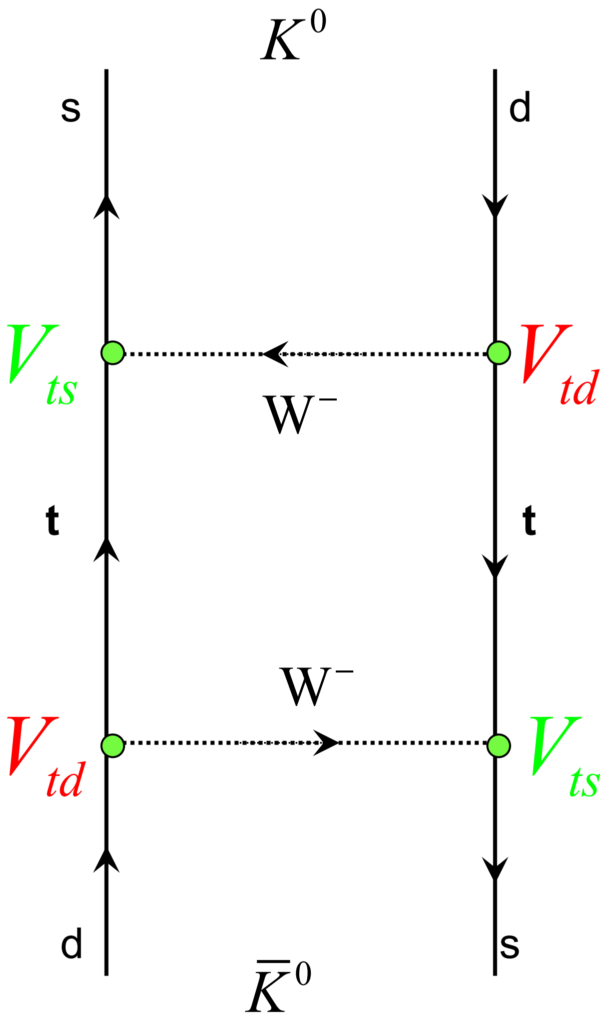
There are also equivalent diagrams with c and t quarks replacing the u quarks in the loop (and diagrams with the quarks going across and the Ws going up – see slide 3).

CP Violation in the K System



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CP Violation in the K System



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

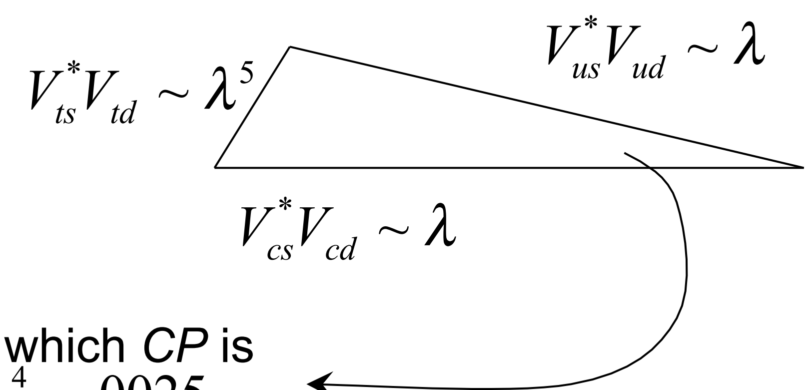
Unitarity Constraints

$$\begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ \lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$$

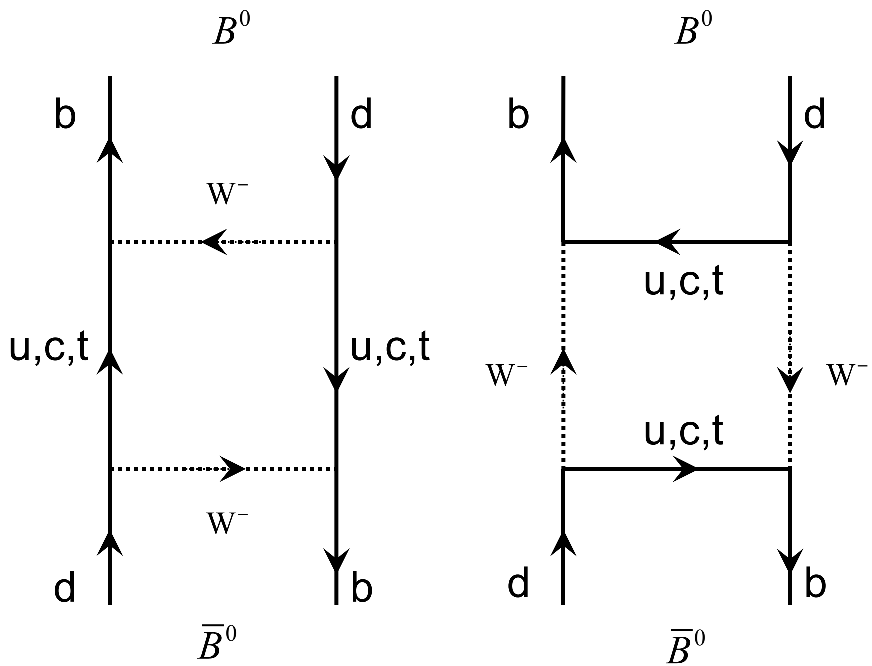
"Unitarity" triangle in complex plane



This angle is a measure of the degree to which *CP* is violated in the kaon system $\sim \lambda^5 / \lambda = \lambda^4 \sim .0025$

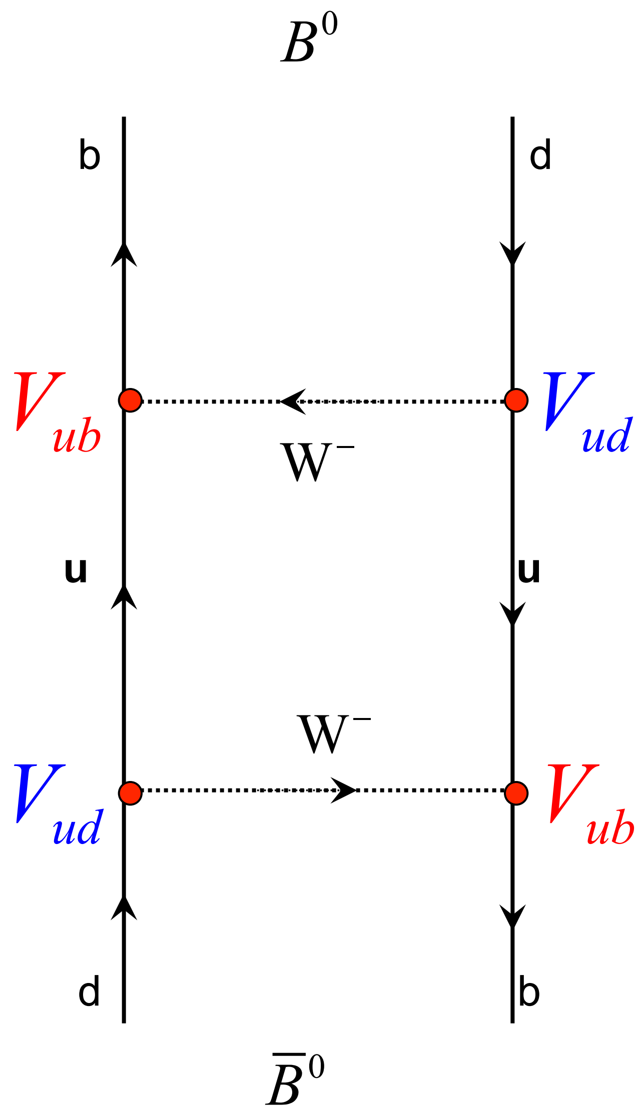
There are six such unitarity triangles, but all have the same area

CP Violation in the B System



Mixing of neutral B meson occurs in the same way as in the kaon system

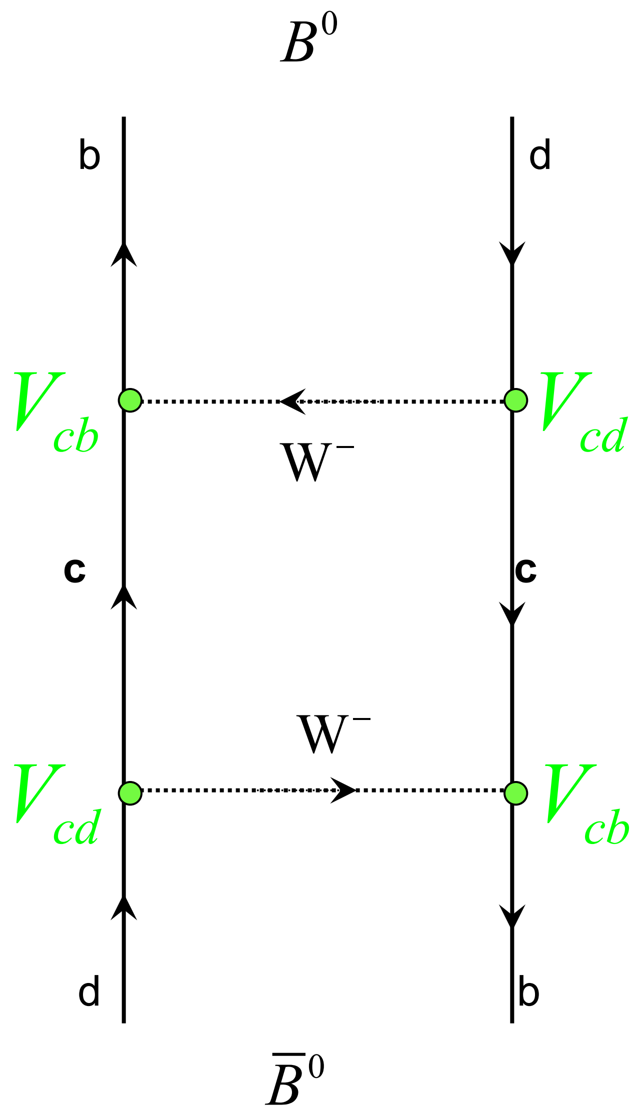
CP Violation in the B System



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

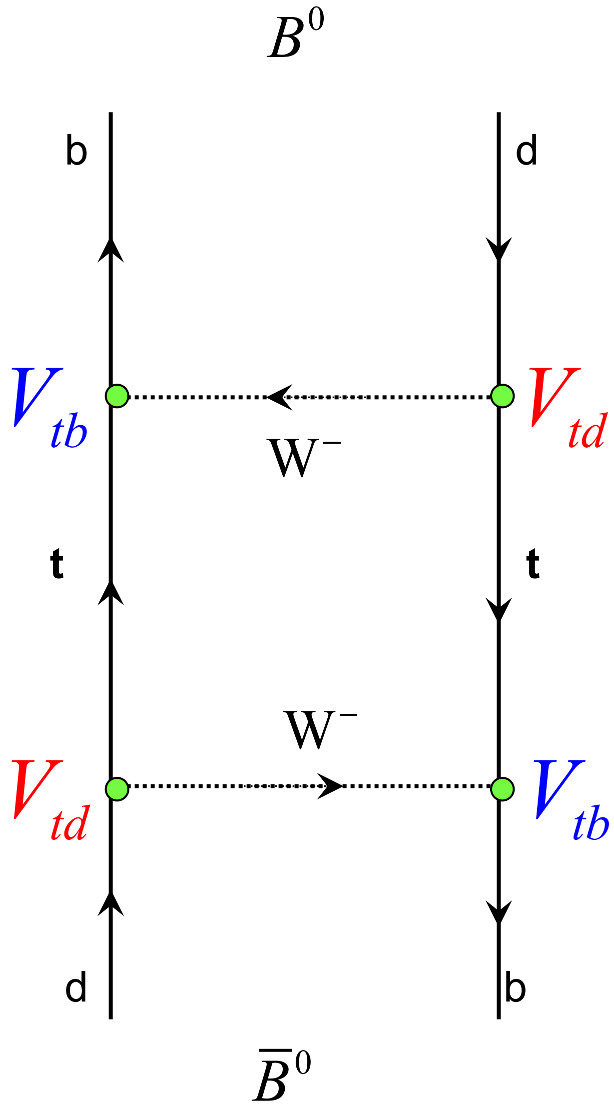
There are also equivalent diagrams with c and t quarks replacing the u quarks in the loop.

CP Violation in the B System



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CP Violation in the B System



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ \boxed{V_{td}} & V_{ts} & \boxed{V_{tb}} \end{pmatrix}$$

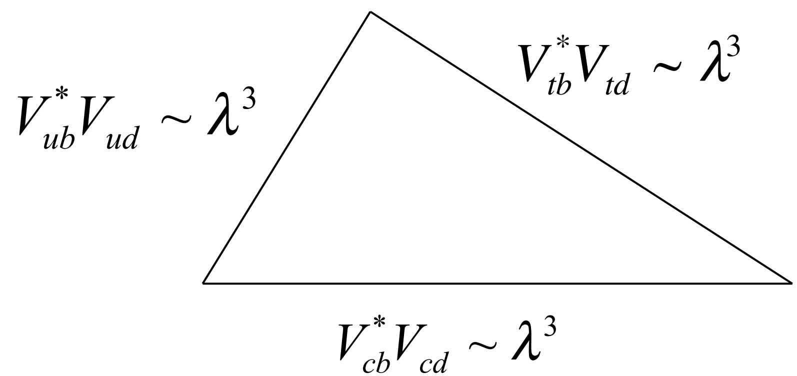
CP Violation in the B Meson System

$$\begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ \lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

“Unitarity” triangle in ρη plane



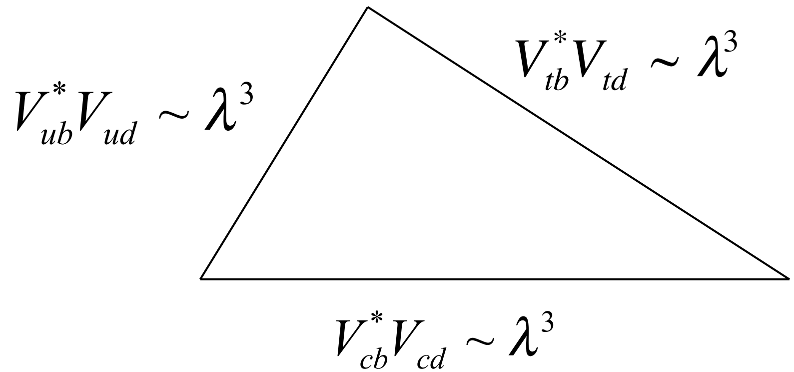
$$\sim \lambda^3 / \lambda^3 = 1$$

CP Violation expected to be large in the B system.

This has been an active field of research at the so-called B factories since the mid-late nineties (and remains so)

CP Violation in the B Meson System

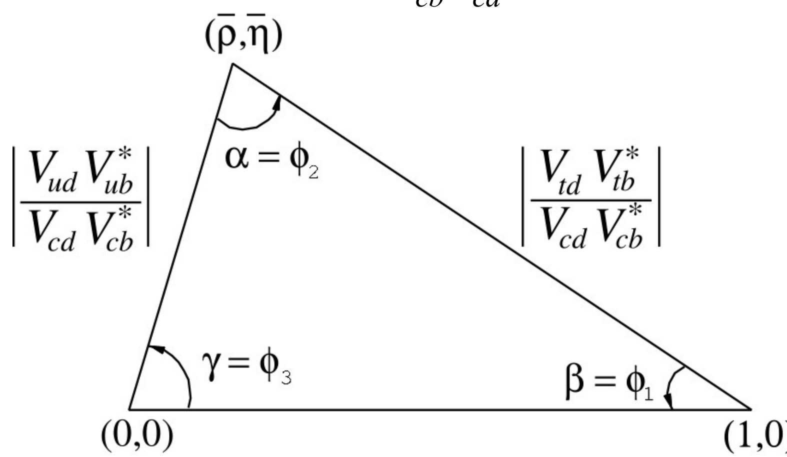
$$V_{ub}^{*}V_{ud} + V_{cb}^{*}V_{cd} + V_{tb}^{*}V_{td} = 0$$



Divide through sides by $V_{cb}^{*}V_{cd}$

⇩

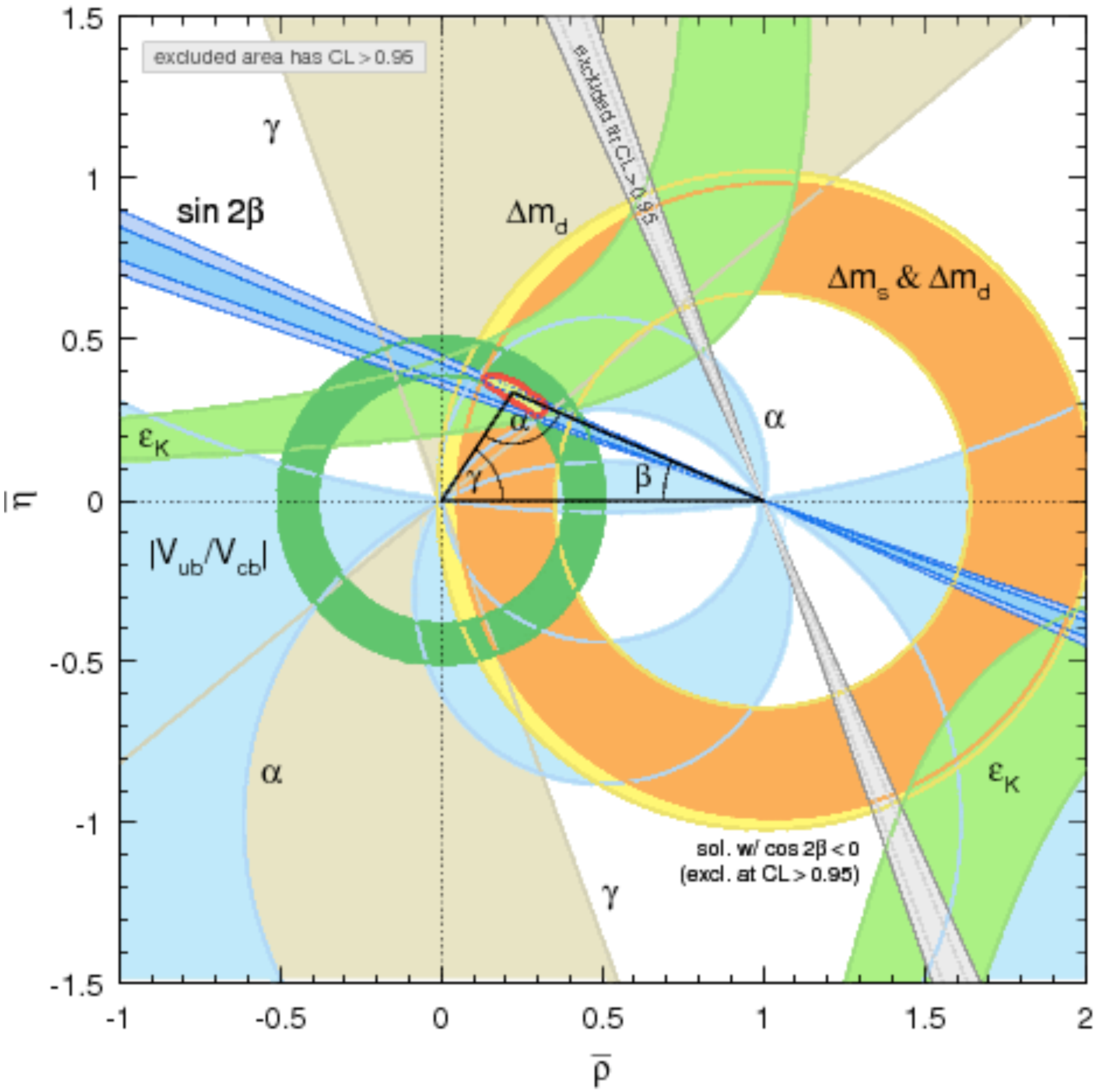
"Unitarity" triangle in ρη plane



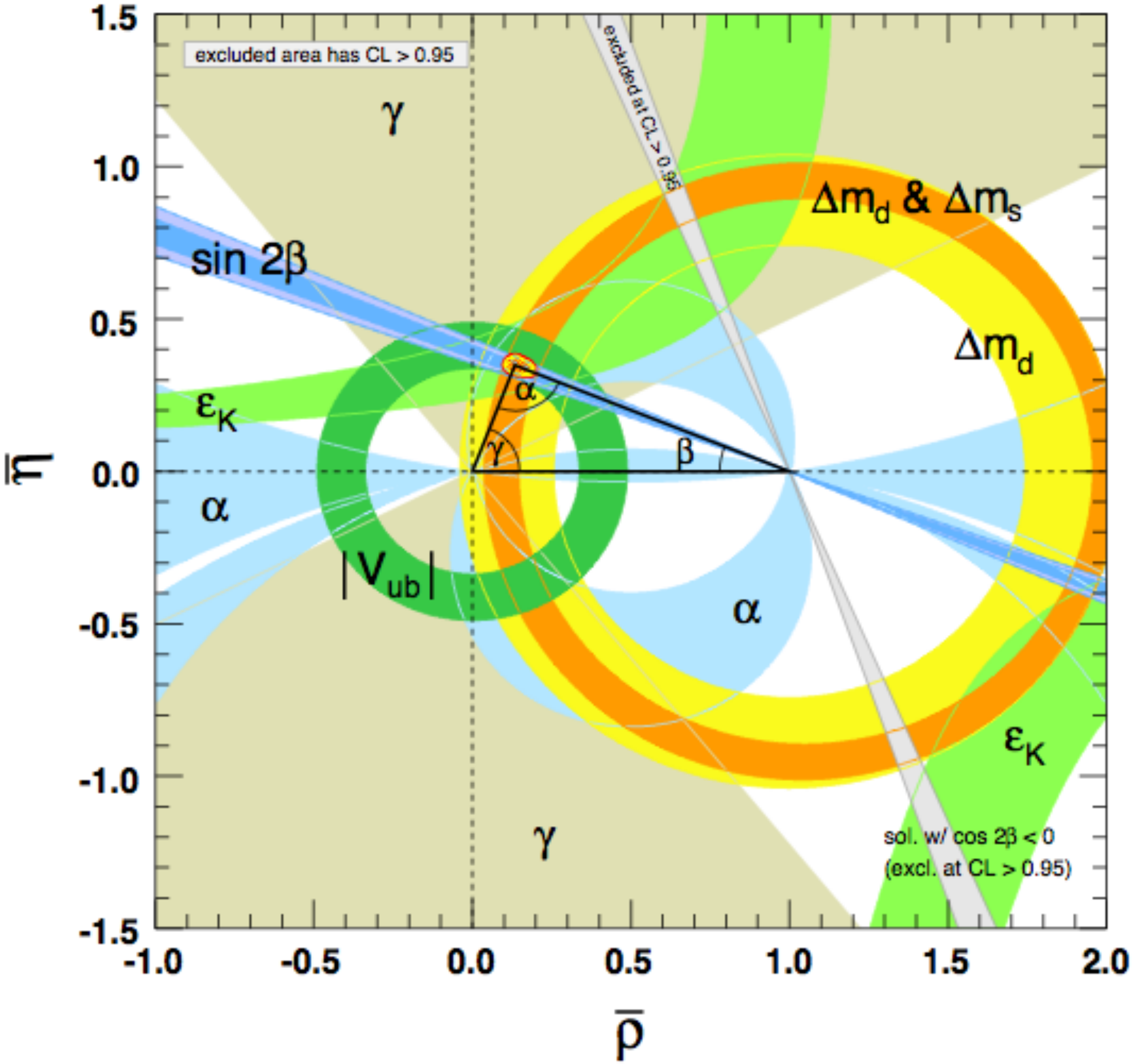
B-factory experiments (BABAR ,BELLE) dedicated to "over-constraining" this unitarity triangle as a test of the SM. 10 years of running have produced precision measurements but no deviations from the predictions of the SM

PDG(2006) sin2β = 0.73 ±0.04 ➔ PDG(2008) sin2β = 0.678 ±0.025

Current Status of the Unitarity Triangle (2006)



Current Status of the Unitarity Triangle (2008)



CP Violation and Baryon Number Asymmetry

Our universe appears to be made of matter only.

Astronomical observations rule out the existence of “domains” of anti-matter dominance

In the Big Bang, matter and antimatter would have been created in equal amounts.

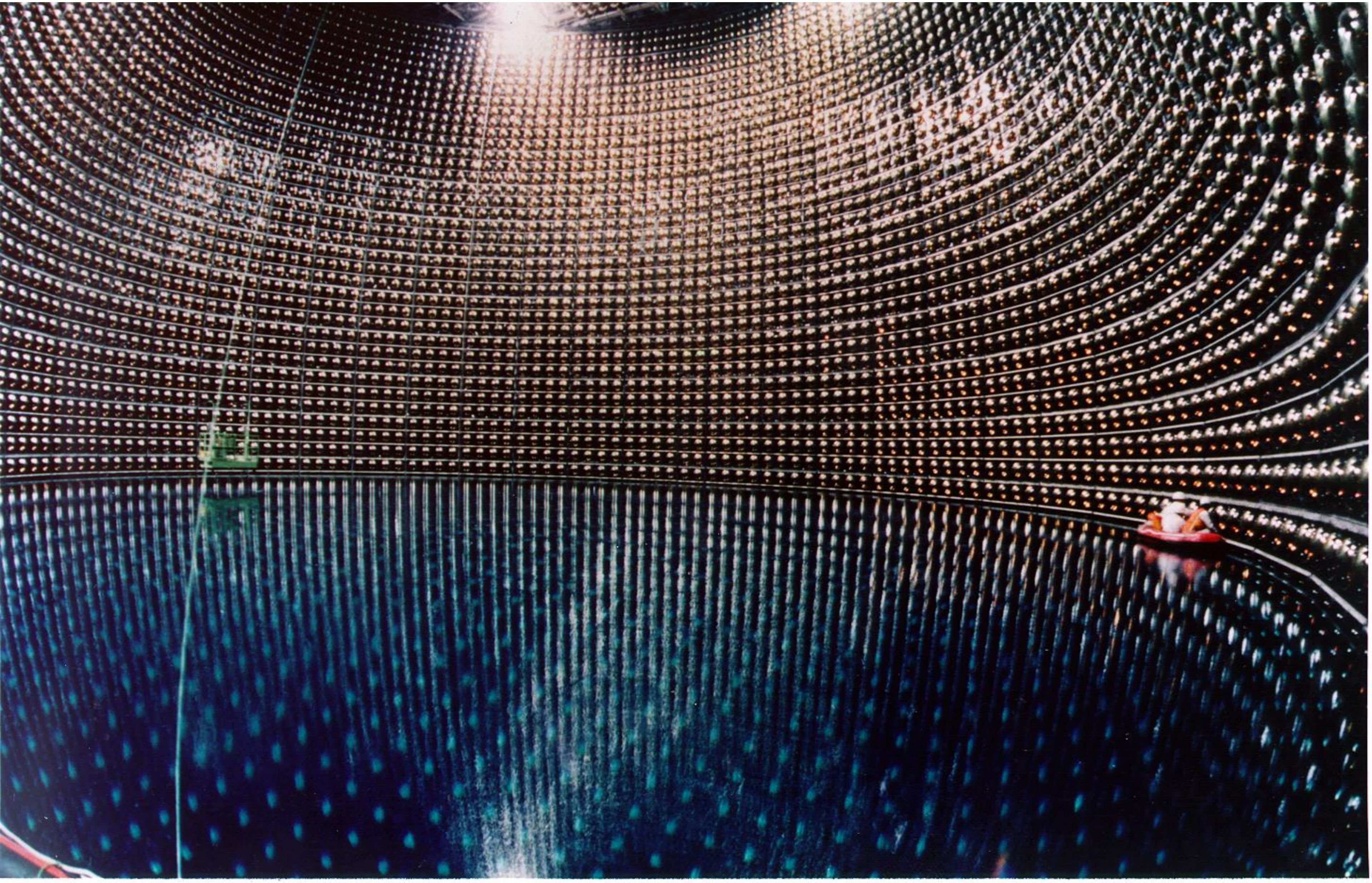
Need some mechanism for creating the observed asymmetry

Need CP violation (which is an asymmetry between matter and anti-matter)

Mechanism of CP violation in Standard Model not enough to explain this.

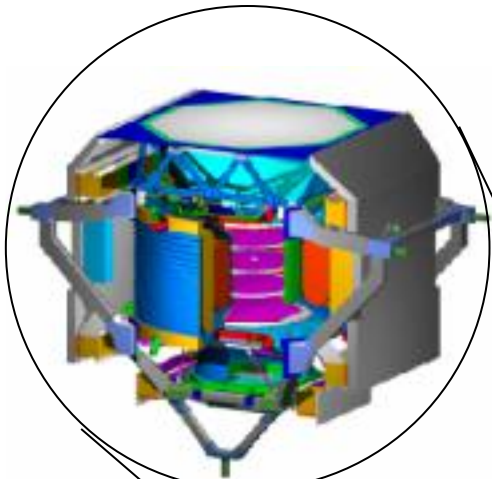
Also need baryon number non-conserving processes (*e.g*. GUTs)

Search for Proton Decay (violates baryon number conservation)

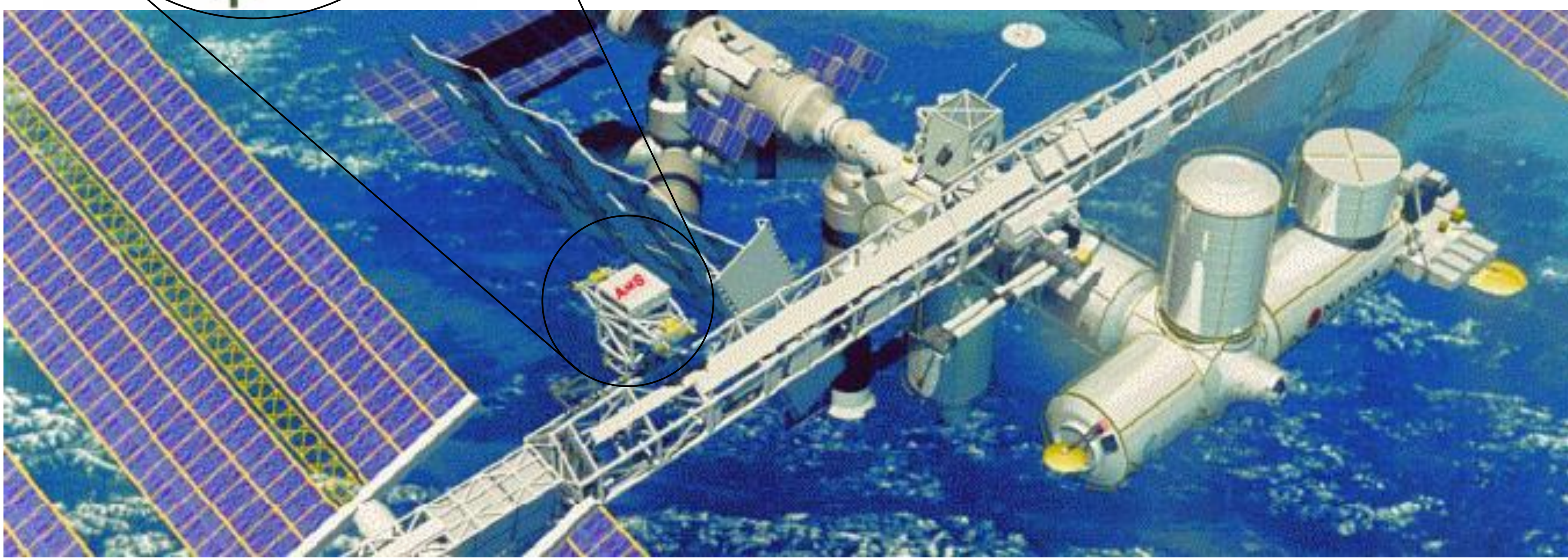


SuperKamiokande Detector, Japan

Alpha Magnetic Spectrometer (on Space Station)



Amongst other studies, AMS will look for anti-matter potentially left over from the Big Bang (or in regions of anti-matter dominance (anti helium nuclei or anti-carbon nuclei etc…)



Time Reversal Invariance

Quantum Field Theory (QFT): *CPT* theorem says that in any theory consistent with Lorentz invariance, quantum mechanics and the idea that interactions are carried by fields, the combined operations of *C*, *P* and *T* (in any order) form an **exact** symmetry of any interaction.

Since *CP* is known to be violated by weak interaction, *T* must be as well

Experimental tests of this are hard: can't easily run a weak interaction backwards (principle of detailed balance)

Instead, look at quantities that are identically zero if *T* is conserved

Popular example is electric dipole moment of the neutron

Current limit from PDG 2008: $d_n < e \cdot 0.29 \times 10^{-25}\ cm$

Question

Why are $K^{*0} - \overline{K}^{*0}$ oscillations not observed experimentally ?

(K^{*0} is the same as K^0 but with spin-1 instead of spin-0.)

$$K^0 \quad \bar{s}d \quad spin-0 \quad \uparrow\downarrow \quad M(K^0) \approx 498\, MeV/c^2$$

$$K^{*0} \quad \bar{s}d \quad spin-1 \quad \uparrow\uparrow \quad M(K^{*0}) \approx 892\, MeV/c^2$$