Adjoint Tomography: Theory, Implementations and Applications

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Traveltime tomography



$$\Delta T = \int_{\text{Ray}}$$

 s_1

 S_2

 s_M

 $s\,dl$ Ray theory(射线理论) High-f approximation

Collect all types of data from all events





* for ray theory, K_{ij} is the length of the ray in the M'th block for the I'th data

$$d = Gm$$

LSQ inversion with regularization $m_{\text{est}} = \left[G^T G + \lambda L(m) \right]^{-1} G^T d$



Finite-frequency kernels





Montelli et al 2004 (Science)



Liu & Gu 2012

When Ray Theory meets Finite Frequency Approximation



Ray vs. Finite-frequency kernel debate

*Dahlen and Nolet, 2005; Montelli et al., 2006a

* de Hoop and van der Hilst, 2004; van der Hilst and de Hoop, 2005, 2006

Effect of regularization/smoothing

Other finite-frequency tomography studies: Hung et al 2006, 2008, 2010 Obrebski et al 2011, 2012, 2014, etc

How to further improve seismic imaging

- More seismic data (improved coverage):

- * more earthquake data, seismic array data
- * ambient noise data, scattered waves
- * waveform, frequency-dependent measurements
- *more accurate sensitivity kernels* K_m(x)

* accurate Green's functions \rightarrow normal mode for 1D (Zhao & Chevrot 2011), numerical simulations for 1D (Nissen-Meyer et al 2007) and 3D models (Liu & Tromp 2006, 2008)

* ambiguity from regularization still exists

- Iterative inversion for a nonlinear inverse problem

* accurate 3D Green's functions, sensitivity kernels \rightarrow 3D numerical solvers

* full waveform inversion (FWI, Tarantola 1984) / adjoint tomography (Tromp et al 2005) 全波形反演 / 伴随层析成像

Adjoint tomography: 3D numerical solvers

- Finite difference
- Finite element
- Spectral element





SPECFEM3D,

Komatitsch et al (2004), Peter et al (2011)

SPECFEM3D_GLOBE Komatitsch & Tromp (2002 a, b), Tromp et al (2010)

Advantages:

- * Weak formulation \rightarrow topography, internal surfaces, complex geology
- * Interpolation/integration of wavefield on local GLL points \rightarrow diagonal mass matrix
- * Meshing by doubling schemes \rightarrow efficient
- * Diagonal mass matrix, local communications \rightarrow Parallel computation
- * Community-supported/developed, open-source; GPU-enabled.

Computational Infrastructure for Geodynamics (CIG): http://www.geodynamics.org/cig/software/packages/seismo/

Local/Global Sensitivity Kernels



SPECFEM3D package, SmS/Sn waves (Liu & Tromp 06)

Two main advantages 1. Works on 3D reference models! 2. No prerequisite knowledge on the pulses measured.

Disadvantage: numerically expensive!!



SPECFEM3D_GLOBE package, Pdiff kernels (Liu & Tromp 08)

Nonlinear Inverse problem $\phi(m) = \sum [T(m) - T_{obs}]^2 = \sum \Delta T^2$ Misfit function $\frac{\partial \phi_e}{\partial m} \sim \int s(x,t) \cdot s_{\rm event}^{\dagger}(x,T-t) \, dt$ Two numerical simulations .. Event kernel (gradient) Unfortunately, computing the 2nd order derivative, i.e, the Hessian matrix $H = G^T G$ is too costly

Traditional $H\Delta m = -g$ **Gauss-Newton Method** Tomography (Ray, FF) Adjoint tomography $\phi(m^j), g = \frac{\partial \phi}{\partial m}(m^j)$ Iterative nonlinear inversions

Adjoint tomography: Examples

We take the adjoint tomography of Europe (Zhu et al 2015) as an example. This is a comprehensive study that can be considered the paradigm of adjoint tomography.

Here is a key summary:

- * 190 earthquakes, 745 stations
- * initial model: Epcrust + S360ANI (called EU00)
- * source inversions: centroid moment tensor and locations
- * adjoint tomographic inversions in three stages
 - ** stage 1: T(f) --> radially anistropic parameters
 - ** stage 2: T(f), A(f) --> elastic parameters + anelastic parameters
 - ** stage 3: T(f), A(f) --> radial and azimuthal anistropy

Note: first two stages uses long-period surface wave (50/20-150 s) + shortperiod body waves (15-50 s), and the last stage uses only surface waves.

** after 60 iterations --> EU60

Study Area: Events & Stations





Moho is honored by one layer element when d < 15 km, 2-3 layer of elements when d is between 25-45 km.



Stage I: Inversion Steps

Define misfit function

$$=\chi^{\phi}, \qquad \qquad \chi^{\phi}=\sum_{c=1}^{N_c}w_c\sum_{m=1}^{N_m}\int w_m\left[\frac{\Delta\tau_m(\omega)}{\sigma_m^{\phi}(\omega)}\right]^2\mathrm{d}\omega,$$

Sensitivity kernels

 $\delta \chi_{\rm I} = \int_{V} \mathbf{K}_c \,\delta \ln c + \mathbf{K}_{\beta_{\rm v}} \,\delta \ln \beta_{\rm v} + \mathbf{K}_{\beta_{\rm h}} \,\delta \ln \beta_{\rm h} + \mathbf{K}_{\eta} \,\delta \ln \eta \,\mathrm{d}V, \quad \delta \ln \rho = 0.33 \,\delta \ln \beta, \quad \beta = \sqrt{\frac{2\beta_{\rm v}^2 + \beta_{\rm h}^2}{3}}.$

Preconditioner: approximate diagnonal of Hessian

$$P(\mathbf{x}) = 1 / \int \partial_t^2 \mathbf{s}(\mathbf{x}, t) \cdot \partial_t^2 \mathbf{s}^{\dagger}(\mathbf{x}, T - t) \, \mathrm{d}t,$$

Kernel smoothing (regularization)

Xι

$$K(\mathbf{r}) = \frac{1}{W(\mathbf{r})} \int_{V} K(\mathbf{r}') \exp[-(r'\Delta)^{2}/(2\sigma_{\Delta}^{2})]$$
$$\times \exp[-(r-r')^{2}/(2\sigma_{r}^{2})] d^{3}\mathbf{r}',$$

Conjugate gradient methods for nonlinear inversion

$$\mathbf{d}_i = -\mathbf{g}_i + \beta \mathbf{d}_{i-1}, \qquad \beta = \frac{\mathbf{g}_i^T \cdot (\mathbf{g}_i - \mathbf{g}_{i-1})}{\mathbf{g}_{i-1}^T \cdot \mathbf{g}_{i-1}}. \qquad \ln \frac{\mathbf{m}_{i+1}}{\mathbf{m}_i} = \alpha \, \mathbf{d}_i,$$

Alternatively, one can use other optimization techniques, e.g., Limited-memory BFGS methods

Inversion Steps (cont.)

Model update in the search direction (d is normalized)

 $\ln \frac{\mathbf{m}_{i+1}}{\mathbf{m}_i} = \alpha \, \mathbf{d}_i,$

Step length alpha is determined through compute misfit of a set of representative earthquakes as a function of step length (alpha = 0.01 - 0.05) corresponding to maximum 5% velocity perturbation, and monitor the reduction of misfit for all categories of data





Stage 2: inversion steps

Stage 2: Misfit function including both phase and amplitude

$$\chi_{\mathrm{II}} = w_{\phi} \chi^{\phi} + w_{A} \chi^{A}, \qquad \chi^{\phi} = \sum_{c=1}^{N_{c}} w_{c} \sum_{m=1}^{N_{m}} \int w_{m} \left[\frac{\Delta \tau_{m}(\omega)}{\sigma_{m}^{\phi}(\omega)} \right]^{2} \mathrm{d}\omega,$$
$$\chi^{A} = \sum_{c=1}^{N_{c}} w_{c} \sum_{m=1}^{N_{m}} \int w_{m} \left[\frac{\Delta \ln A_{m}(\omega)}{\sigma_{m}^{A}(\omega)} \right]^{2} \mathrm{d}\omega,$$

Sensitivity kernels

$$\delta \chi_{\mathrm{II}} = \int_{V} \mathbf{K}_{c} \,\delta \ln c + \mathbf{K}_{\beta_{\mathrm{v}}} \,\delta \ln \beta_{\mathrm{v}} + \mathbf{K}_{\beta_{\mathrm{h}}} \,\delta \ln \beta_{\mathrm{h}} + \mathbf{K}_{\eta} \,\delta \ln \eta + \mathbf{K}_{\mathcal{Q}^{-1}} \,\delta \mathcal{Q}^{-1} \,\mathrm{d}V,$$

$$\mathbf{K}_{\mathcal{Q}^{-1}} = -\int_{0}^{T} 2\mu \,\mathbf{D}^{\dagger}(T-t) : \mathbf{D}(t) \,\mathrm{d}t,$$

$$\tilde{f}_{i}^{\dagger}(t) = \frac{1}{2\pi} \,\int_{-\infty}^{+\infty} \left[(2/\pi \,\ln(|\omega|/\omega_{0} - i\,\mathrm{sgn}(\omega)]^{*} f_{i}^{\dagger}(\omega) \,\mathrm{exp}(i\,\omega t) \,\mathrm{d}\omega \right]$$

Inverse Shear

Separate simulations for elastic and anelastic kernels

Stage 3: inversion steps

Stage 3: misfit function including both phase and amplitude (surface wave only)

$$\chi_{\rm III} = w_{\phi} \chi^{\phi} + w_A \chi^A,$$

In weakly anisotropic medium,

 $c(\omega, \theta) = c_0(\omega) + c_1(\omega)\cos(2\theta) + c_2(\omega)\sin(2\theta) + c_3(\omega)\cos(4\theta) + c_4(\omega)\sin(4\theta)$

C_0: radially anisotropy C_1,2: related to G, H, B_{s,c} C_3,4: related to E_{s,c} Total 13 parameters (5+8)

Select to invert for only radial anisotropy (L,N) and azimuthal anisotropy (G_c, G_s)

$$\delta \chi_{\mathrm{III}} = \int_{V} \mathbf{K}_{L} \delta L + \mathbf{K}_{N} \delta N + \mathbf{K}_{G_{c}} \delta G_{c} + \mathbf{K}_{G_{s}} \delta G_{s} \, \mathrm{d}V,$$

$$= \int_{V} \mathbf{K}_{\beta_{v}} \delta \ln \beta_{v} + \mathbf{K}_{\beta_{h}} \delta \ln \beta_{h} + \mathbf{K}_{G_{c}'} \delta G_{c}' + \mathbf{K}_{G_{s}'} \delta G_{s}' \, \mathrm{d}V,$$

$$\zeta = \frac{1}{2} \operatorname{arctan}(G_{s}/G_{c}).$$
Fast axis & Strength

Misfit Reduction



Traveltime anomaly Histogram

Final Images

Vs

0.95 1.00 1.05

EU60: depth cross-sections

EU60: azimuthal anisotropy

2% peak to peak anisotropy

Resolution test: point spread functions

0.000 0.002 0.004 0.006 0.0 δm

Stage 1: point spread function, Vsh, Vsv

1e-06 2e-06 3e-0 Ηδm (m⁻³) Stage 2: Vsh, Vsv, 1/Q

0 2e-07 4e-07 6e-07 8e-0 Hδm (m⁻³)

Point spread function (cont)

Stage 3: L, N, Gc, Gs

0.000

0.002

0.004

δm

0.006

0.008

1e-10 2e-10 3e-10 4e-10 5e-10 Hδm (m⁻³)

Coverage: Approximate Hessian Diagonal

1 2 3 4

Adjoint tomography of East Asia 60° 70° 80° 90° 100° 110° 120 130 140 150° 60° Sea of Okhotsk Russia Sayar King'an-Eas 50° Kazakhstan Altay ast Gobi Ba Kuril T. Japan Sea angbai V. Nansha Tarim Basin Basin 40 Ordos Bohai Bay Pacific Plate Fold Block China Blo Qiangtang Block Izu-Bonin T. 30° Philippine Sea Plate ndia Plate outh China Sea Bay of Mariana Arabian Sea ippine Basi Topography (m) 1000 2000 3000 4000 5000 6000

* continental-continental collision * subduction * basin formation

* intracontinental rift Intraplate volcanism

* large variation in

topography

* inversion of both body and 20° surface waves for 3D radially anisotropic models of both the crust ^{10°} and mantle

eliminates crustal corrections

Chen et al (2015a,b)

* 227 earthquakes (Mw=5-7, red: 20 subset; green: 39 extra)

* 1869 stations (majority are broadband stations) from F_net, CEArray, NECESSArray, INDEPT IV array

* 1.7 million frequency-dependent traveltime measurements from body and surface waves

Adjoint tomography Procedures

* 3D initial model: S362ANI (Kustowski et al 2008) + CRUST 2.0 (Bassin et al 2000)

* forward modelling based on SPECFEM3D_GLOBE package (E-topo2, Moho, 410,660 topography, 8km/5km h/v grid spacing in the crust, accurate up to 10 sec)

* Updating CMT source parameters (CMT3D, Liu et al 2004): reinverted based on 3D initial model and seisograms filtered at 30-60 s (body), 50-100 s (surf.), and 80-150 s (surf) \rightarrow depth mostly becomes shallow (computational very costly, eqv. 15 structural iterations, only done once)

* frequency-dependent measurements between data and 3D synthetics made at automatically picked windows: FLEXWIN (Maggi et al 2009) for three passbands 15s-40s (body-wave-only), 30-60s, and 50-100 s for the first 12 iteration, and the lower bound is lowered to 12, 20 and 40 s.

* total ~ 1.2 M measurements for the initial model, and 1.7M measurements for the final model

Windowing

Adjoint tomography Procedures

* Define misfit function
$$\phi = \frac{1}{N_c} \sum_c \phi_c$$
, $\phi_c = \sum_{pc} \int \left(\frac{\Delta T_{pc}(\omega)}{\sigma_{pc}(\omega)} \right)^2 d\omega$

* Parameterization in terms of Vc, Vsh, Vsv and eta (dln(rho) = 0.33dln(Vs)):

$$\delta T = \int_{V} \left(K_{V_c} \delta \ln V_c + K_{V_{SH}} \delta \ln V_{SH} + K_{V_{SV}} \delta \ln V_{SV} + K_{\eta} \delta \ln \eta \right) \, dV$$

* adjoint sources are computed by assimilating measures

* Event sensitivity kernels are computed by the adjoint methods: interaction of forward and adjoint wavefields), and then summed to obtain gradient vector $\frac{\partial \phi}{\partial m}$

* Apply approximate diagonal of Hessian matrix as preconditioner and smoothed with 5km (v)/ 100-60 km (h) Guassian function

* optimal model in the gradient direction is obtained through line search: step length determined by a subset of 20 events

- * model is updated based on gradient vector and step length,
- * Next iteration

Total 20 iterations (8 million CPU) Final model: EARA2014 (East Asia Radially Anisotropic Model 2014)

Line Search & Misfit reduction

Model Quality Assessment

Mean/std of measurements:

Initial model: -0.12+/-3.88 s Final model: -0.03+/-2.32 s

39 extra earthquakes: Initial model: -0.10+/-4.24 s Final model: -0.24+/-3.11 s

Vsv Model updates

Vsv model comparison

With high-resolution global shear velocity model SL2013sv from Schaeffer & Lebedev (2013)

Similarities:

* 50-100 km: low V beneath Tibetan plateau and Altay-Sayan Mnt range
* 200 km: high-V beneath central/southern Tibet down to TZ
* 50-200 km, high V beneath Ordos and Yangtze platform

A more refined model:

* narrow slabs from 200-400km at Pacific margin

* confined low V to Qiangtang and Songpan Ganzi blocks

Vsv model comparison

With high-resolution global shear velocity model SL2013sv from Schaeffer & Lebedev (2013)

Between 400 - 700 km

- * narrower slabs
- * hainan plume

Challenges

- * Wave simulations are generally performed on HPC
- * Data and synthetic processing for a large number of events and stations
- * Preprocessing requires running a series of software packages sequentially (windowing, measurement, adjoint source calculation)
- * Optimization strategies (step length, CG, BFGS, etc)
- * monitoring waveform and traveltime fits at each iteration
- * Many iterations ...

 \rightarrow community based effort to develop more general inversion frameworks that utilize Advanced Data Format, Scientific Workflow, etc

ObsPy: data acquisition and processing

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<> Code	(!) Issues 101	្រ៉ា Pull requests 44	🗐 Wiki	-∳~ Pulse	III Graphs		

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ObsPy is an open-source project dedicated to provide a **Python framework for processing seismological** data. It provides parsers for common file formats, clients to access data centers and seismological signal processing routines which allow the manipulation of seismological time series.

The goal of the ObsPy project is to facilitate rapid application development for seismology.

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- News
- Getting Started

Krischer et al 2015

Multiscale collaborative global Earth model

A framework that combines model parameterization, numerical solver, model refinement (workflow)

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DmBorisov fixe muting			Latest commit	4e54267 14 days ago	
docs	updated public_html paths			17 days ago	
scripts	updated scripts/sfexamples			a month ago	
seisflows	fixe muting			14 days ago	
tests	refactored import_all			21 days ago	
.gitignore	removed unused coverage metrics			2 months ago	
🖹 .travis.yml	removed unused coverage metrics			2 months ago	
	edited docs			a year ago	

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SeisFlows

SeisFlows is an open source seismic inversion package that

- Delivers a complete, customizable waveform inversion workflow
- · Provides a framework for research in regional, global, and exploration seismology

Smith et al 2015

Alternative Approaches

* 3D greens functions (numerical solver) for misfit functions, asymptotic (NACT) kernels.

French & Romanowicz (2015)

FWI applied around the globe

Adjoint tomography for the whole globe

Slabs and plumes/hotspots enhanced in GLAD-M15 (Bozdag et al 2016) New-generation GLAD-M25 (Lei et al 2019) with six-fold increase of data

Work-flow based Automatic FWI

Automated Large-Scale Full Seismic Waveform Inversion: The Collaborative Seismic Earth Model (Fichtner et al, 2018)

Opportunities and Challenges

- Taking advantage of the ever-growing seismic array deployment and leveragi
- However, FWI at the scale of continents and the globe is still numerical extremely

It requires continued community-based effort on software development and maintenance, and methodology advances (optimization, inversion strateg

Future development of FWI for earthquake seismology

- Continued deployment of regional stations and arrays
- New datasets (ambient-noise)
- Special signals (scattered waves)
- Apply full numerical simulations to improve source characterization.

More data: Array deployments

USArray Transportable Array deployment currently at Alaska

Canadian Cordillera Array (CCArray) www.ccarray.org

GNSS (CCArray proposed)

GNSS (existing Canada)

GNSS (planned Canada)

Seismic (CCArray proposed)

Episodic GNSS (CBN)

GNSS (US)

Existing Seismic

FWI of ambinent-noise empirical Greens function

25⁰N

-0,8

50–100 s

Anisotropic adjoint tomography in southern California

Interpretation of mid-lower crustal radial anisotropy

Oblique zone of negative radial anisotropy (-6%) bounded by Pacific coast and San Andreas Fault probably due to steeply dipping amphibole or horizontally aligned plagioclase associated with compressional tectonics.

□ Positive radial anisotropies might result from subhorizontally alignment of minerals in schists or genesis associated with extensional tectonics.

Vs radial anisotropies, $\gamma = (Vsh - Vsv)/Vs$, plotted as a function of rotation angle ϑ (Xie et al., 2013)

Wang et al (2019)

What about linear arrays?

USArray: Flexible array experiments

Linear Array Ambient Noise Adjoint Tomography Reveals Intense Crust-Mantle Interactions in North China Craton: using a 3D/2D transformation for ambinent-noise EGFs.

Other Dataset: teleseismic body waves

T = 10 - 20 S b a 96 104 106 MC20 MC22 MC24 32 T = 15 - 30 SС MC20 SCB 30° MC22 28 MC24 T = 20 - 40 Sd 26 MC20 24 MC22 MC24 1000 2000 3000 4000 5000 6000 Time Topography (m)

* Noise cross-correlate functions

Chen et al. (2014), SE Tibet

scattered waves of teleseismic waves

Rondenay (2008) GRT of scattered/converted waves of teleseismic P phase.

Scattering imaging based on FWI and hybrid methods

SEM-FK hybrid methods (Tong et al 2014) External solver: Finite-wavenumber for 1D background model Internal solver: full 3D SEM

FWI of teleseismic body waves based on DSM-SEM hybrid methods

The deep roots of the western Pyrenees revealed by full waveform inversion of teleseismic P waves along PYROPE seismic transect (Wang et al 2016)

Conclusions

* Seismic imaging is transitioning from traditional tomographic methods (raybased traveltime) to full-waveform inversions (FWI) based on sensitivity kernels computed by full numerical simulations of seismic wave propagation.

* In combination with the expanding deployment of global and regional seismic network as well as temporary arrays, FWI provides unprecedented high-resolution and robust images of the Earth's interior at global, regional and industrial scales.

* The continued incorporation of innovative datasets (ambient-noise surfacewaves, teleseismic scattered waves, etc) will further improve the coverage and hence the seismic image quality.

* The future of high-resolution imaging: High-performance computing + more seismic dataset + innovative inversion strategies + continued software development by the community

"Full-waveform inversion (FWI) has emerged as the final and ultimate solution to the Earth resolution and imaging objective."

--Announcement of the 2013 SEG workshop on FWI

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The End