JPE2605 Advanced Seismology

Lecture 1: A concise Review of Seismology

January 20, 2021

University of Toronto

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Introduction

Course Content (2020-2021)

- A quick review of seismology (2 weeks)
- Finite-difference method (FD) (3 weeks)
- Finite-element method (FEM) (2 weeks)
- Spectral element methods (SEM) (3-4 weeks)
- Full waveform inversions (FWI) (2-3 weeks)

Course Administration

Course Administration

- course website:
  https://www8.physics.utoronto.ca/ liuqy/courses/jpe2605h.html
- course assignments: 4 assignments (15/30/30/25)
- computing language: Python (Jupyter), matlab/octave, Fortran 90 (for the last two assignments) or any other language of your choice
- Lecture hours / office hours (TBA)
- contact email: liuqy@physics.utoronto.ca

Main Textbook

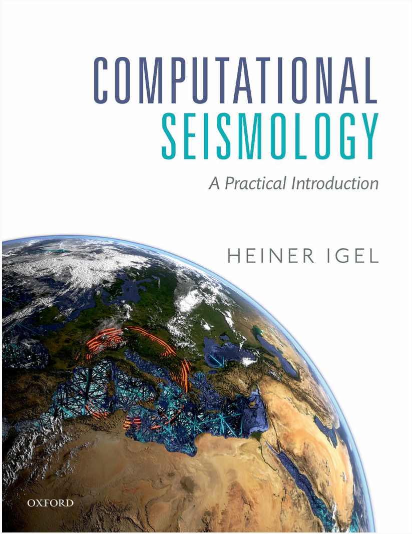
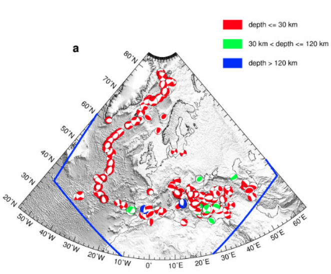


Figure 1: Main text book: Link through the uoft library

Motivations: why numerical methods?

FWI: Source-receiver distributions



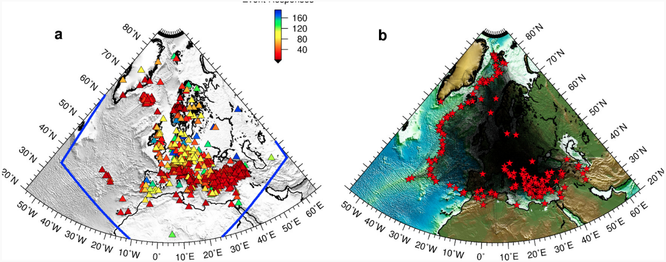


Figure 2: Source–receiver raypaths for earthquake-receiver configuration for Europe (Zhu et al 2015)

Waveform fits on seismograms

Seismic imaging: invert the subsurface structures by reducing the misfit between observations (black traces) and theoretical seismograms (red traces).

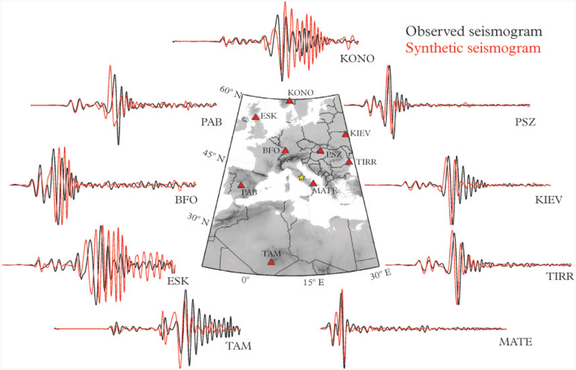


Figure 3: Seismograms for the damaging 2009 M6.3 l'Aquila, Italy earthquake

Full waveform inversion

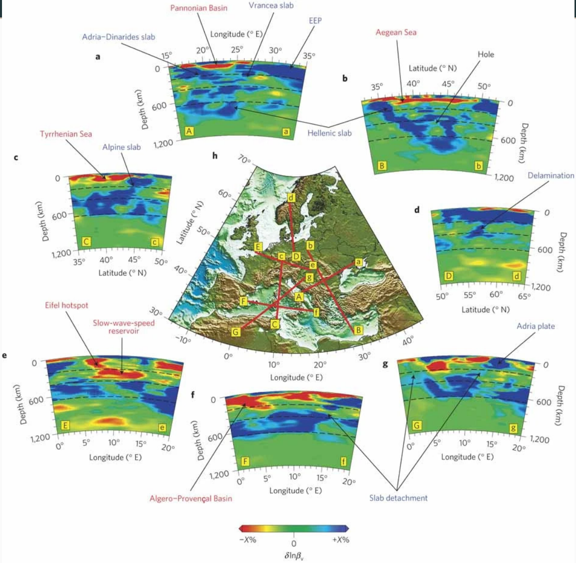
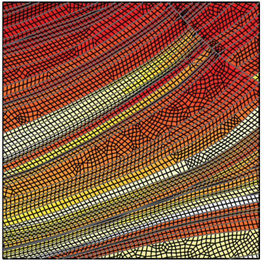


Figure 4: Subsurface structures beneath Europe from *adjoint tomography* (Zhu et al 2015)

Compute the synthetics

One of the most fundamental tasks in seismology is to **compute the synthetics**. But beyond the analytical methods for simple models (e.g., ray-theory, reflectivity, F-k, normal modes, etc), we need to resort to **numerical methods** to simulate seismic wave propagation.

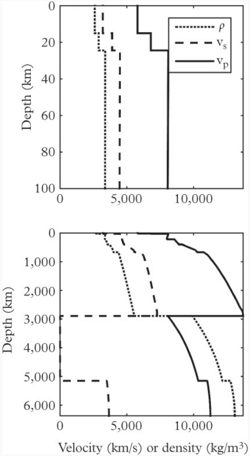
(a) global seismic wave propagation based on SEM.

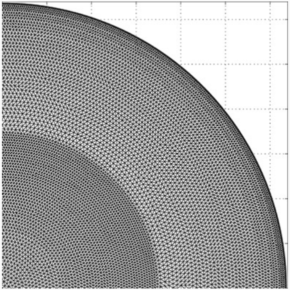


(b) Rectangular mesh for the Marmousi benchmark model for reservoir simulations.

Numerical simulations: what to consider

- accuracy (seismic phases to be modelled; wavelength $\lambda = V \cdot T$, npts-per-wavelength, v increase with depth)
- choice of numerical methods: time-domain vs frequency domain; FD vs FEM/SEM
- meshing (heterogenities, discontinuities), simulation stability





HPC Power

Numerical cost of simulations: number of elements/grid points.

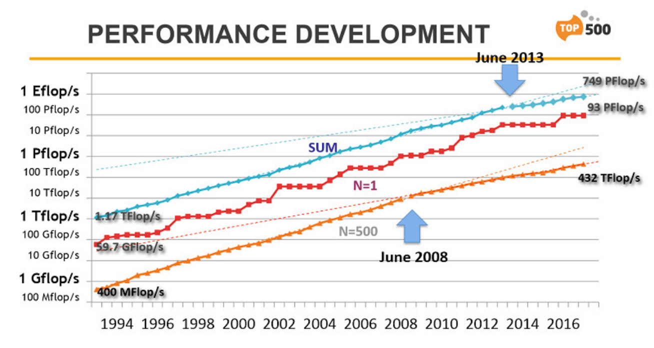


Figure 6: TOP500 aggregate performance

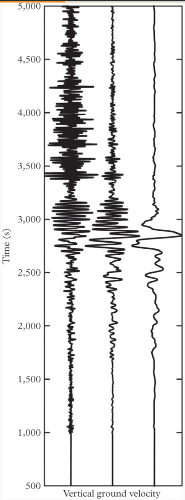
Application of Comp. Seismology

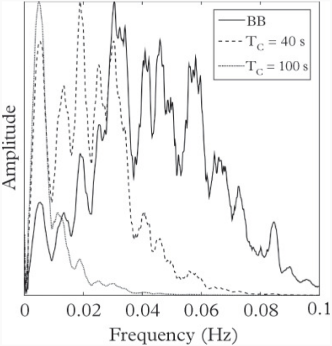
1. ground-shaking **hazard analysis**: high-f building response, shaking characteristics
2. **earthquake physics**: proper implementation of friction laws on faults.
3. **exploration geophysics**: mostly use body waves, finite-difference method (difficult with free surface), complicated constitutive laws (anisotropy, poroelasticity) for reservoirs.
4. **Full waveform inversion** (FWI, also known as adjoint tomography) using full seismogram instead of a few bytes of info (e.g., travel time); global/continental-scale seismology (>1000km) vs. regional/local scales (tens-to-hundreds-of-km)

“An excellent preparation for a research project in Computational Seismology with any method is to start by coding the WE from scratch in 1D and explore its capabilities and traps..."

A concise Review of Seismology

What do seismograms look like?





Vertical-component velocity seismogram of the M9.1 Sumatra-Andaman earthquake of 26 December 2004, recorded at WET station with an STS-2 seismometer. Low-pass filtered seismograms with corner period 40 and 100 s. Right: normalized spectra

Seismic wave equation

The **seismic wave equation** (elastodynamic equation)

$$\rho \ddot{u}_i = \sigma_{ij,j} + M_{ij,j} + f_i \tag{1}$$

constitutive relation for **general anisotropy**

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \tag{2}$$

or for **isotropic medium**

$$\sigma_{ij} = \lambda \epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} \tag{3}$$

- material properties: density $\rho(x)$, elastic constants $C_{ijkl}(x)$
- input source term: moment tensor $M_{ij}(x,t)$, or volumetric force $f_i(x,t)$ or excitation through I.C.
- boundary condition (B.C.)
- solution as displacement wavefield $u_i(x,t)$

Acoustic/scalar wave equation

For **pressure-wave propagation**

$$\ddot{p} = c^2 \nabla^2 p + s \tag{4}$$

or SH wave propagation ($\mathbf{u} = [0, u_y, 0]$)

$$\rho \ddot{u}_y = (\mu u_{y,x})_{,x} + f_y \tag{5}$$

Or velocity-stress formulation (important for FD) for the elastodynamic equation ($\dot{\epsilon}_{ij} = (v_{j,i} + v_{i,j})/2$):

$$\begin{aligned} \rho \dot{v}_i &= \sigma_{ij,j} + f_i \qquad (6)\\ \dot{\sigma}_{ij} &= \lambda \dot{\epsilon}_{kk} \delta_{ij} + 2\mu \dot{\epsilon}_{ij}, \qquad (7) \end{aligned}$$

Properties: **superposition** principle (linear wrt source); **reciprocity** (linear elasticity); symmetry in time (**time reversal**, only 2nd order derivative in t, ω^2, replace $t \to -t$, $u(x,t) -> u(x,-t)$, the equation invariant)

Viscoelasticity and attenuation

Intrinsic attenuation describes the fractional energy loss (conversion to heat) per cycle as waves propagate through medium, defined by the inverse of the **quality factor** Q

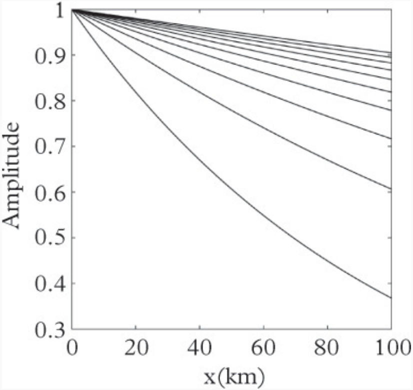
$$\frac{1}{Q}(\omega) = -\frac{\Delta E}{2\pi E} \tag{8}$$

which gives amplitude decay over distance for plane-wave propagation along x:

$$A(x) = A_0 exp(-\frac{\omega x}{2cQ}). \tag{9}$$

Q is usually considered **f-independent** in the seismic period band. Therefore, constant Q implies that amplitude of high-f waves gets attenuated more than those of short-f over the same distance, resulting in diminishing high-f waves away from the source (e.g., in teleseismic waves).

Attenuation: amplitude decay



Amplitude decay due to attenuation only as a function of propagation distance for an initial unity amplitude. The curves illustrate the behaviour for $Q = 10$ (lowest curve, strongest attenuation) to $Q = 100$ (top curve, weakest attenuation) at a frequency of 1 Hz.

Anisotropy

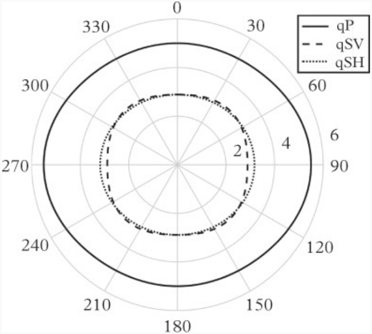
The elastic tensor C_{ijkl} can be reduced to 6×6 matrix with the Voigt notation (11$\rightarrow$1, 22$\rightarrow$2, 33$\rightarrow$3, 32$\rightarrow$4, 13$\rightarrow$5, 12$\rightarrow$6).

Under hexagonal symmetry (e.g., **transverse isotropy, or radial anisotropy** with symmetry axis in the $\hat{z}$ or $\hat{r}$ direction), only five independent elastic moduli $\mathbf{A} = C_{11} = \rho\alpha_h^2$, $\mathbf{C} = C_{33} = \rho\alpha_v^2$, $\mathbf{L} = C_{44} = \rho\beta_v^2$, $\mathbf{N} = (C_{11} - C_{12})/2 = \rho\beta_h^2$, $\mathbf{F} = C_{13}$ (sometimes $\eta = F/(A - 2L)$ is used) are needed to define the full matrix:

$$C_{pq} = \begin{pmatrix} A & A-2N & F & 0 & 0 & 0 \\ A-2N & A & F & 0 & 0 & 0 \\ F & F & C & 0 & 0 & 0 \\ 0 & 0 & 0 & L & 0 & 0 \\ 0 & 0 & 0 & 0 & L & 0 \\ 0 & 0 & 0 & 0 & 0 & N \end{pmatrix} \tag{10}$$

S-wave splitting

S waves propagating through a homogeneous medium with hexagonal symmetry split into orthogonally polarized qS_1 **and** qS_2 propagating with different velocities. P waves polarized in the longitudinal direction are replaced by *qP* waves. Note non-spherical wavefronts.



Phase velocity variation in x-z plane for TI medium with vertical symmetry axis ($\theta = 0$).

Weak anisotropy

For weak general anisotropy, it can be described by three positive parameters, fractional qP anisotropy ϵ, fractional qSH anisotropy γ, and δ (Thomsen 1986).

$$V_P(\theta) = V_{P_0}(1 + \delta \sin^2\theta \cos^2\theta + \epsilon \sin^4\theta) \tag{11}$$

$$V_{SV}(\theta) = V_{S_0}\left[1 + (V_{P_0}/V_{S_0})^2(\epsilon - \delta)\sin^2\theta\cos^2\theta\right] \tag{12}$$

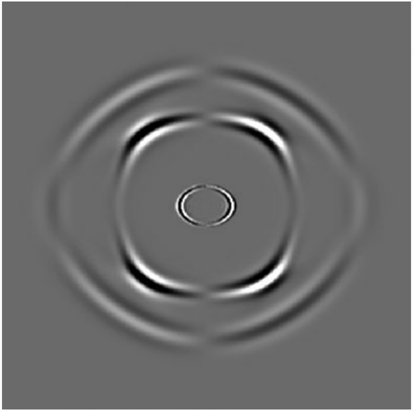
$$V_{SH}(\theta) = V_{S_0}(1 + \gamma\sin^2\theta), V_{P_0} = \sqrt{C_{33}/\rho}, V_{S0} = \sqrt{C_{44}/\rho}$$

where θ is the angle between the axis of symmetry and the wave propagation direction.

Quasi-analytical solutions can be obtained based on *anisotropic reflectivity method*

Poroelasticity

When rock mass is fractured and pore space is (partially) filled with fluids or gas, the constitutive relation needs to be developed using continuum mechanics. In a homogeneous **poroelastic** medium, there are two compressional waves (fast and slow one) in addition to the classic S waves

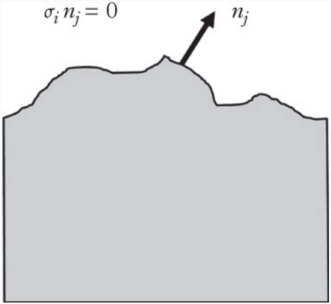


I.C. and B.C.

I.C. can be given either as u, v at $t = 0$, or v, σ at $t = 0$ (velocity-stress formulation).

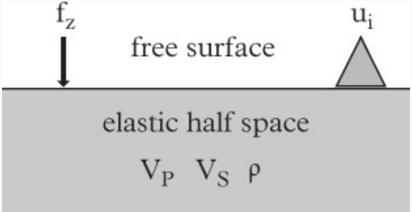
Two types of B.C.

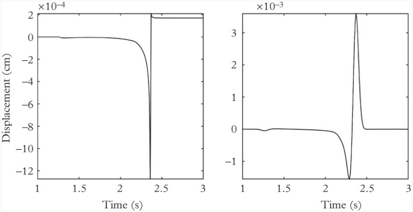
Free-surface B.C.: traction $t_i = \sigma_{ij} n_j = 0$ at the surface ($\Rightarrow$ surface waves)



Lamb's problem

Lamb's problem commonly used for benchmarking





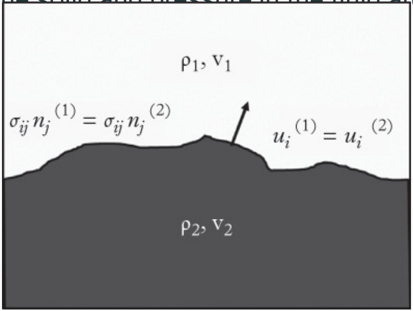
The setup and analytical solutions for Lamb's problem for free-surface bench-marking. The solution is a faint P waves followed by large-amplitude Rayleigh waves (L: Heavside STF; R: Gaussian STF).

Internal boundaries

Material discontinuities, solid-solid (S-S) interface can be treated as perfectly welded interfaces: Continuity of displacement and traction across the interface.

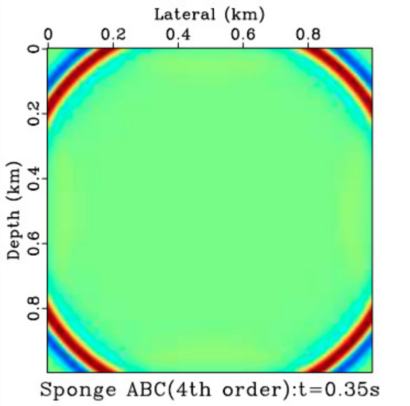
Discussion: do the accurate locations of such interfaces need to be honoured by a computational mesh? Or use homogenization?

For fluid-solid (F-S) interfaces, a different set of equations for displacement in the solid and pressure in the fluid are used.



Absorbing boundary

In practice, simulations are performed in limited regions, requiring waves to pass these virtual boundaries undisturbed (or absorbed). Implementation of **absorbing boundary condition** (ABC) can be challenging (certain f band, directions, wave types).



Solutions: Body waves

For homogeneous medium, two type of body-wave solutions, P and S waves exist

$$V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad \text{and} \quad V_s = \sqrt{\frac{\mu}{\rho}} \tag{13}$$

Plane waves can be expressed as

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{A} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}, \tag{14}$$

A is the polarization vector, **k** is the wave-number vector pointing in the direction of propagation. For P waves $\mathbf{A} \parallel \mathbf{k}$; for S waves $\mathbf{A} \perp \mathbf{k}$. Wavespeed $c = \frac{\omega}{|\mathbf{k}|} = \frac{\lambda}{T}$ (typical values for the crust: $V_p = 6$ km/s, $V_s = 3.5$ km/s, $\rho = 2.5 \times 10^3\ kg/m^3$)

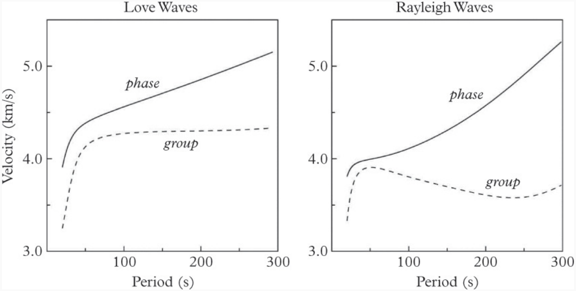
Surface waves

In Homogeneous halfspace, the incident P or S waves results in one type of surface waves, **Rayleigh waves**: polarizes in the S-R plane and propagates horizontally without dispersion (in homogeneous halfspace) with $V_R < V_s$. For layered medium, it becomes dispersive $V_R(T)$ (long-period waves typically arrival earlier).

Love waves (horizontally polarized surface-waves) does not exist in homogeneous half-space. It exists for layered medium and is always dispersive $V_L(T)$. for example it propagates with velocity between $[v_1, v_2]$ for layer (v_1) over half-space model (v_2).

Surface-wave dispersion

Resolving 1D structures based on surface-wave dispersion



Distinguish between physical dispersion, surface-wave dispersion and numerical dispersion.

Greens functions for inhomogeneous wave eqn

The linearity of the WE wrt to source means that we can separate the source terms from the details of wave propagation by introducing the Greens function that captures the complexity of wave propagation as a result of material properties.

Greens function is the solution to the PDE at (x, t) for a delta-function source at (x_0, t_0). ‘System function’ for linear system. The field due to any body-force distribution is a superposition of the Greens functions for individual point sources.

Greens function for the acoustic WE in homogeneous medium:

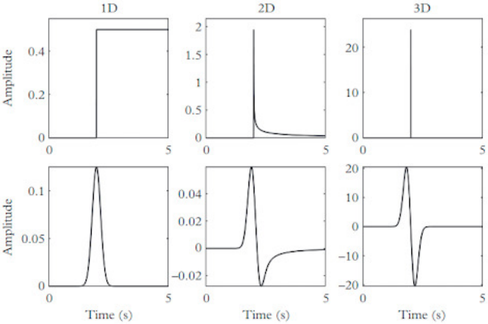
$$p(x,t) = [p_0(x - ct) + p_0(x + ct)]/2 \tag{15}$$

Greens fun for 1D/2D/3D homo.

For the acoustic WE in a homogeneous model, a δ function input source gives

Table 2.1 *Green's functions for the inhomogeneous acoustic wave equation after Rienstra and Hirschberg (2016).*

1D	**2D**	**3D**
$\frac{1}{2c}H(t-\frac{\lvert r\rvert}{c})$	$\frac{1}{2\pi c^2}\frac{H(t-\frac{\lvert r\rvert}{c})}{\sqrt{t^2-\frac{r^2}{c^2}}}$	$\frac{1}{4\pi c^2 r}\delta(t-r/c)$
$r = x$	$r=\sqrt{x^2+y^2}$	$r=\sqrt{x^2+y^2+z^2}$



Seismic source

Two type of forces for the WE:

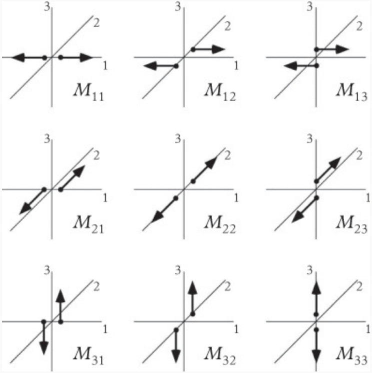
- external force $f_i(x,t)$: external processes such as pressure sources by water waves, vibroseis sources, hammer hitting the ground.
- stress perturbation as **moment-tensor** $M_{ij}(x,t)$: spontaneous processes such as natural and induced earthquakes.

$$M = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{pmatrix} \tag{16}$$

M is a symmetric second-order tensor.

Moment-tensor

$M_{jk} = f_j d\hat{k}$ represent a force couple with force vectors f_j separated by a distance d in the $\hat{k}$ direction. The diagonal elements, M_{ii}, represent a vector dipole while pairs of off-diagonal elements, $M_{ij} + M_{ji}$ $(i \neq j)$ represents **double couple force** system. Unit same as pressure N/m^2.



Scalar moment

For earthquakes as slip on a rupture plane, the equivalent point body force is a force couple (plus the corresponding auxiliary force couple forming the **double couple**) with the angular momentum of M_0, which is the **scalar moment**, defined by

$$M_0 = \mu A d \tag{17}$$

where μ is the shear modulus at the source, A is the slip surface area, and d is the average slip on the fault.

This is one of **the most important results in seismology** because it relates physical properties of the earthquakes source to the DC source model and ultimately the observed seismic waves. It also determines the moment magnitude of eqk (hence radiated energy): $M_w = \frac{2}{3} \log_{10} M_0 - 6$.

DC Source model

For a double-couple source with only M_{12} and M_{21} elements,

$$M^{dc} = M_0 \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{18}$$

where the scalar moment is defined as

$$M_0 = \frac{1}{\sqrt{2}} \left(\sum_{ij} M_{ij}^2 \right)^{1/2} \tag{19}$$

The definition is different for explosive source

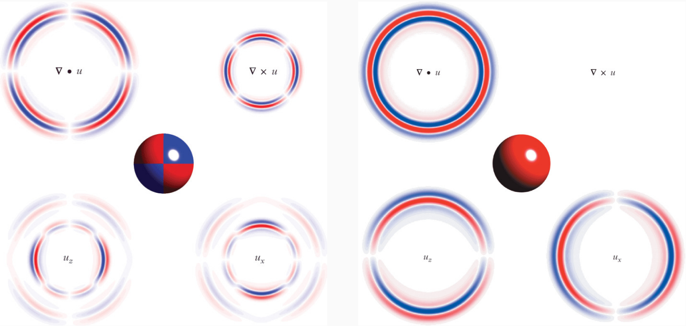
$$M^{expl} = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{20}$$

Displ. due to DC source

For MT source M placed at (x_0, t_0),

$$u_i(x,t) = \partial_{(x_0)_k} G_{ij}(x,t;x_0,t_0) M_{jk}(x_0,t_0) \tag{21}$$

A DC MT source leads to anisotropic radiation pattern (how amplitude of waves change as a function of direction from the source) compared to an explosive source



P and S radiation pattern

Kinematic source studies: linking earthquake sources to observable such as far-field seismograms or crustal deformation through analytical/numerical solutions. For a DC source, M_{xz}+M_{zx}, in an infinite homogeneous medium, **far-field** displacement

$$u(x,t) = \frac{A^{FP}}{4\pi\rho\alpha^3 r}\dot{M}_0(t - r/\alpha) + \frac{A^{FS}}{4\pi\rho\beta^3 r}\dot{M}_0(t - r/\beta) \tag{22}$$

where $A^{FP} = \sin 2\theta \cos\phi\,\hat{r}$ and $A^{FS} = \cos 2\theta \cos\phi\,\hat{\theta} - \cos\theta \sin\phi\,\hat{\phi}$.

- $u \propto \frac{1}{\rho c^3 r}$, geometrical spreading factor of $1/r$.
- amplitude of S waves are $\sim$ 5 times of P waves
- near-field ($1/r^3$), and intermediate field ($1/r^2$) terms are ignored
- displacement waveform follows the moment-rate function.

Radiation patterns for P and S

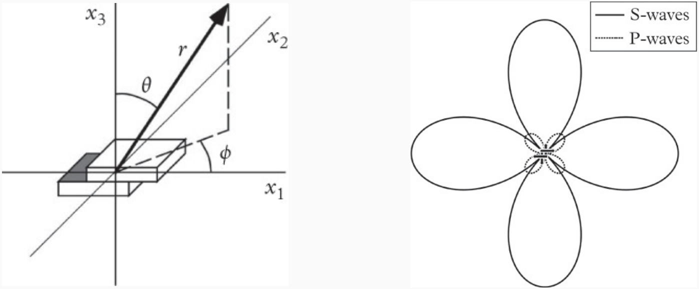
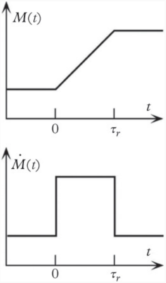


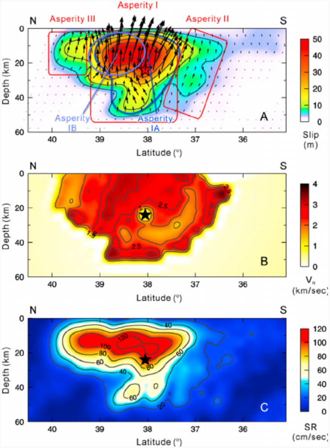
Figure 7: Far-field radiation pattern of a shear-dislocation source. The figure illustrates the strongly varying radiation of P-waves (dotted) and shear waves (solid) for a double-couple point source (indicated at the centre). Note approximately five times ($\sim 1/(3^{3/2})$) larger peak amplitude of S-waves compared to P-waves.

Source time function

$M_0(t)$ is the defined as the moment time function (or $M_0(t) = M_0 s(t)$, source time function); $\dot{M}_0(t)$ is defined as the moment-rate function. Rise time τ_r determines the slip velocity across the fault plane $V = D/\tau_r$. Far field waveform $\propto M_0(t)$.



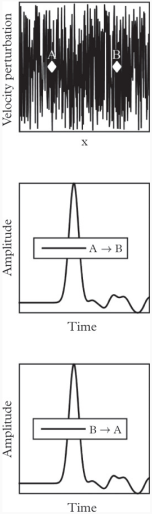
Finite-fault simulations



(a) Fault slip distribution of the Tohoku Oki earthquake, asperities are shown with red open squares, (b) distribution of rupture velocity (V_R), and (c) distribution of maximum slip rate (S_R) (from Lee et al 2011).

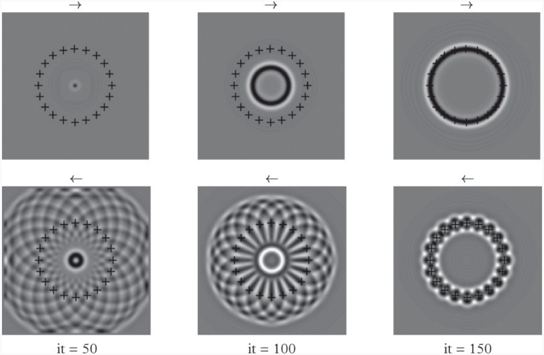
Source receiver reciprocity

The concept of **reciprocity** is a direct result of the linear constitutive relationship, $G_{ij}(x, t, x_0) = G_{ji}(x_0, t; x)$



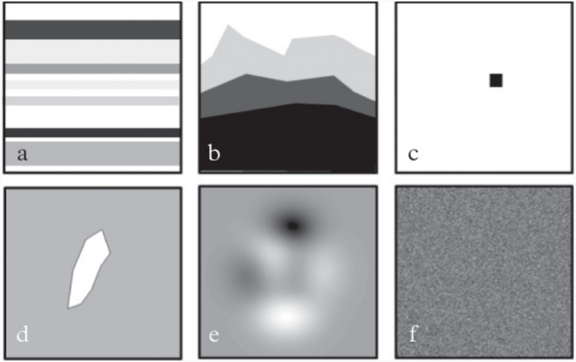
Time reversal

- The symmetry of elastic wave equation in time (replace t with $-t$ does not change the equation) makes the propagation reversible (ignoring anelasticity).
- Recorded seismograms at receivers can be simultaneously injected to back-propagate and focus at the original source locations.
- time-reversal source imaging: back-projection methods; reverse-time migration, FWI



Scattering

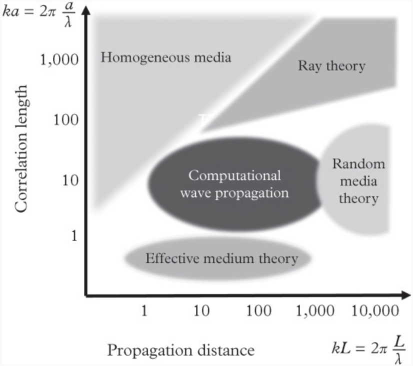
Characterize heterogeneity inside the Earth: choose computational methods?



layered; single scatter; cavity; smooth; all scales. Should mesh honor all interfaces? Homogenization?

Scattering Regime

Investigate **scattering regimes**: correlation length of a medium change, *a*, vs the dominant wavelength λ of the propagating field.



Small scatters ($ka \ll 1$), effective medium theory; large-scale heterogeneity ($ka \gg 1$), locally homogenious, ray theory; strong scattering regime $a \sim \lambda$, or $ka \sim 1$: numerical methods or radial transfer theory.