Factorization and effective field theory (or How to Finesse the Strong Interactions)

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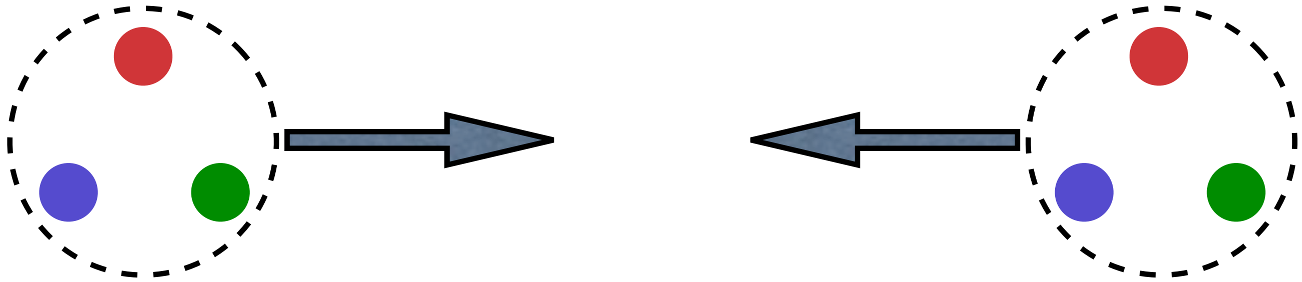
Outline:

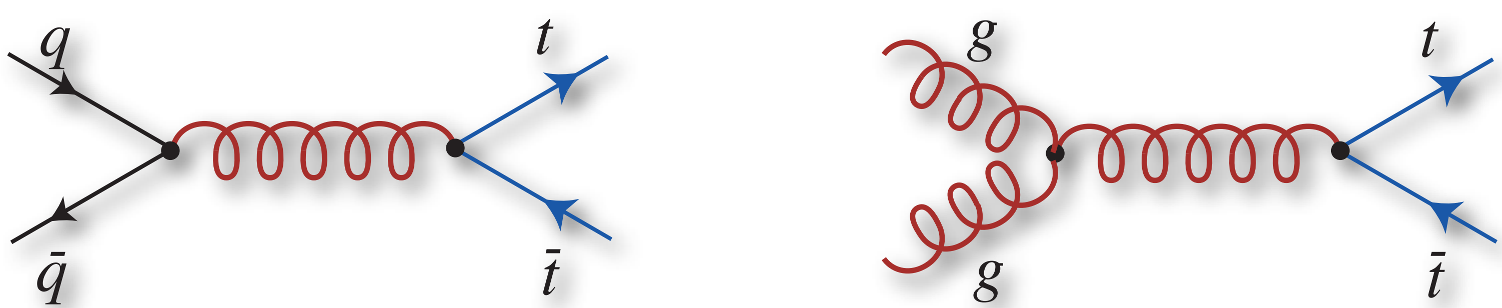
1. The problem
2. Factorization - 70's & 80's (partons)
3. Effective Field Theory - classic, modern and postmodern
4. Some applications

The Probem: **How do we do physics at proton colliders at all?** (i.e. Tevatron, LHC)



Colliding protons → Colliding quarks and gluons

i.e. top production at Fermilab:



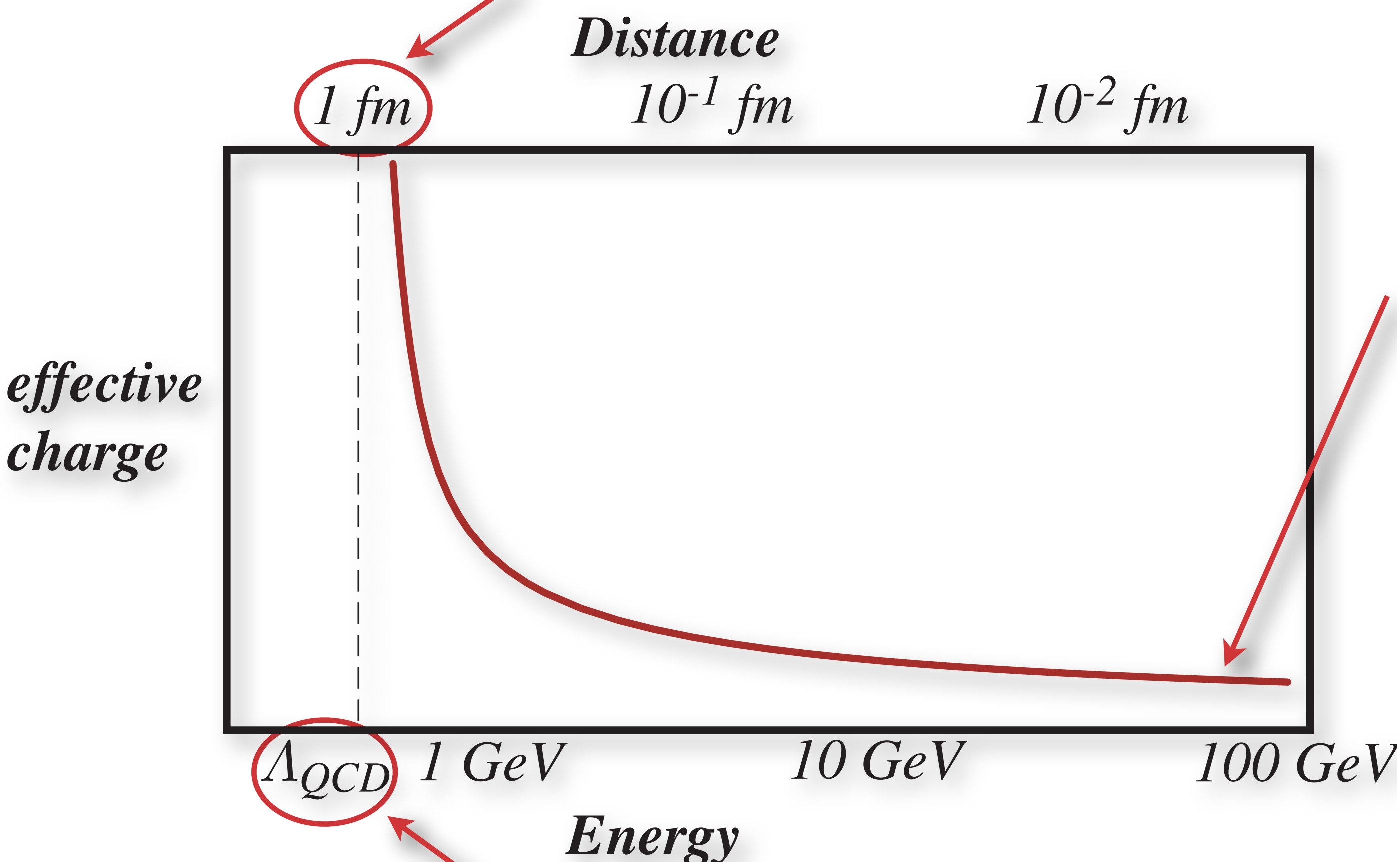
... this is the physics we want to study

... but protons aren't so simple ...

"Quantum Chromodynamics" (QCD)

(Gross, Politzer, Wilczek - Nobel Prize, 2004)

1 fm = 10^{-15} m ~ radius of proton



"asymptotic freedom":

effective QCD CHARGE of quarks/gluons under is small at SHORT distances (large energies), large at LONG distances (low energies)

Λ_{QCD} ~300 MeV sets the scale for nonperturbative effects

$$\Lambda_{\mathrm{QCD}} \sim 300\,\mathrm{MeV} \sim \frac{1}{3} m_{\mathrm{proton}} \qquad \frac{1}{\Lambda_{\mathrm{QCD}}} \sim 1\,\mathrm{fm} \sim r_{\mathrm{proton}}$$

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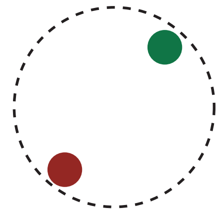
(1) sets the maximum size of a hadron

$1\ \mathrm{fm}$



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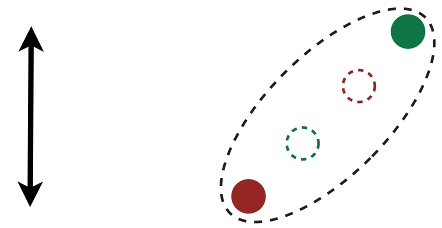

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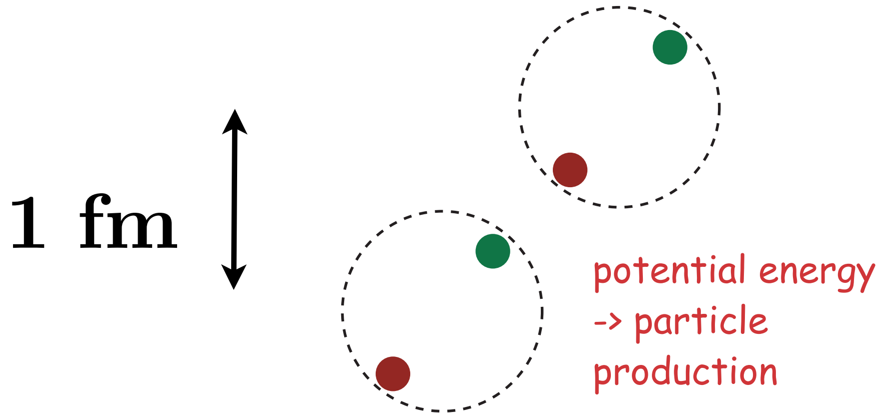

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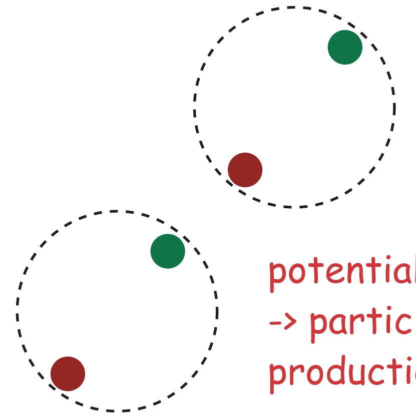
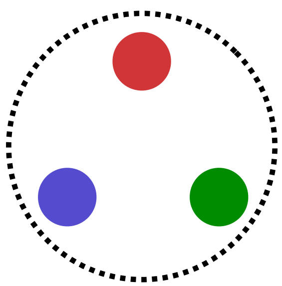


potential energy
-> particle
production

(2)



$m_{\mathrm{up}} \sim 5\ \mathrm{MeV}$

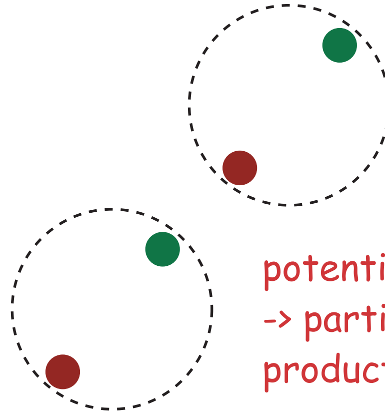
$m_{\mathrm{down}} \sim 10\ \mathrm{MeV}$

$\ll \Lambda_{\mathrm{QCD}}$

$$\Lambda_{\rm QCD} \sim 300\,{\rm MeV} \sim \frac{1}{3} m_{\rm proton} \qquad \frac{1}{\Lambda_{\rm QCD}} \sim 1\,{\rm fm} \sim r_{\rm proton}$$

(1) sets the maximum size of a hadron

1 fm $\updownarrow$



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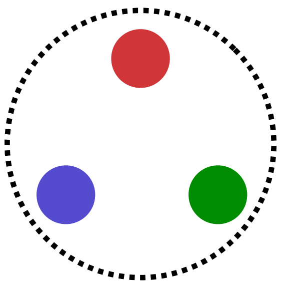

$$m_{\rm up} \sim 5\ {\rm MeV}$$
$$m_{\rm down} \sim 10\ {\rm MeV}$$
$$\ll \Lambda_{\rm QCD}$$

but Heisenberg: $\Delta p \sim \dfrac{1}{\Delta x} \sim \Lambda_{\rm QCD} \gg m_{u,d}$

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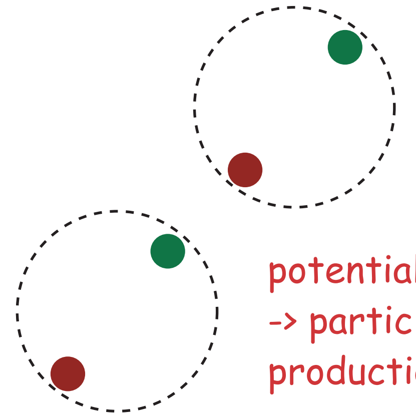

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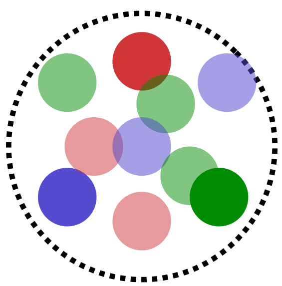
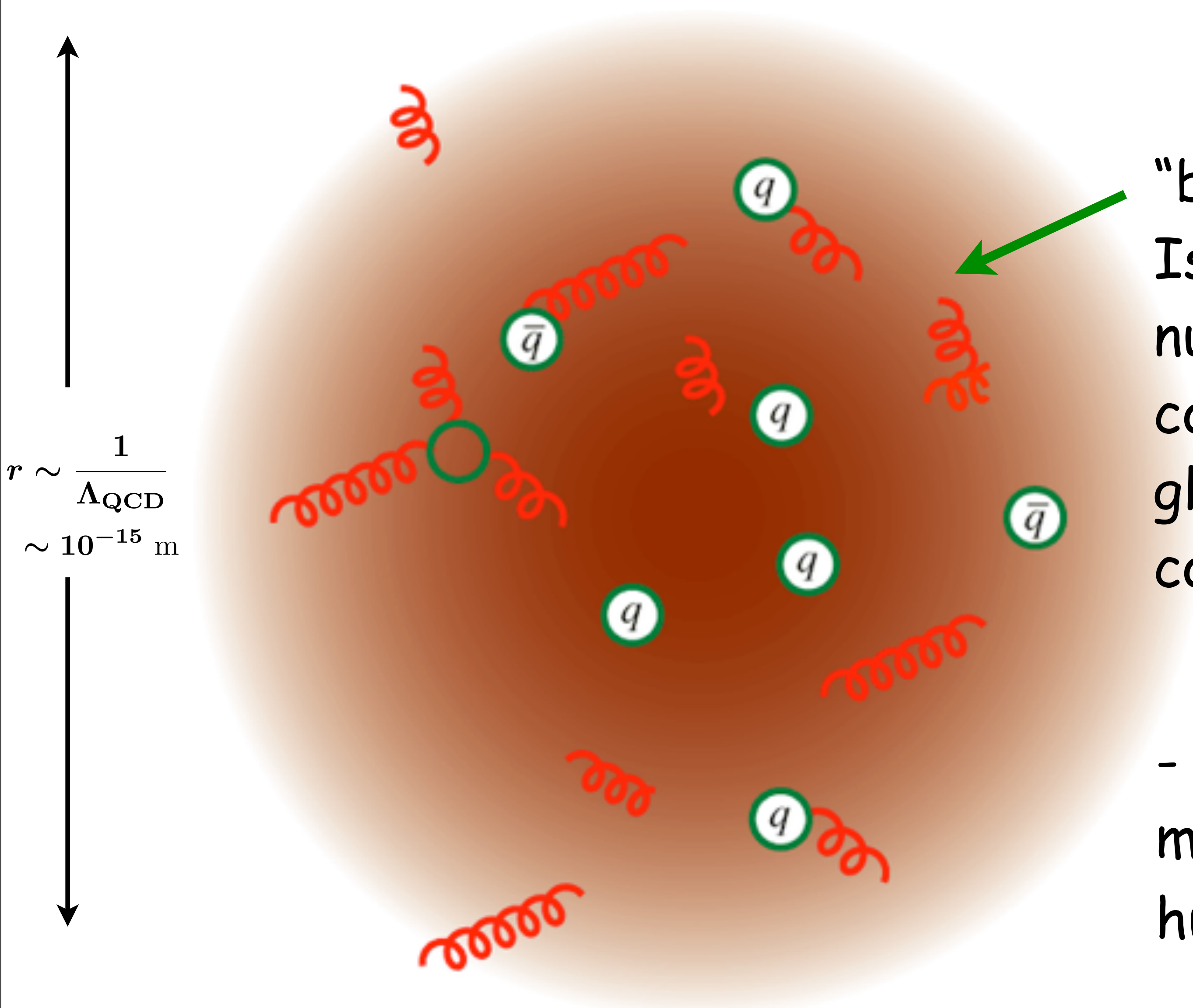


$$\left.\begin{array}{l} m_{\mathrm{up}} \sim 5\ \mathrm{MeV} \\ m_{\mathrm{down}} \sim 10\ \mathrm{MeV} \end{array}\right. \ll \Lambda_{\mathrm{QCD}}$$

but Heisenberg: $\Delta p \sim \dfrac{1}{\Delta x} \sim \Lambda_{\mathrm{QCD}} \gg m_{u,d}$

-> particle production! Indeterminate number of quarks in proton

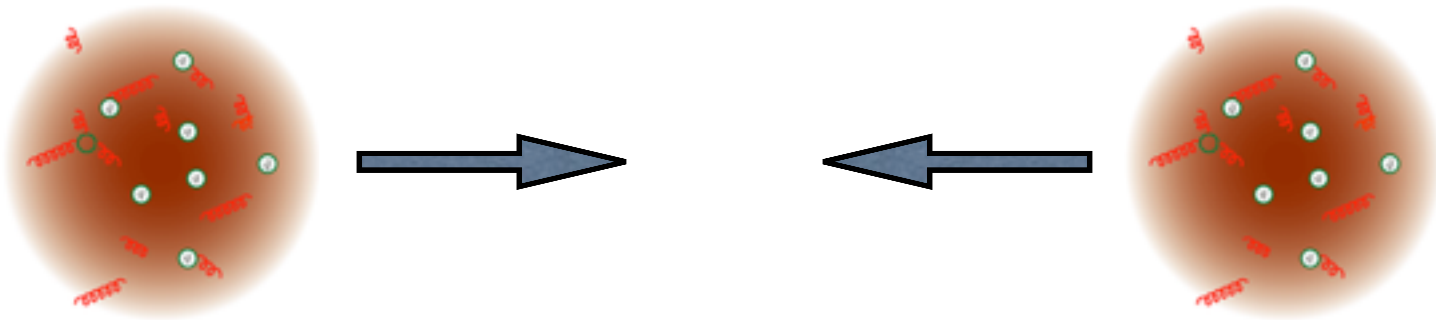
So a proton looks something like this:



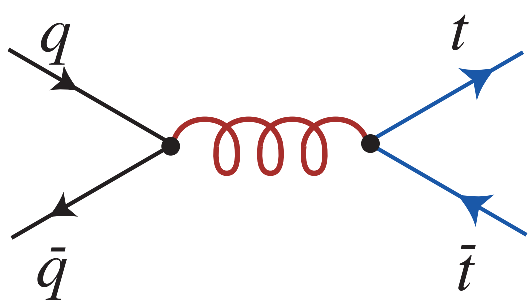
"brown muck" of QCD (N. Isgur) - an indeterminate number of strongly coupled light quarks and gluons (horrible strongly-coupled mess)

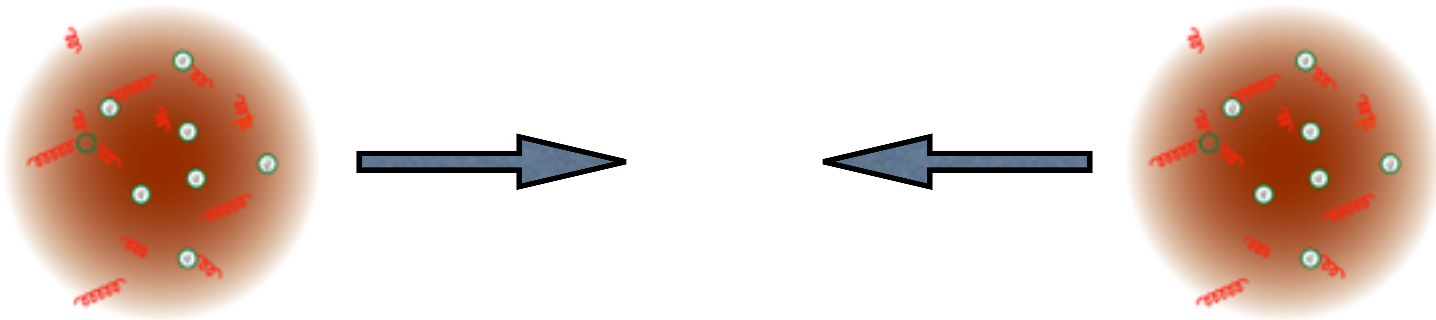
- quarks & gluons all have momentum ~Λ_{QCD}~few hundred MeV

(Actually, it's a linear superposition of all these states ...)



... so our simple quark-level process





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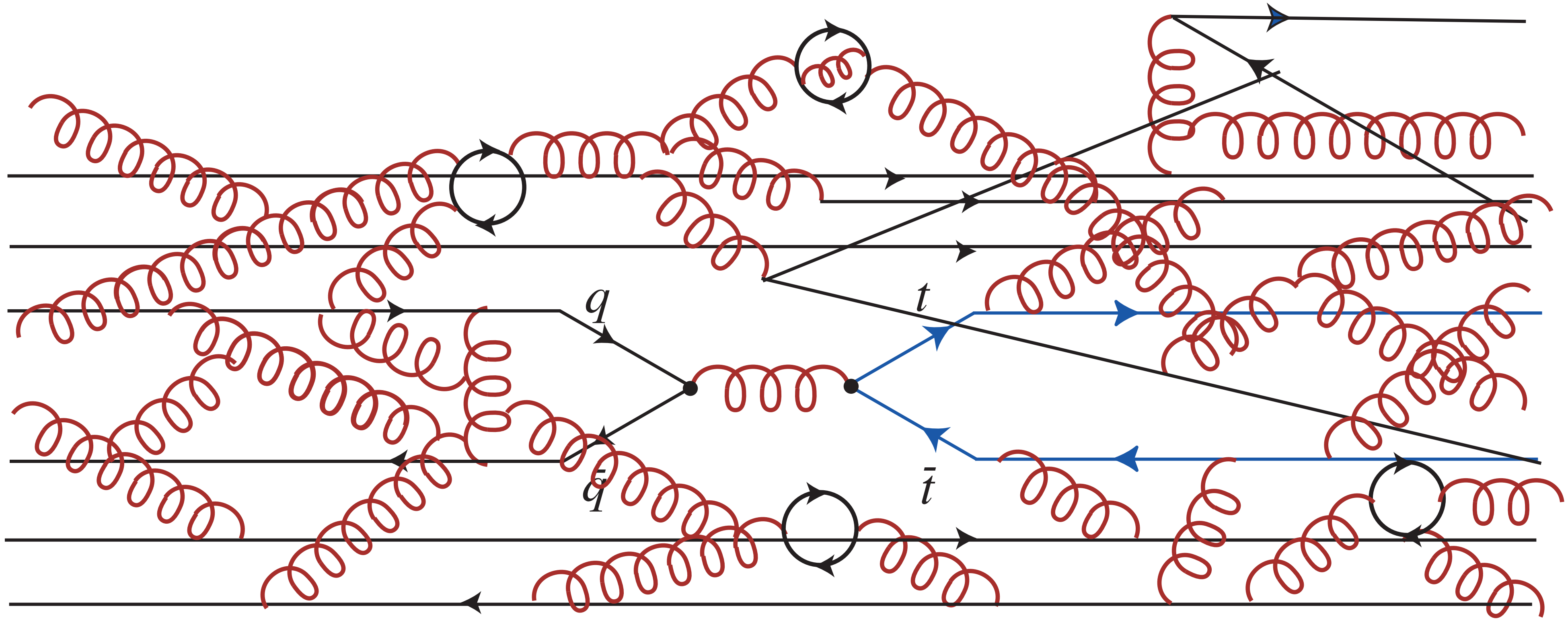


... is buried in the muck.

How can we calculate anything without solving QCD?

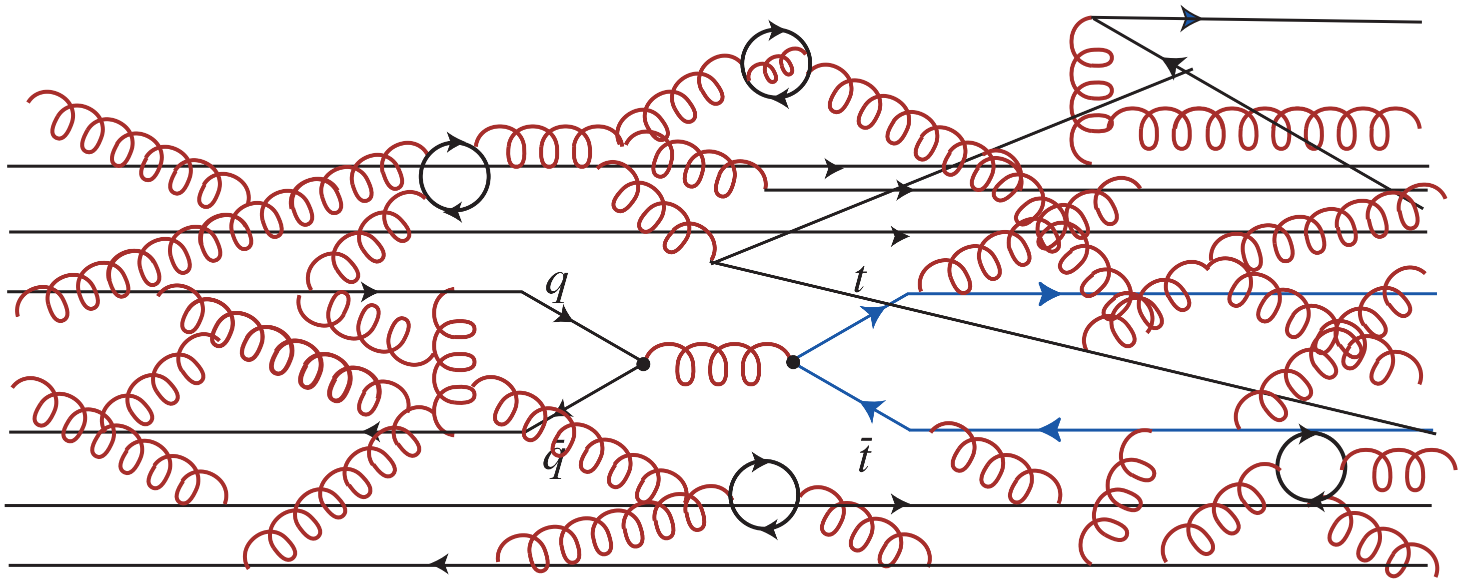
A miracle occurs .... "Factorization"

$$\sigma(p(P_1) + p(P_2) \to t\bar{t} + X)$$

(NB for simplicity, neglecting top quark decay)

$$= \int_0^1 dx_1 dx_2 \sum_f f_f(x_1) f_{\bar{f}}(x_2) \cdot \sigma(q_f(x_1 P) + \bar{q}_f(x_2 P) \to t\bar{t})$$

$$+O\left(\frac{\Lambda_{\mathrm{QCD}}}{2m_t}\right)$$



(Feynman, Bjorken)

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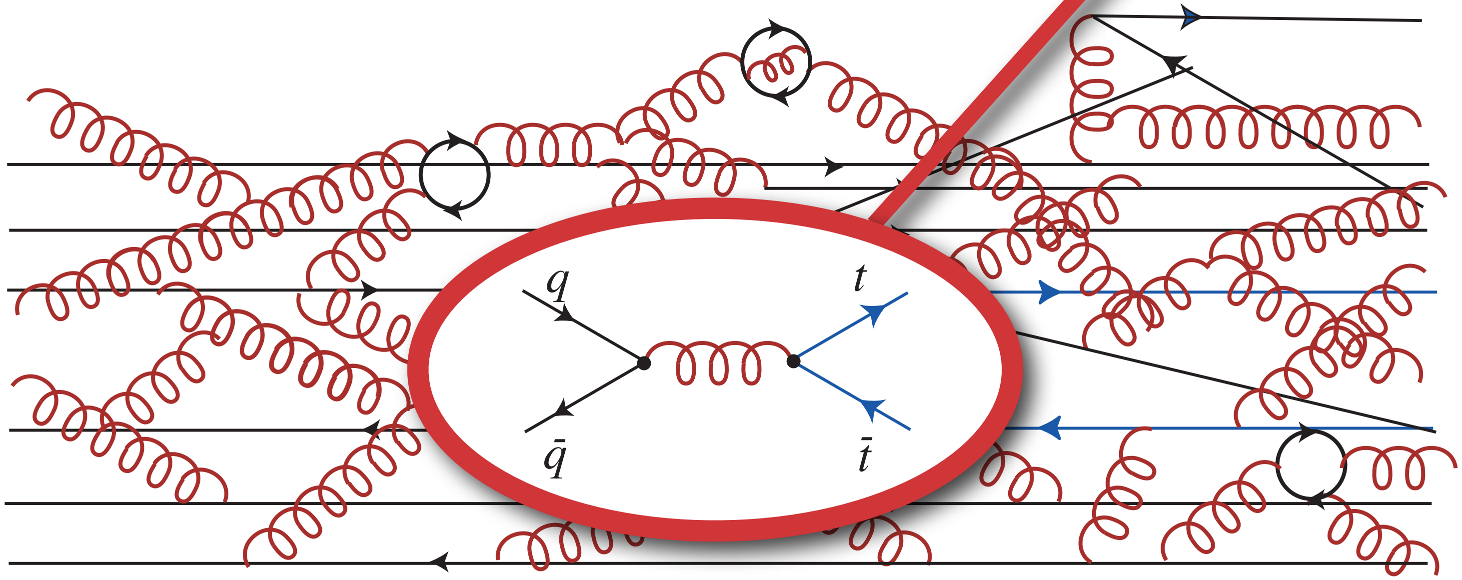

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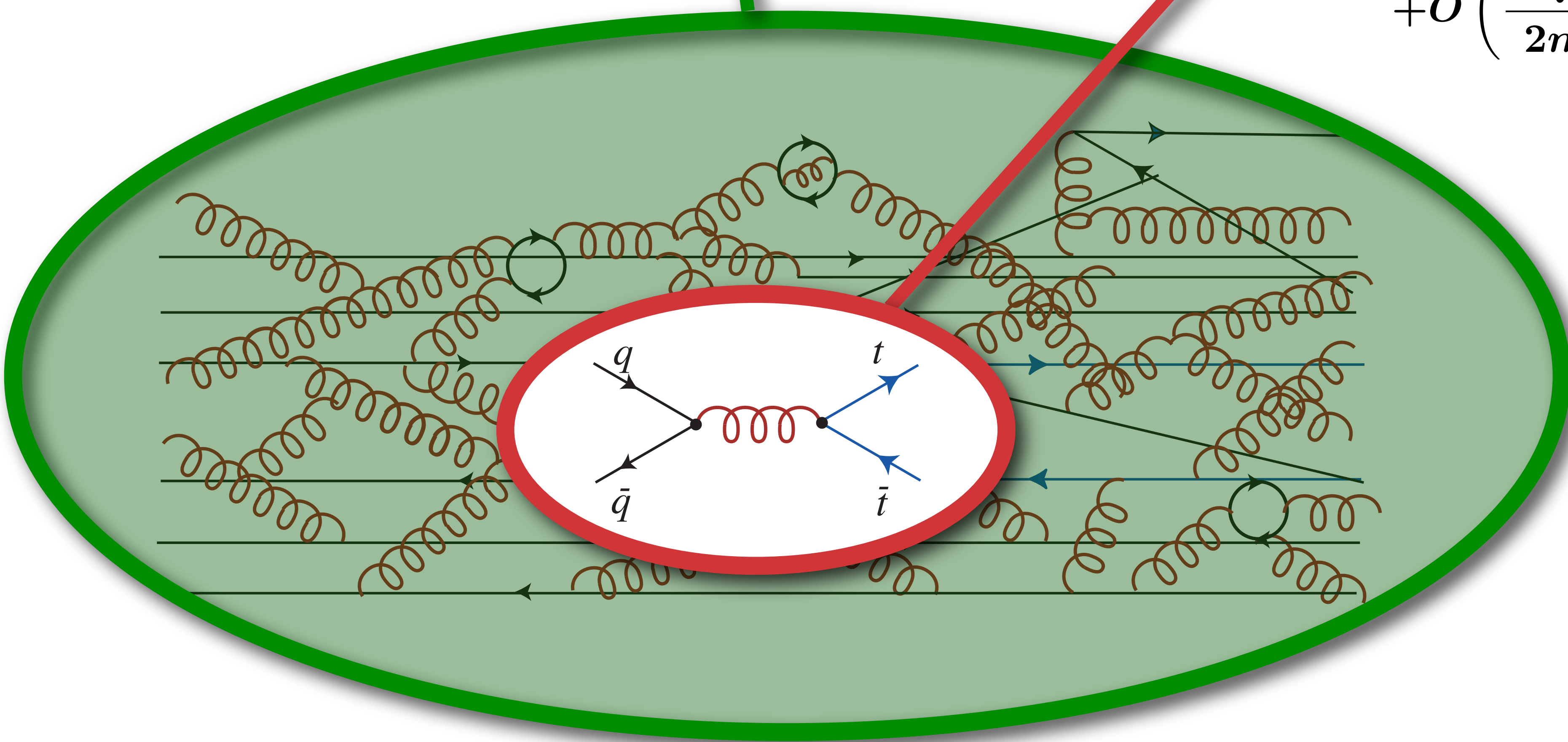

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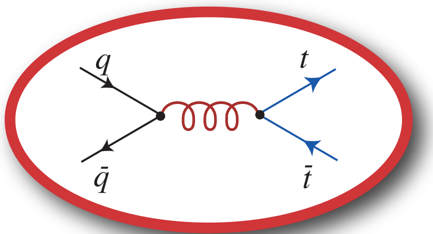
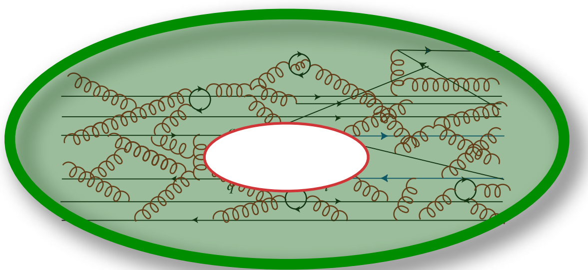


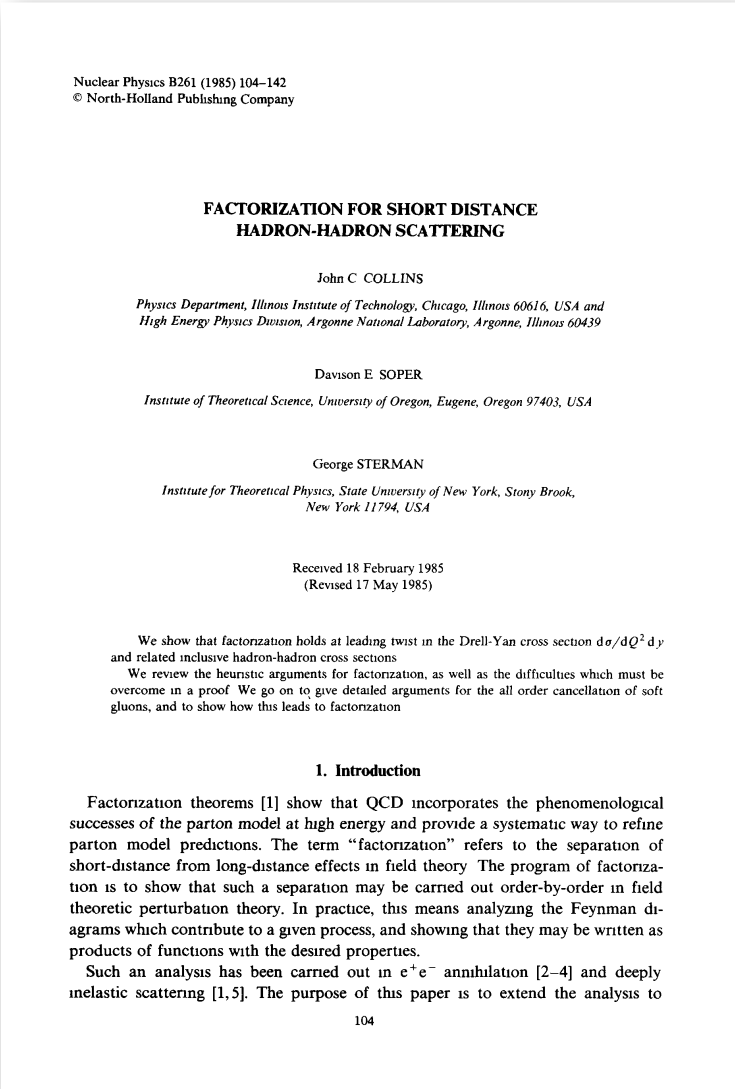
SHORT DISTANCE: cross section for free quarks (and gluons) - can calculate in perturbation theory



LONG DISTANCE: $f_f(x_1)$:probability to find parton f with fraction x_1 of longitudinal momentum of proton ("parton distribution function") - property of the PROTON - can't calculate ... but UNIVERSAL (can measure in another process)

The proofs of factorization are long and complicated

(and based on exhaustive analysis of Feynman diagrams ...)



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FACTORIZATION FOR SHORT DISTANCE HADRON-HADRON SCATTERING

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We show that factorization holds at leading twist in the Drell-Yan cross section $d\sigma/dQ^2\,dy$ and related inclusive hadron-hadron cross sections

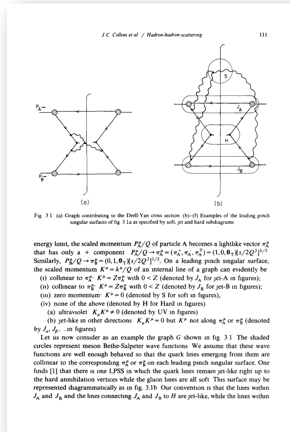
We review the heuristic arguments for factorization, as well as the difficulties which must be overcome in a proof We go on to give detailed arguments for the all order cancellation of soft gluons, and to show how this leads to factorization

1. Introduction

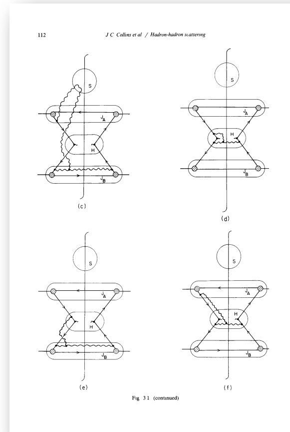
Factorization theorems [1] show that QCD incorporates the phenomenological successes of the parton model at high energy and provide a systematic way to refine parton model predictions. The term "factorization" refers to the separation of short-distance from long-distance effects in field theory The program of factorization is to show that such a separation may be carried out order-by-order in field theoretic perturbation theory. In practice, this means analyzing the Feynman diagrams which contribute to a given process, and showing that they may be written as products of functions with the desired properties.

Such an analysis has been carried out in e^+e^- annihilation [2–4] and deeply inelastic scattering [1,5]. The purpose of this paper is to extend the analysis to

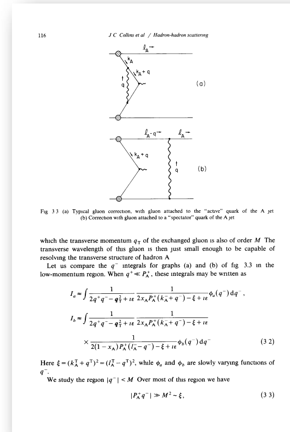
104



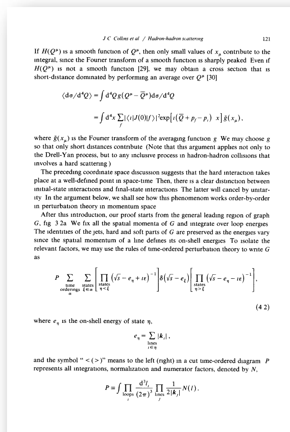
page 8



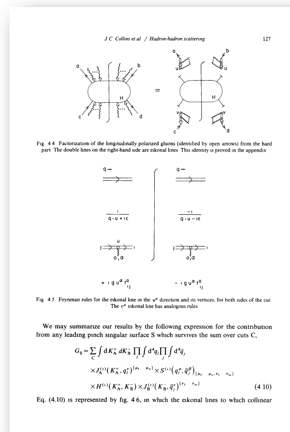
page 9



page 13



page 18



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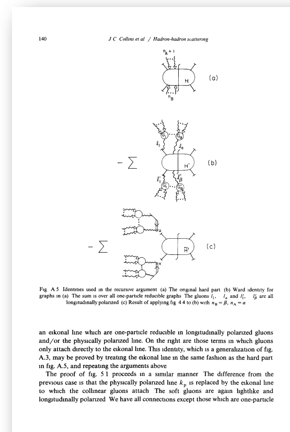
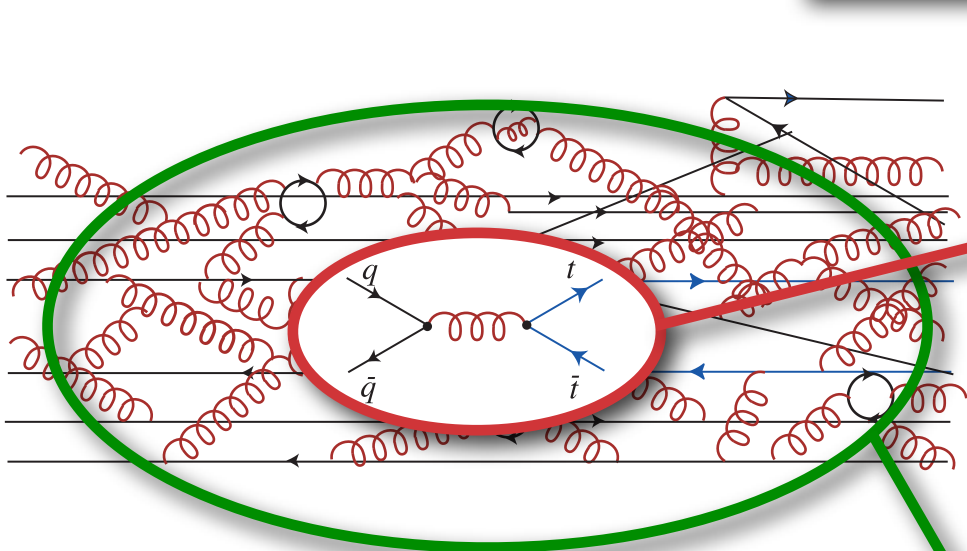
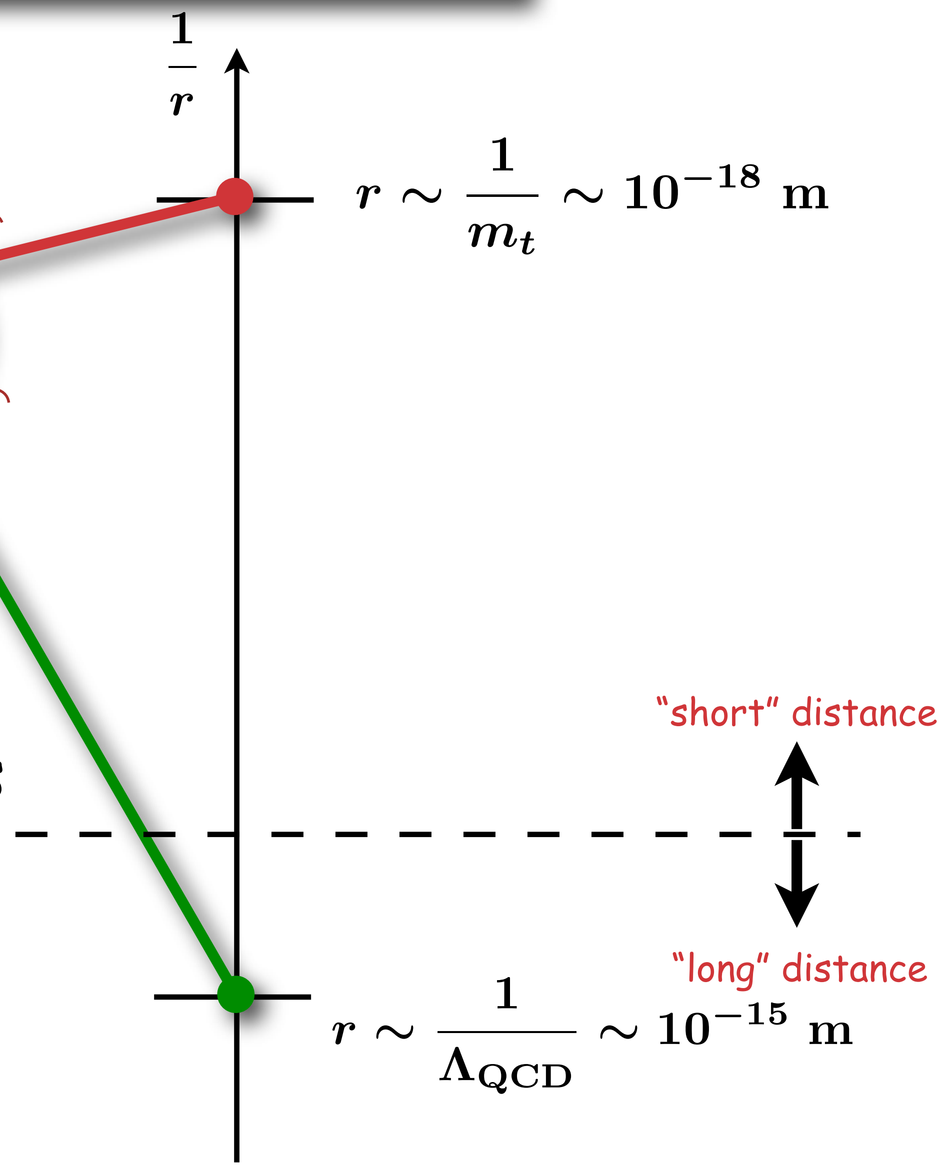


page 37

(Collins, Soper, Sterman, 1980's)

... but the physics is simple: **Separation of Scales**





- top quark production is a short-distance process, hadronic physics is long-distance
- hadronic physics cannot resolve details of short-distance physics - hadronization is independent of details of scattering (so parton distributions are universal)

COMMENTS:

$$\sigma(p(P_1)+p(P_2)\to t\bar{t}+X) \;= \int_0^1 dx_1 dx_2 \sum_f f_f(x_1) f_{\bar{f}}(x_2)\cdot\sigma(q_f(x_1P)+\bar{q}_f(x_2P)\to t\bar{t}) + O\left(\frac{\Lambda_{\rm QCD}}{2m_t}\right)$$

- form of the factorization formula (convolution over light-cone momentum fraction) is non-trivial

- final hadronic state unspecified - sum over all of them ("+X") - probability to hadronize = 1! **"inclusive"**

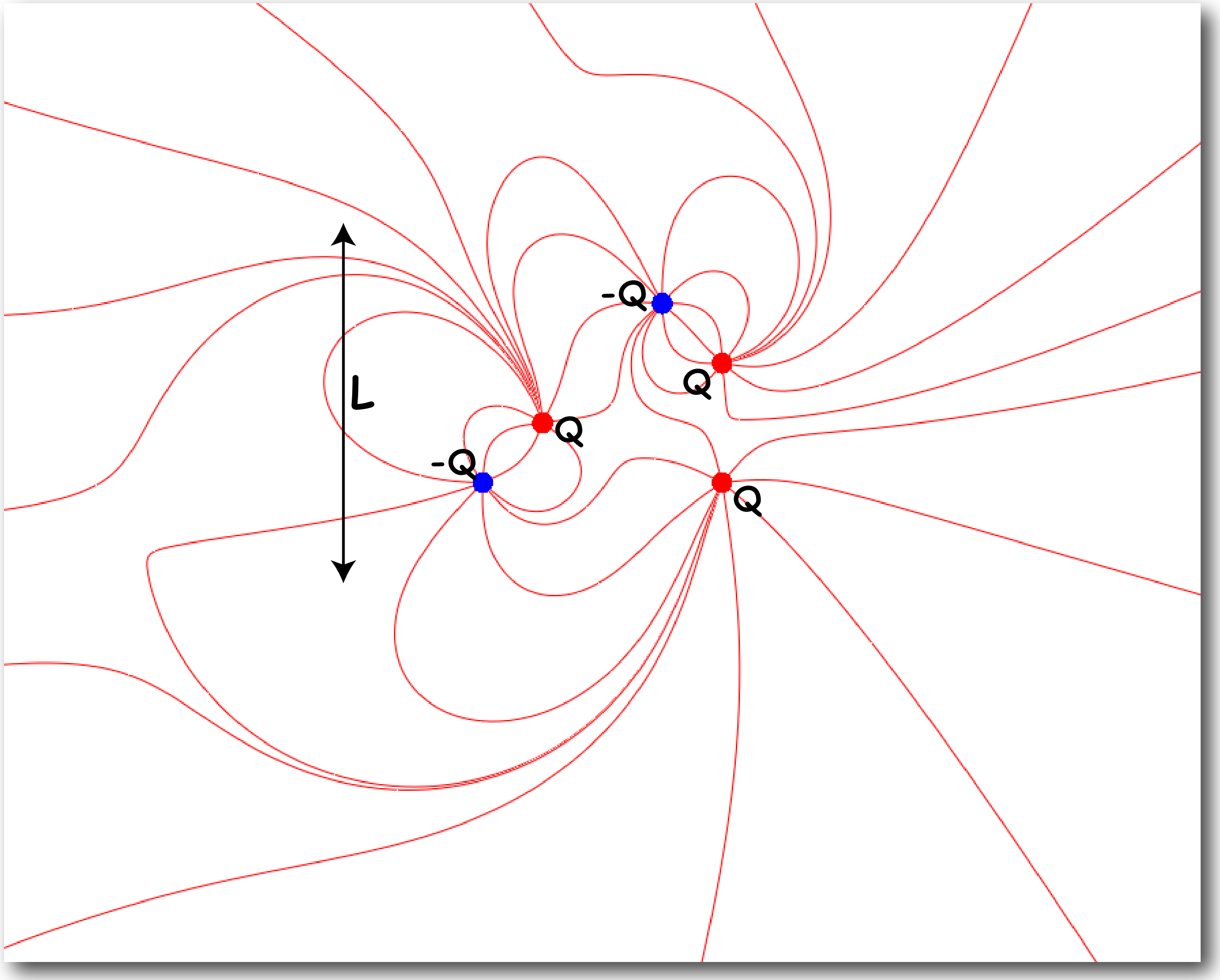
- subleading ($O(\Lambda_{QCD}/Q)$) terms (**"power corrections"**) don't factorize in this way ... fortunately, these are small for $Q\sim 2m_t$ - don't generally worry about going to higher orders

More generally, multi-scale problems are complicated theoretically:

- Perturbation theory breaks down - terms in perturbation theory are enhanced by powers of $\log(m_1/m_2)$ - if ratio is large, perturbation theory breaks down even at weak coupling
- Perturbative and nonperturbative physics is hard to separate
- QCD factorization theorems and the like have power corrections proportional to the ratios of scales - need a systematic expansion to go beyond leading order
- You shouldn't use quantum gravity to calculate projectile motion!

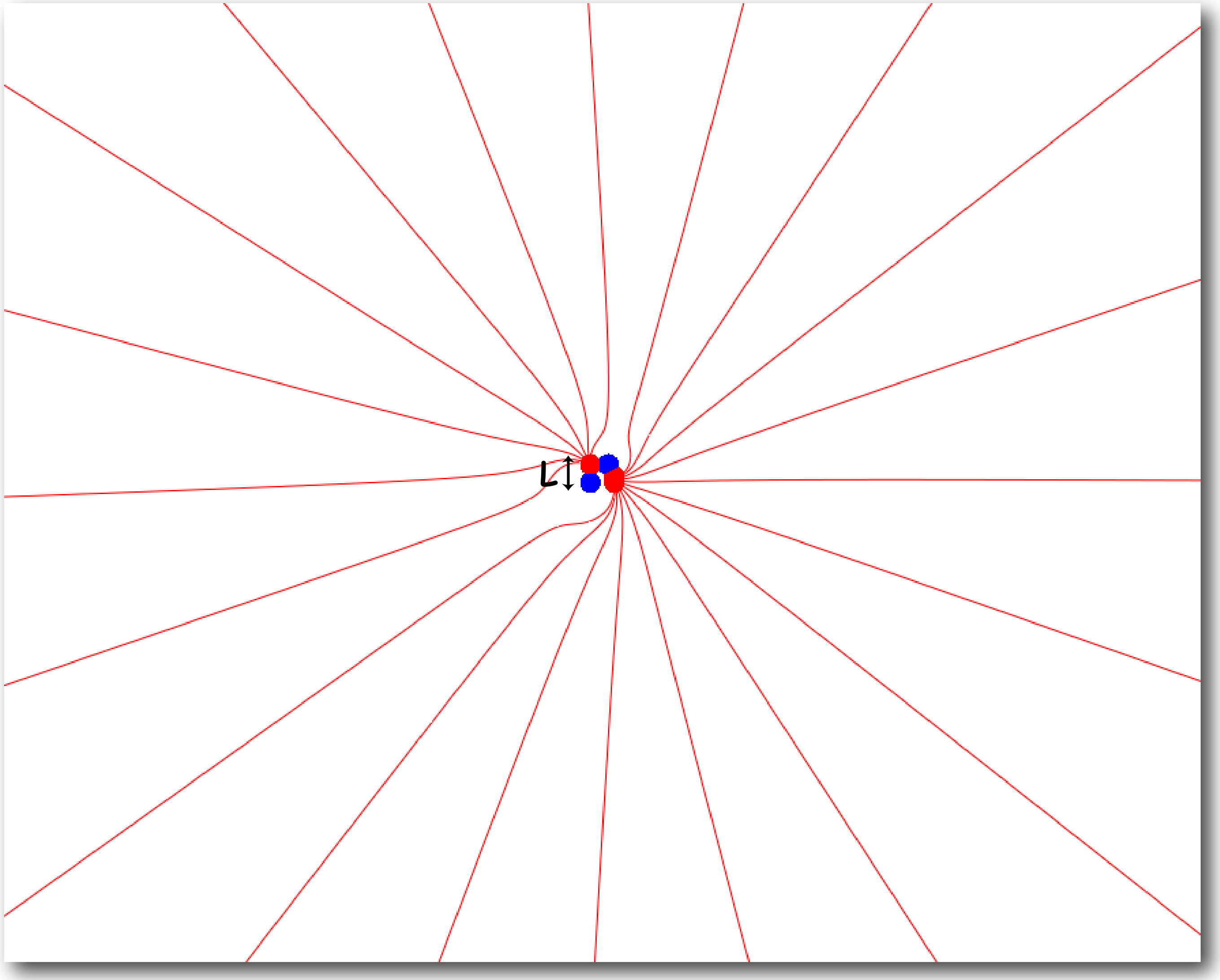
Particle physics is full of important multi-scale problems ... i.e. GUT-scale physics, b-quark decays, Standard Model extensions ... how can we deal with this problem systematically?

We can do this in classical electrodynamics:



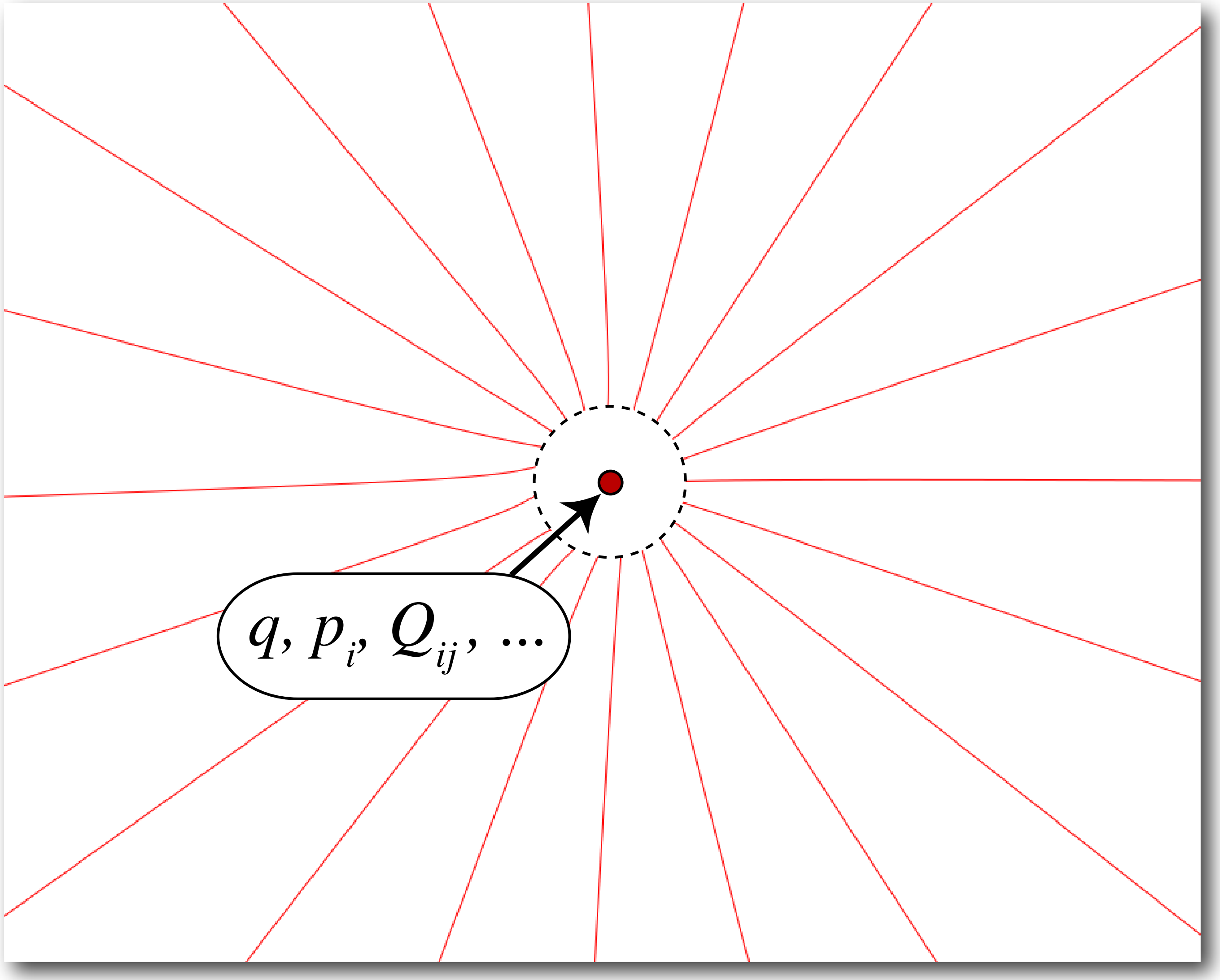
Physics at r~L is complicated - depends on details of charge distribution

We can do this in classical electrodynamics:



BUT ... if we are interested in physics at r>>L, things are much simpler ...

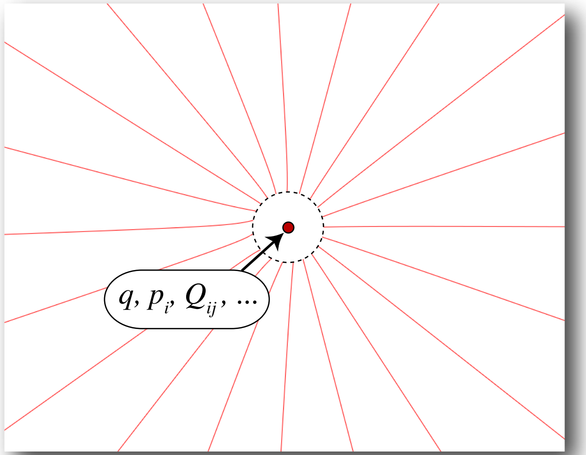
We can do this in classical electrodynamics:



... can replace complicated charge distribution by a POINT source with additional interactions (multipoles)...

Multipole expansion:

$$\boxed{V(r) = \frac{q}{r} + \frac{\vec{p}\cdot\vec{x}}{r^3} + \frac{1}{2}Q_{ij}\frac{x_i x_j}{r^5} + \cdots}$$



$q,\ p_i,\ Q_{ij},\ \ldots$: short distance quantities (depend on details of charge distribution)

$\left\langle \frac{1}{r} \right\rangle, \left\langle \frac{x_i}{r^3} \right\rangle, \left\langle \frac{x_i x_j}{r^5} \right\rangle, \cdots$: long distance quantities (independent of short distance physics)

FACTORIZATION!

higher multipole moments <-> new effective interactions from "integrating out" short distance physics .. effects are suppressed by powers of L/r

Field Theory generalization: Effective Field Theory

-at low momenta p<<Λ, a theory can be described by an effective Hamiltonian where degrees of freedom at scale Λ have been "integrated out":

$$H_{\text{eff}} = H_0 + \underbrace{\sum_i \frac{C_i}{\Lambda^{n_i}} \mathcal{O}_i}_{\text{corrections determined by matrix elements of operators } O_i \text{ - power counting determined by dimensional analysis}}$$

Hamiltonian in Λ->∞ limit

C_n's : short distance quantities (in QCD: perturbatively calculable if Λ >>Λ_{QCD})

$\langle \mathcal{O}_n \rangle$'s : long distance quantities (in QCD: nonperturbative ... need to get them elsewhere)

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corrections determined by matrix elements of operators O_i - power counting determined by dimensional analysis

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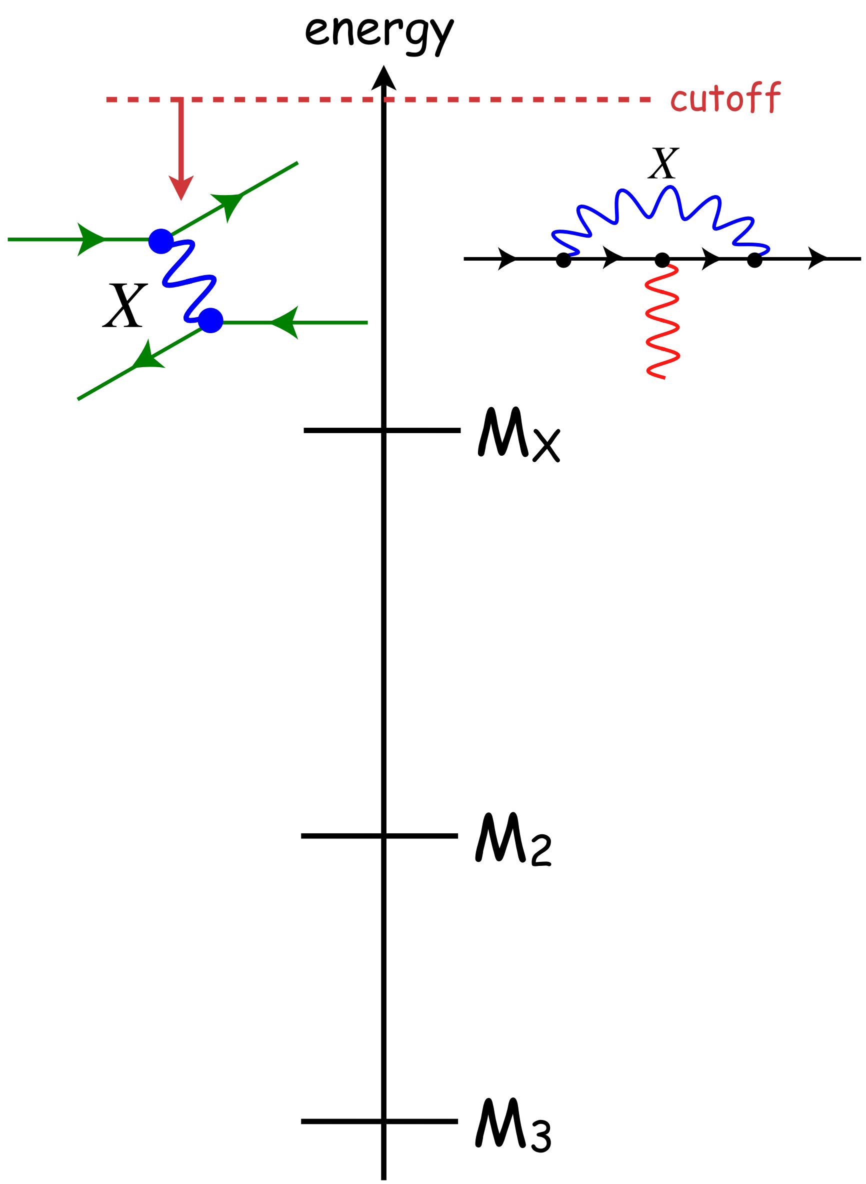
$\langle \mathcal{O}_n \rangle$'s : long distance quantities (in QCD: nonperturbative ... need to get them elsewhere)

- Effective Field Theory automatically factorizes the calculation
- by keeping more terms, can work to arbitrary accuracy in 1/Λ

(1) "Classic" Effective Field Theory (4-fermi theory and the like):

- lowering cutoff - effects of virtual excitations removed from dynamics, incorporated into parameters of theory (Renormalization Group)



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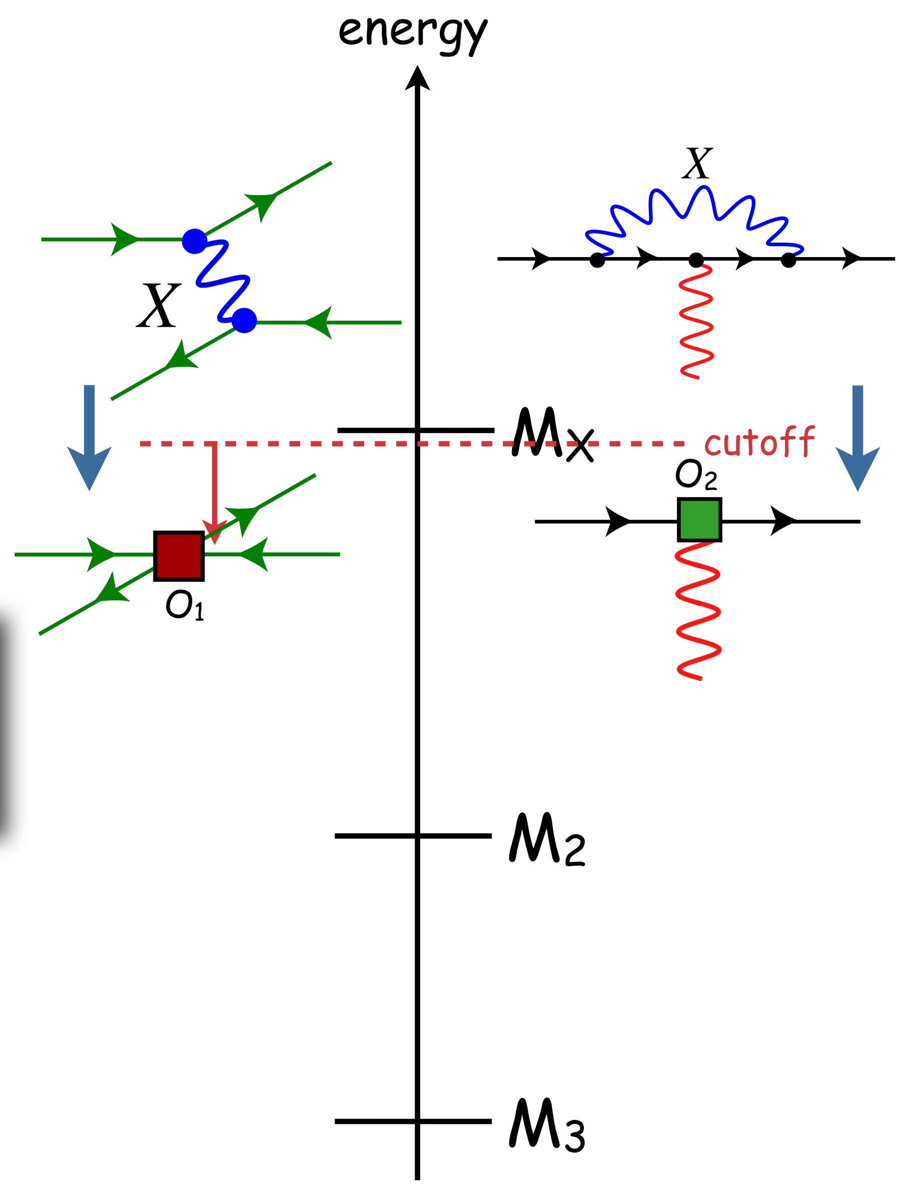
- at thresholds, heavy particles removed from theory ("integrated out"), effects incorporated into local operators

$$\boxed{H(\Lambda < m_X) \sim H(\Lambda > m_X) + \sum_i \frac{C_1}{M_X^{n_i}} \mathcal{O}_i}$$



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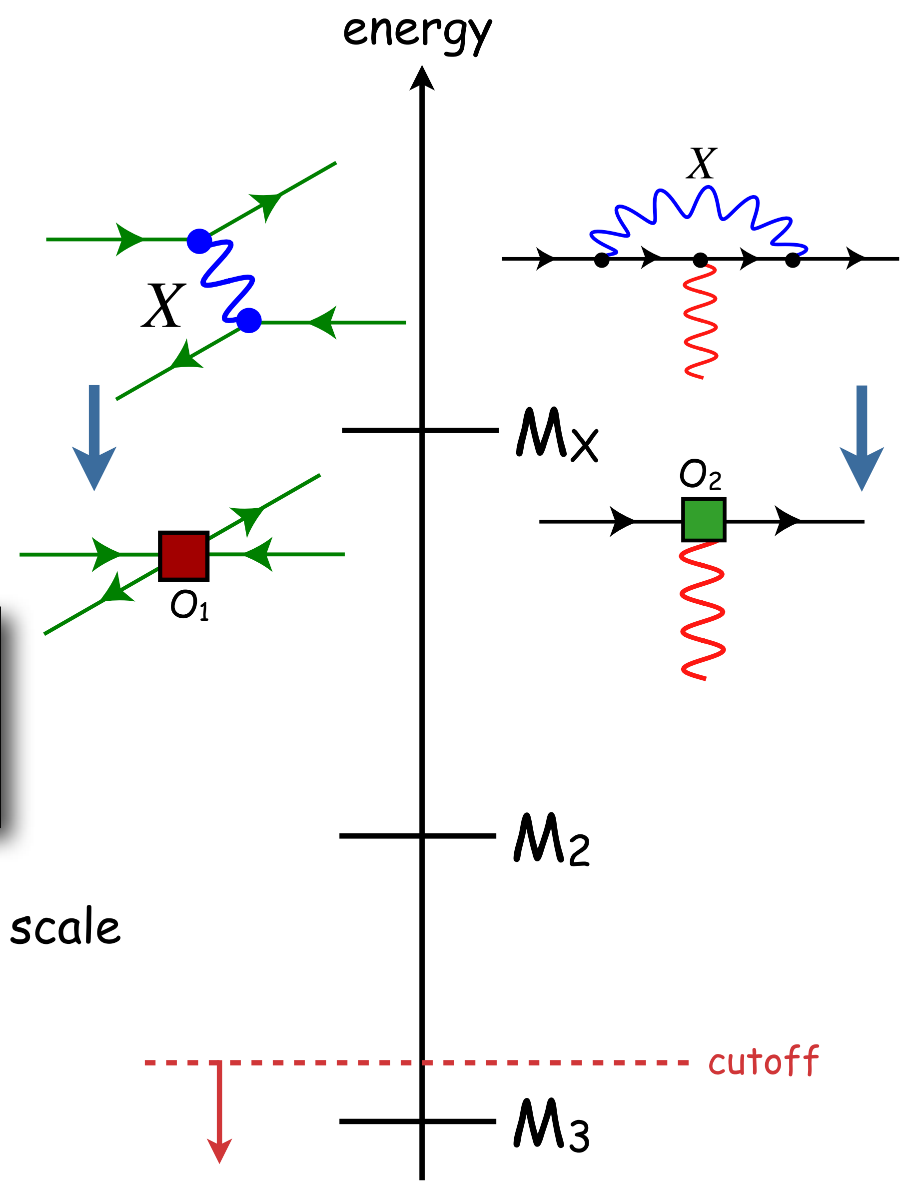
- ideally, keep lowering cutoff until only a single scale is left ... all short-distance physics is now in the coefficients C_i of local operators, long distance physics is in their matrix elements - **FACTORIZATION**

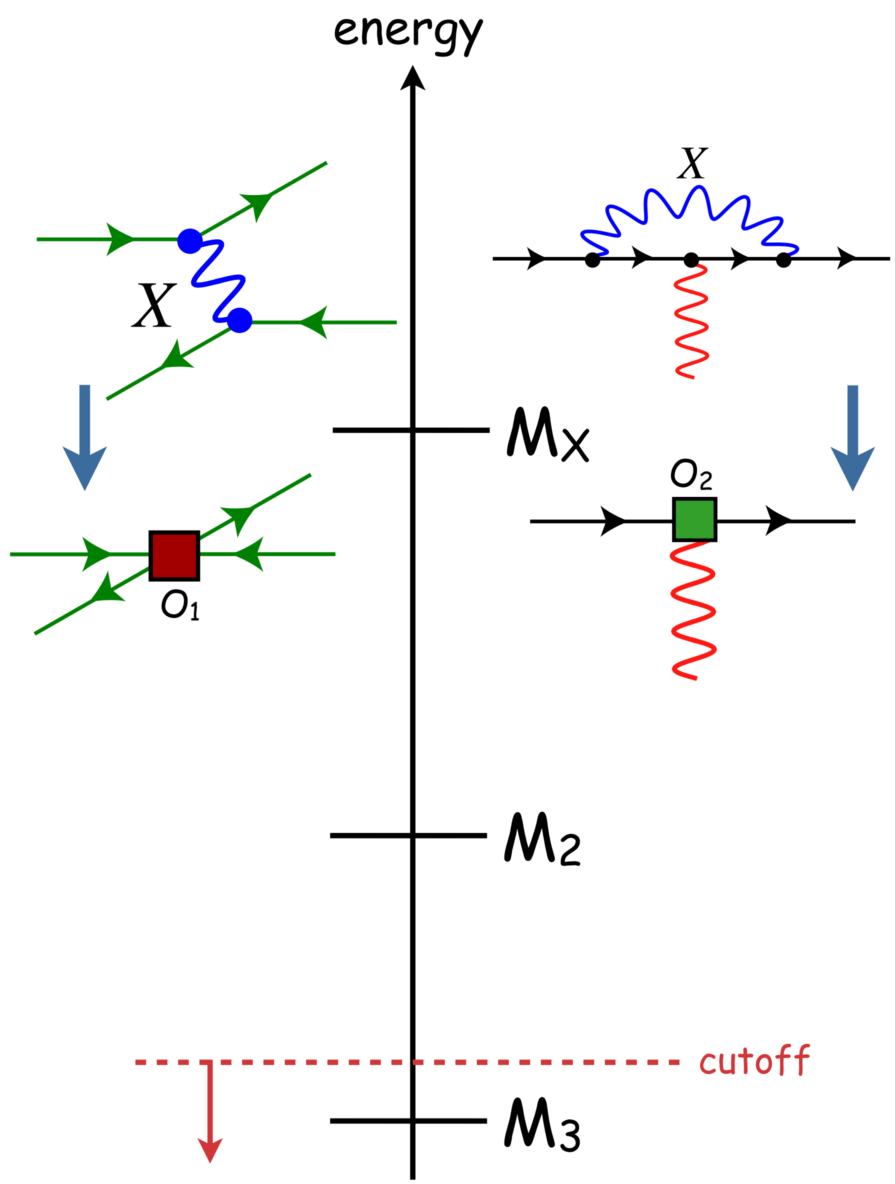


(1) "Classic" Effective Field Theory (4-fermi theory and the like):

- classic example: K-$\bar{\text{K}}$ mixing in the Standard Model (Gilman, Wise, '83)

- W, Z and successive quarks integrated out, renormalization group used to sum terms of order

$$\alpha_s^n \log^n \frac{m_c}{m_{t,W}}$$



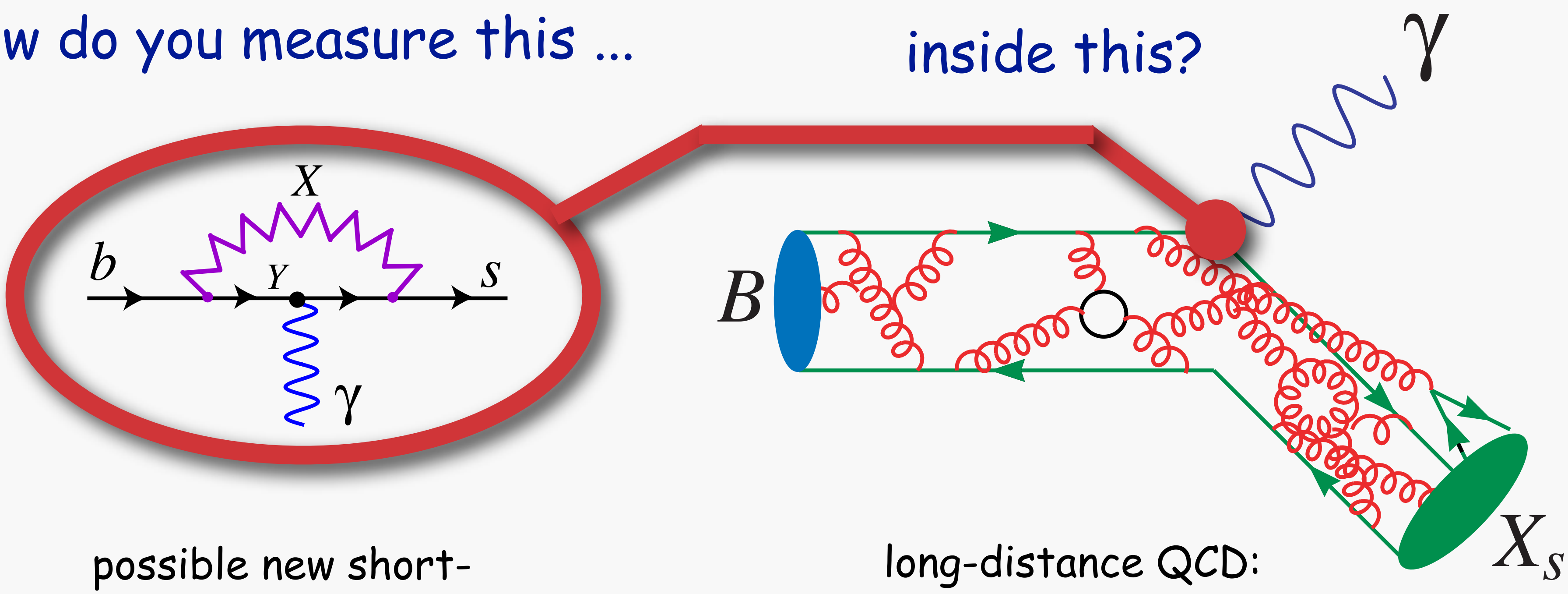
(2) “Classic” -> “Modern”: Heavy Quark Effective Theory (“HQET”)

Qu: how do you lower the cutoff of an EFT below the mass of a particle in the initial state? (i.e. not virtual)

(2) "Classic" -> "Modern": Heavy Quark Effective Theory ("HQET")

- precision b quark decays provide a powerful tool to probe new physics virtually ... but QCD muddies the waters: (Isgur, Wise, Georgi, Voloshin, Shifman, ...)

how do you measure this ... inside this?

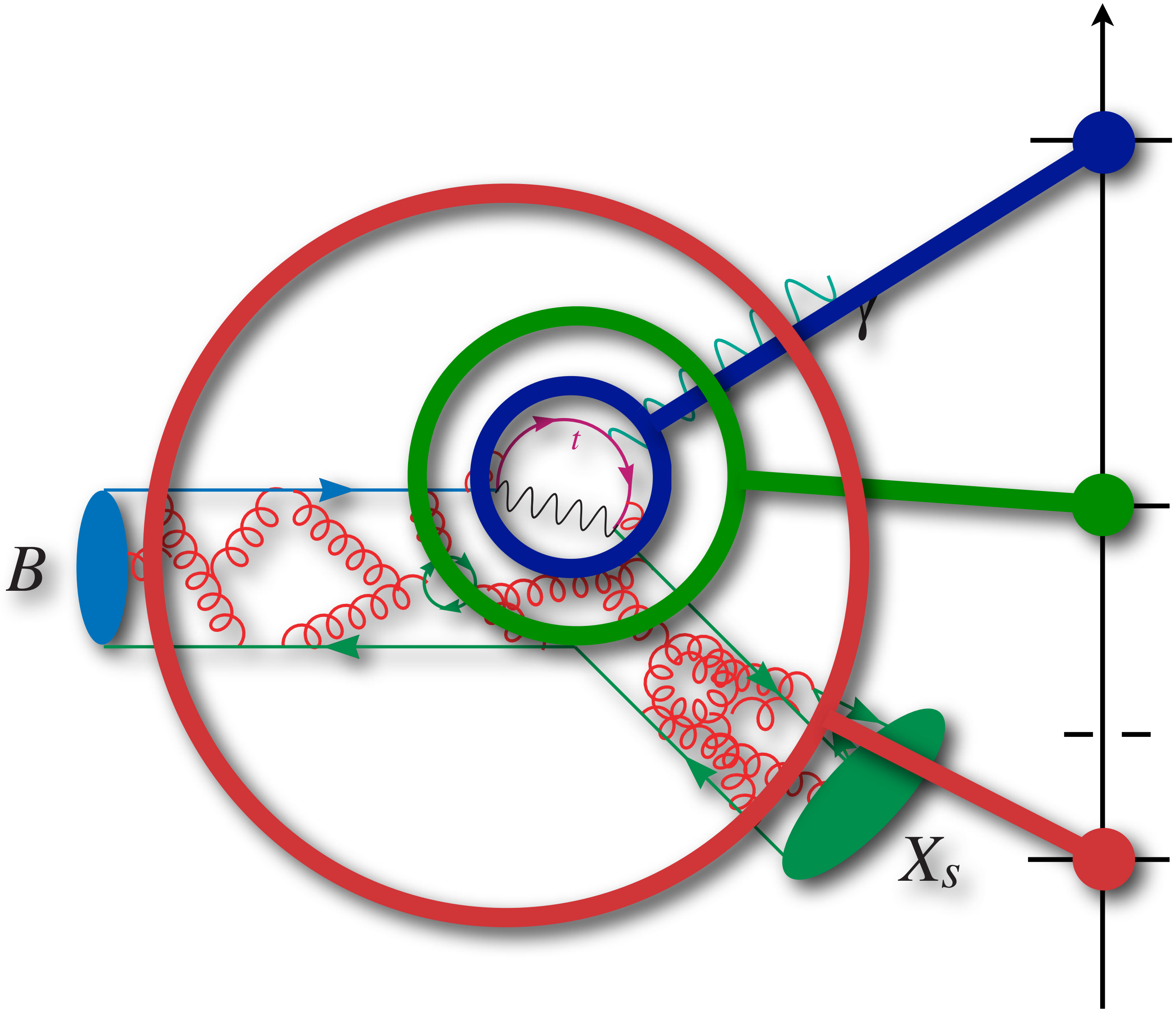


possible new short-distance physics mediated by X particle

long-distance QCD: hadrons, nonperturbative form factors ...

(and to believe small discrepancy = new physics, need model independent predictions - challenge for theory!)

(2) "Classic" -> "Modern": Heavy Quark Effective Theory ("HQET")



$$r \sim \frac{1}{m_W} \sim 4 \times 10^{-18}\ \mathrm{m}$$

- physics mediating decay (what we're interested in ...)

$$r \sim \frac{1}{m_b} \sim 6 \times 10^{-17}\ \mathrm{m}$$

- decay occurs

"short" distance

$$r \sim \frac{1}{\Lambda_{QCD}} \sim 10^{-15}\ \mathrm{m}$$

"long" distance

- hadronization

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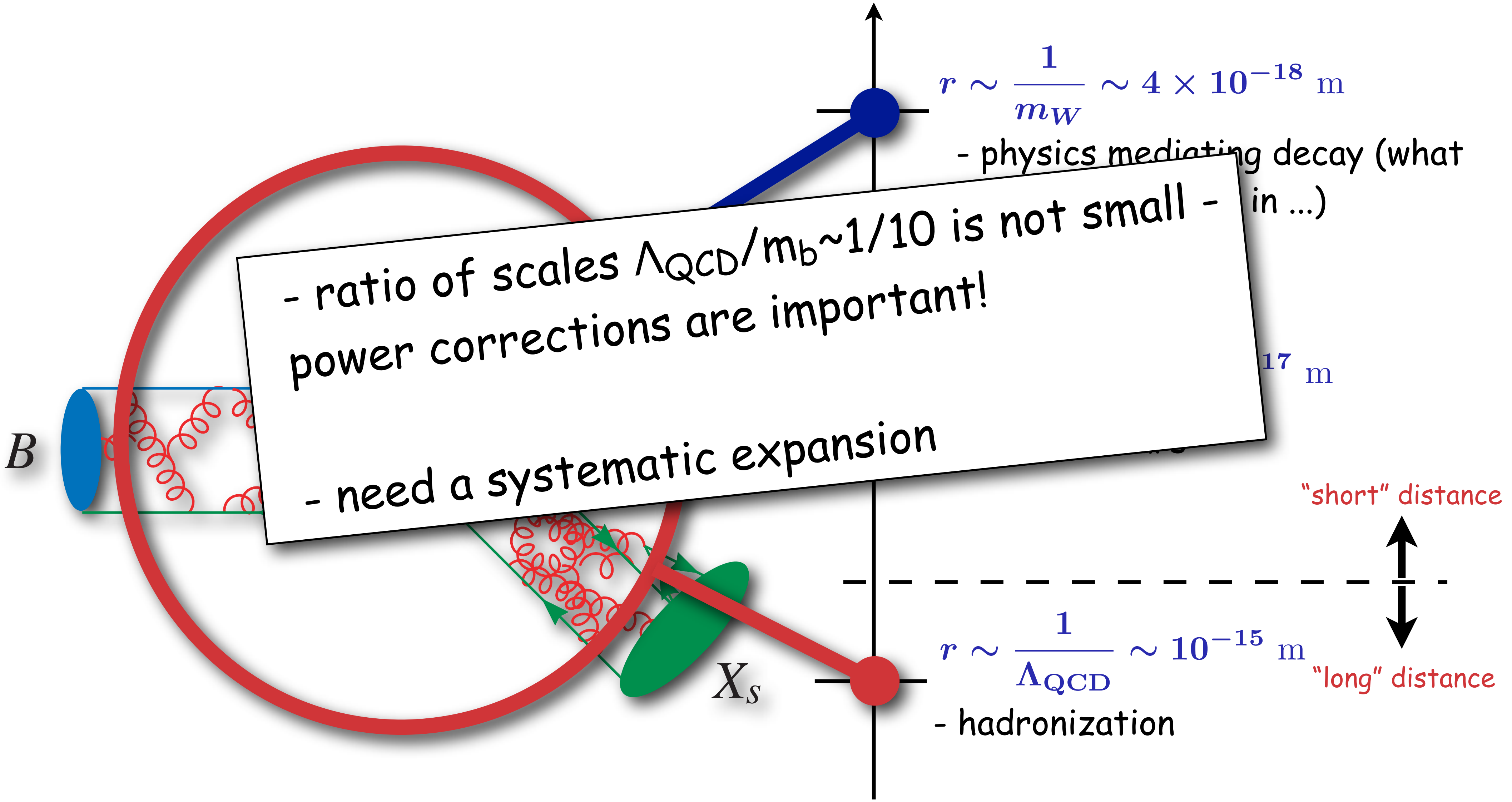
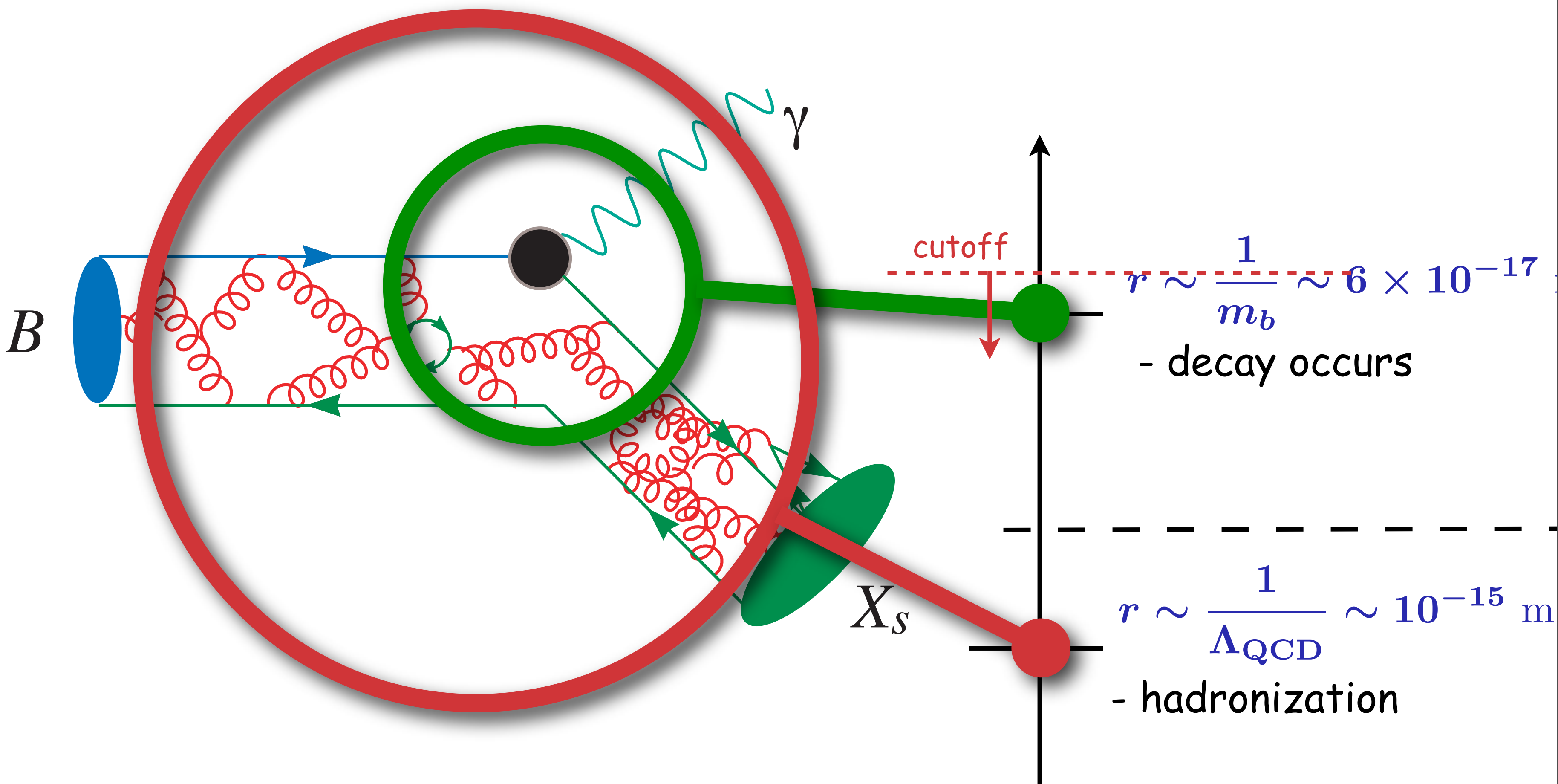


- ratio of scales Λ_{QCD}/m_b~1/10 is not small - power corrections are important!

- need a systematic expansion

(2) "Classic" -> "Modern": Heavy Quark Effective Theory ("HQET")

We can use usual EFT methods to integrate out physics above m_b - but what happens when we lower the cutoff BELOW the b mass?



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- unlike virtual excitations, b quark doesn’t get removed from the theory ... instead, the EFT describes the low-energy dynamics of a heavy quark

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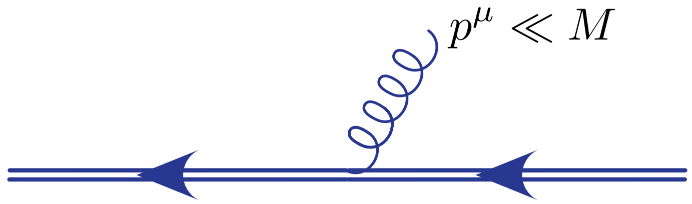

Interactions in the effective theory don't deflect the worldline of the heavy quark

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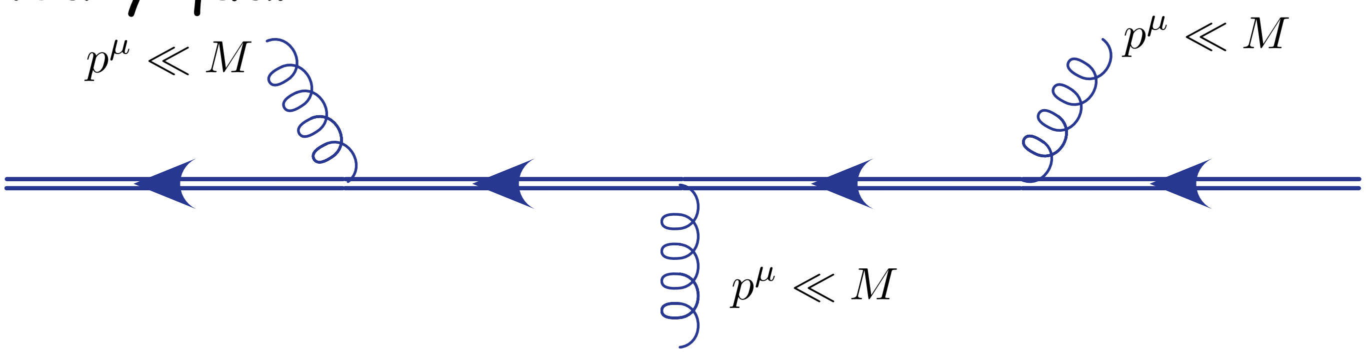

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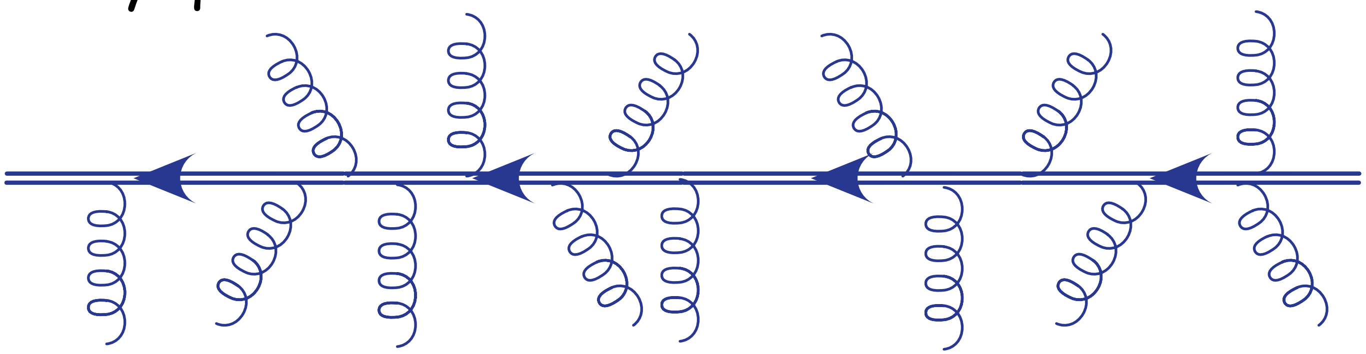

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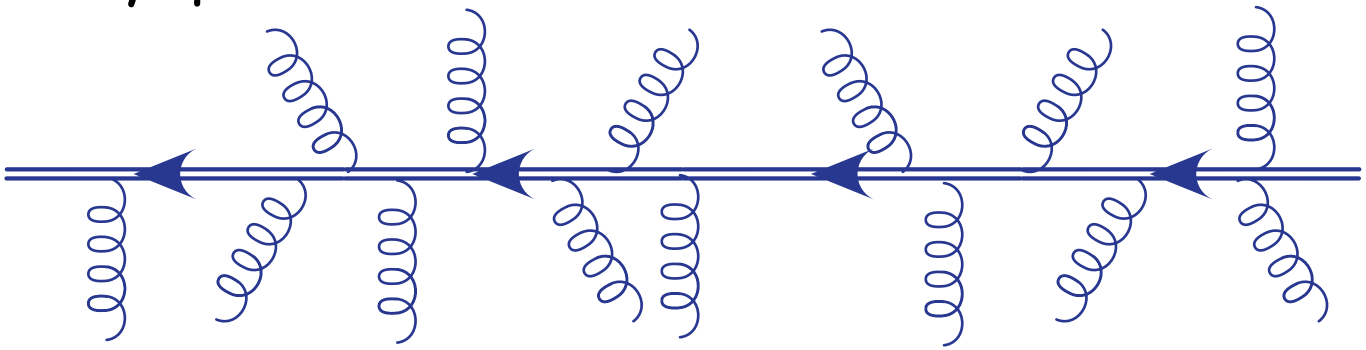


Interactions in the effective theory don't deflect the worldline of the heavy quark

QCD: heavy quark -------> HQET: Wilson line

- appropriate description is a **classical colour charge** moving with a constant velocity - "Wilson line" (timelike)
- other than this, technology is still the same
- NB: the mass, spin of the quark have become irrelevant: extra symmetry in low energy theory! (not manifest in QCD)

(Isgur, Wise, Georgi, Voloshin, Shifman, ...)

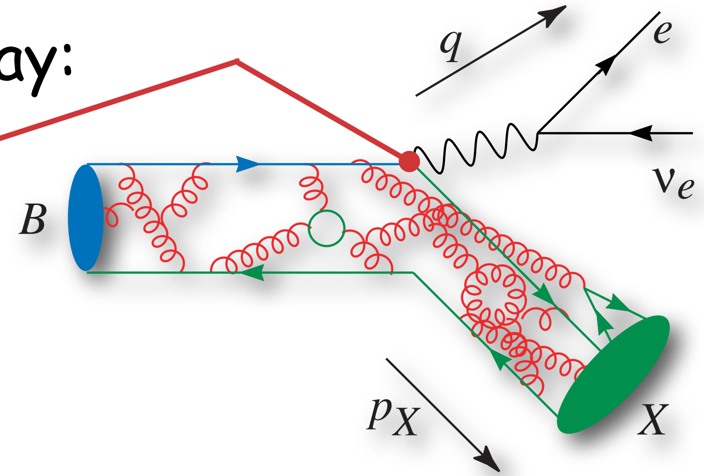
This field became suddenly fashionable in the 1990's ...

- heavy meson spectroscopy
- semileptonic decays (measure parameters of Standard Model - calibration)
  - inclusive (sum over all hadronic states)
  - exclusive (decays to specific final states - particular those with charm quarks - "Heavy Quark Symmetry")
- nonleptonic decays (lifetimes)
- rare (inclusive) decays i.e. $b \to s\gamma$, $b \to s\mu^+\mu^-$

All can be handled in an expansion in Λ_{QCD}/m_b~1/10 ... remarkable success over past decade or so

"Killer App": Inclusive semileptonic b->c decay:

(need to determine b->c weak coupling constant V_{cb})



$$\Gamma(B \to X_c \ell \bar{\nu}) = \frac{G_F^2 \, |V_{cb}|^2}{192\pi^3}(0.534)\left(\frac{m_\Upsilon}{2}\right)^5 \times$$

$$\Big[1$$

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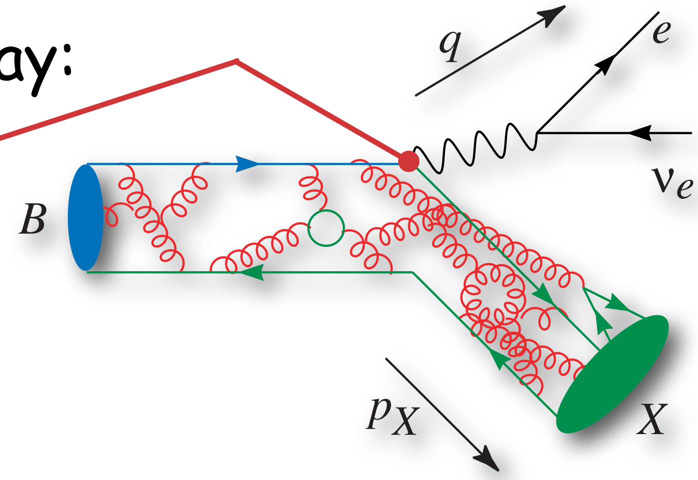

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$O(\Lambda_{QCD}/m_b)$: ~20% correction

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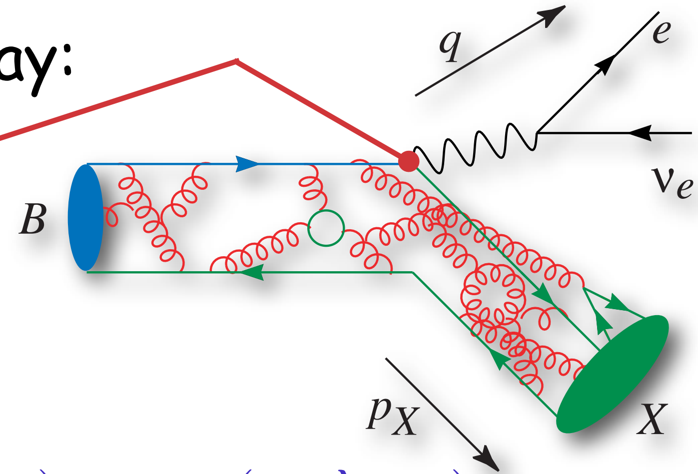

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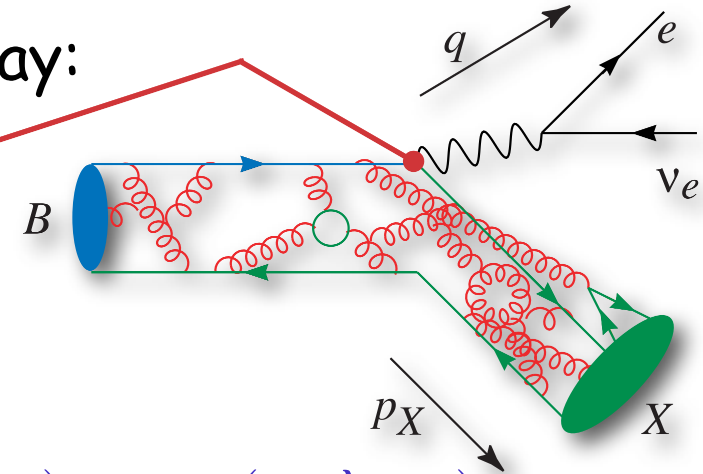

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$$+0.011\left(\frac{T_1}{(500\ \mathrm{MeV})^3}\right) + 0.002\left(\frac{T_2}{(500\ \mathrm{MeV})^3}\right) - 0.017\left(\frac{T_3}{(500\ \mathrm{MeV})^3}\right) - 0.008\left(\frac{T_4}{(500\ \mathrm{MeV})^3}\right)$$

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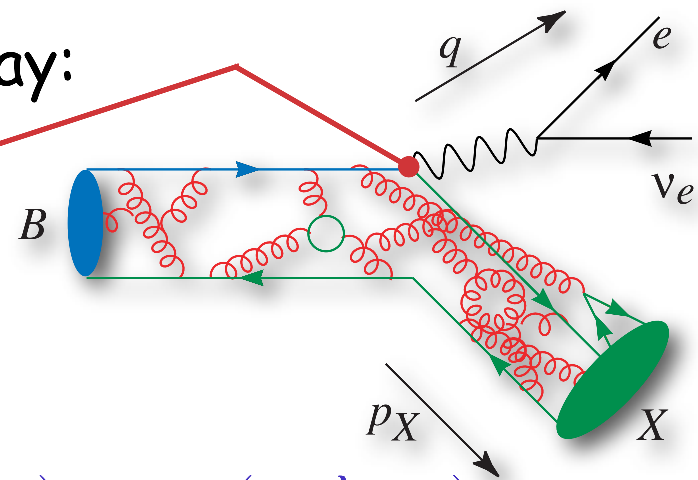

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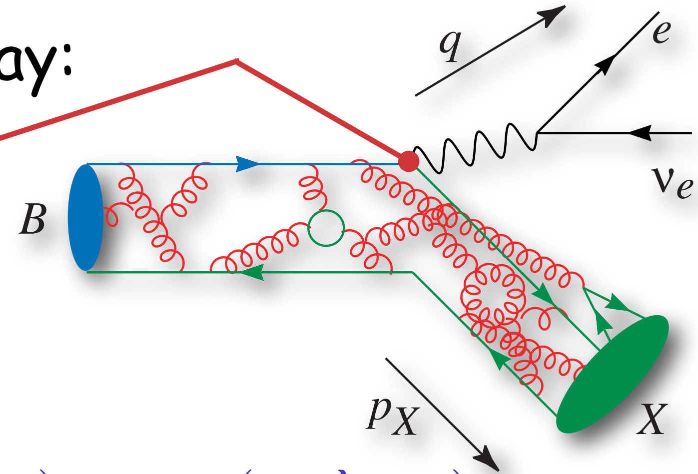

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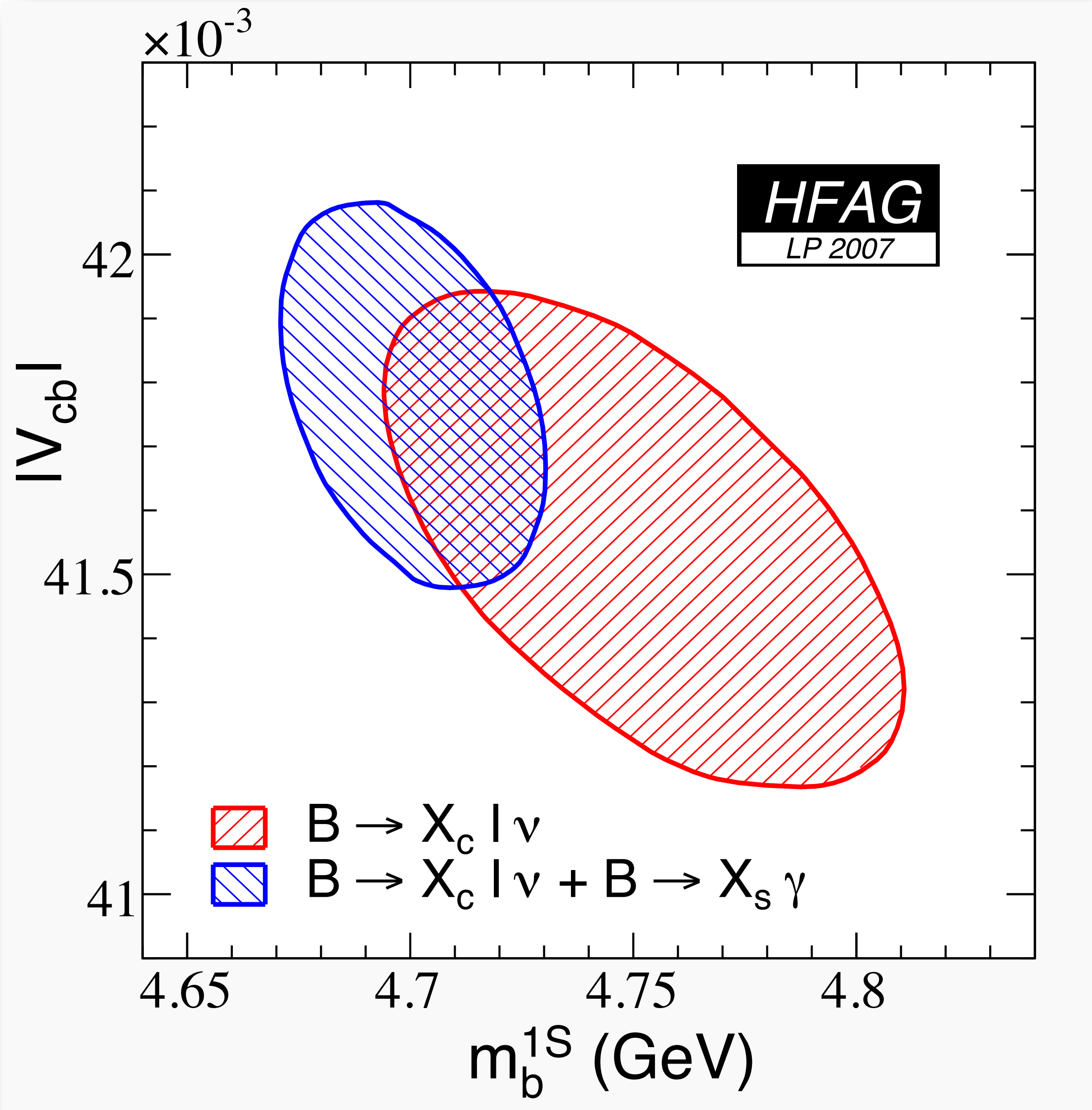
Perturbative: ~few %

-> This is a PRECISION field!

Global fits:

(Bauer, Ligeti, ML, Manohar and Trott)

(up to $1/m^3$)



mass of b quark to 30 MeV!

$$
\begin{aligned}
m_b^{1S} &= 4.601 \pm 0.030 \text{ GeV} \\
\lambda_1 &= -0.313 \pm 0.025 \text{ GeV}^2 \\
|V_{cb}| &= 41.78 \pm 0.30 \pm 0.08
\end{aligned}
$$

b-c weak coupling at % level!

Global fits:

(Bauer, Ligeti, ML, Manohar and Trott)

(up to $1/m^3$)

The fit also allows us to make precise predictions of other moments as a cross-check:

$$D_3 \equiv \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{0.7} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{1.5} \frac{d\Gamma}{dE_\ell}\, dE_\ell} = \begin{cases} 0.5190 \pm 0.0007 & \text{(theory)} \\ 0.5193 \pm 0.0008 & \text{(experiment)} \end{cases}$$

$$D_4 \equiv \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{2.3} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{2.9} \frac{d\Gamma}{dE_\ell}\, dE_\ell} = \begin{cases} 0.6034 \pm 0.0008 & \text{(theory)} \\ 0.6036 \pm 0.0006 & \text{(experiment)} \end{cases}$$

(some fractional moments of lepton spectrum are very insensitive to $O(1/m^3)$ effects, and so can be predicted very accurately)

(C. Bauer and M. Trott)

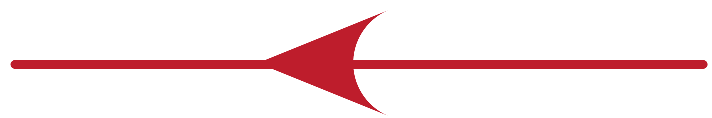
NB: these were REAL PREdictions (not postdictions)

Hadronic physics with < 1% uncertainty!

(3) "Post-Modern": Soft-Collinear Effective Theory ("SCET")

(Bauer, ML, Fleming, Stewart, Pirjol, ...)

What is the correct EFT to describe the dynamics of a very LIGHT, ENERGETIC quark?



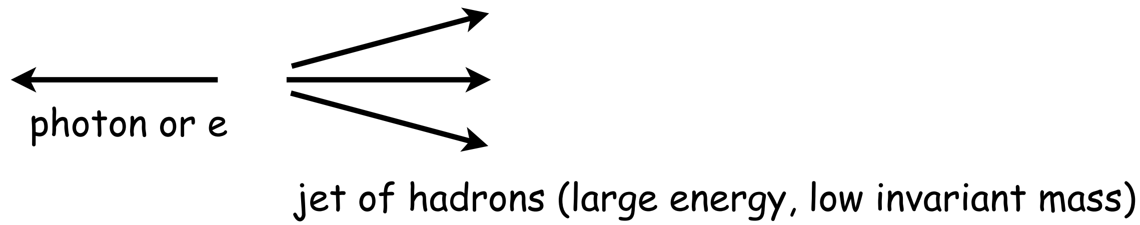
$$p_Q = (p^+, p^-, p_\perp) \sim (Q, \lambda^2 Q, \lambda Q)$$

NB: using light-cone coordinates!

High Energy $\boldsymbol{E \sim Q}$ massless $\boldsymbol{p_Q^2 = 0}$

Why would you want to do this? lots of reasons, i.e.

(1) (original) B decays - to reduce backgrounds, often need to look at restricted regions of phase space - i.e. $b \to s\gamma$ near photon endpoint, $b \to ue\bar{\nu}$ near electron energy endpoint. HQET expansion observed to break down in this region.



(2) collider physics - hard QCD processes - Drell-Yan, jet production, event shapes, ...

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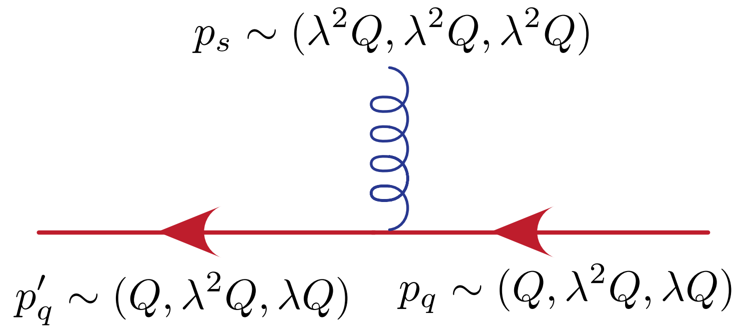

Interactions with soft gluons don't deflect the worldline of the energetic quark

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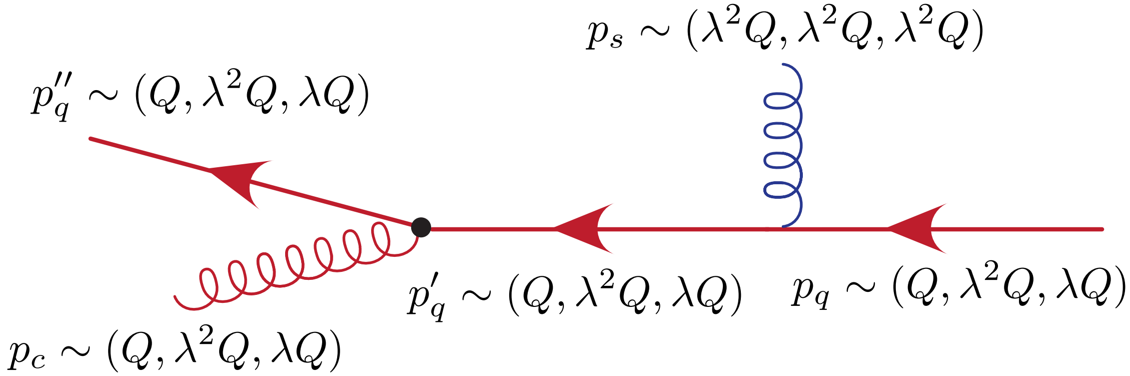

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BUT ... the quark can also emit a hard, collinear gluon

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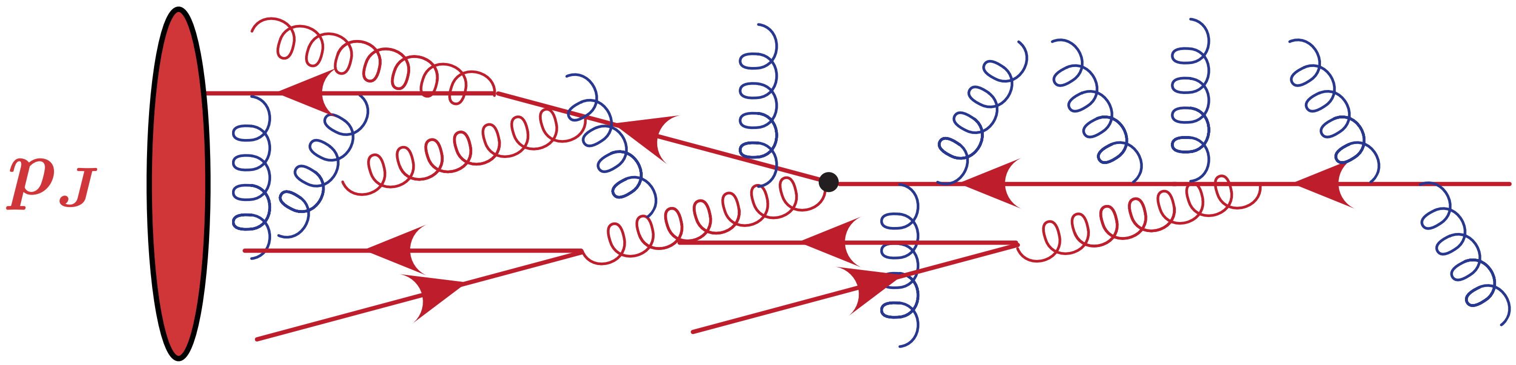


Interactions with soft gluons don't deflect the worldline of the energetic quark

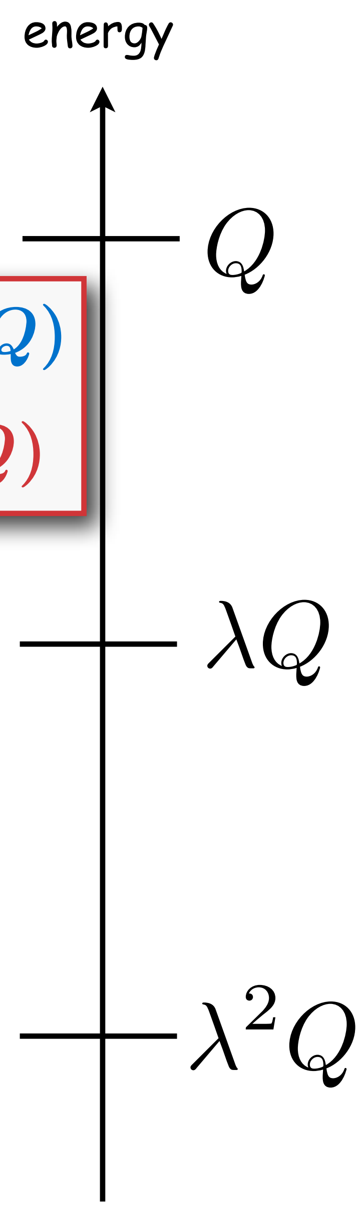
BUT ... the quark can also emit a hard, collinear gluon

- get a JET of final state particles

- jet energy is large, invariant mass is parametrically smaller

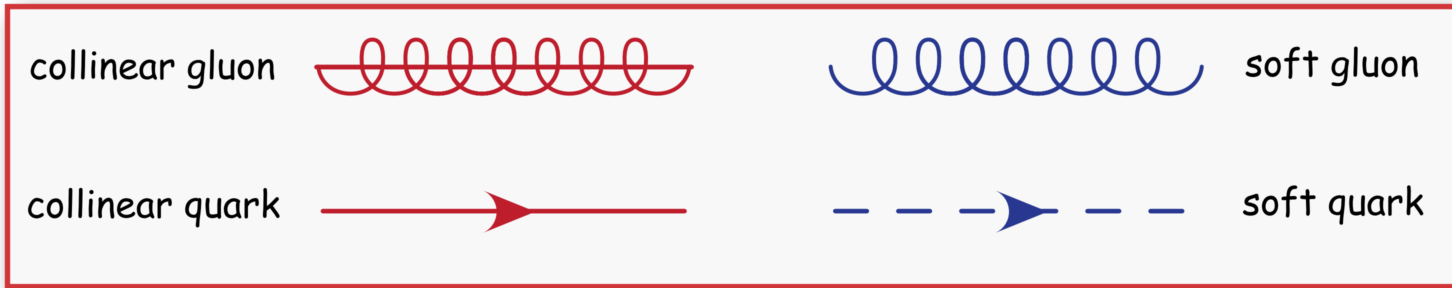
$$E_J \sim Q \qquad p_J^2 \sim \lambda Q \ll Q^2$$

SCET ("soft-collinear effective theory") is an effective theory of JETS



"Soft" particles $p_s^\mu = (p^+, p^-, \vec{p}_\perp) \sim (\lambda^2 Q, \lambda^2 Q, \lambda^2 Q)$

"Collinear" particles $p_c^\mu = (p^+, p^-, \vec{p}_\perp) \sim (Q, \lambda^2 Q, \lambda Q)$

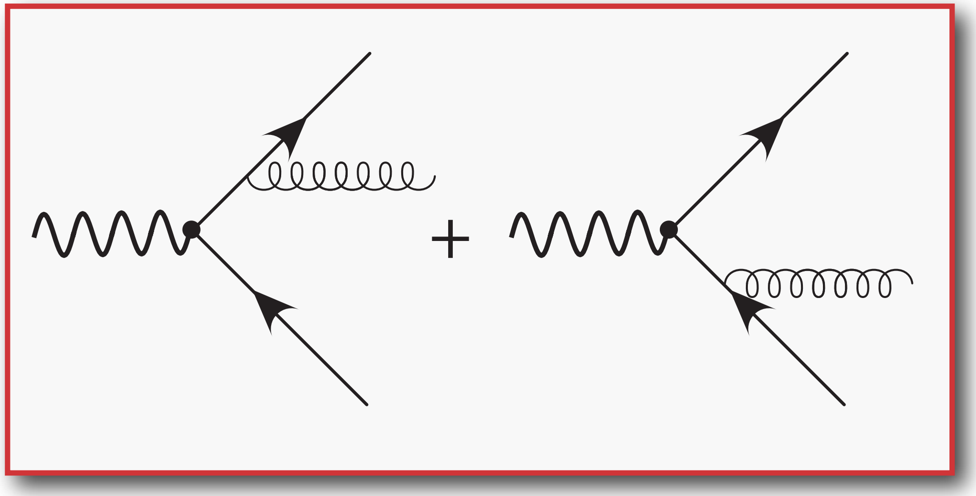


- need a separate field for each momentum scaling (a hallmark of "postmodern" EFT's)
- couplings are interesting, because each field "sees" the others in different ways ...

multiscale .. w/ correlated scales

Ex: $q\bar{q}$ production current:

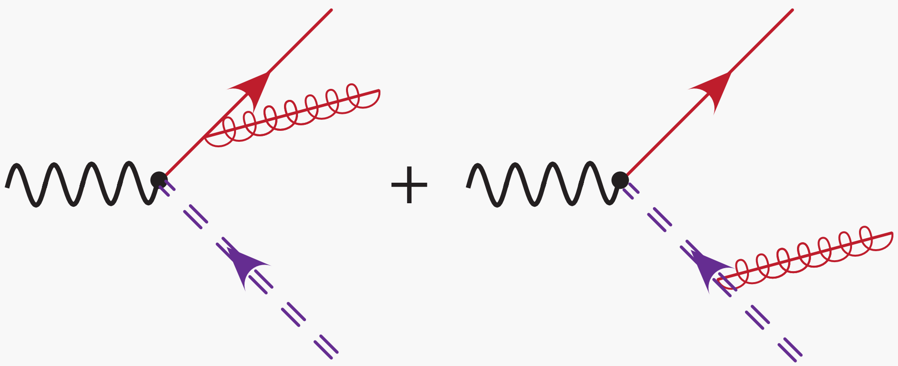
(1) QCD



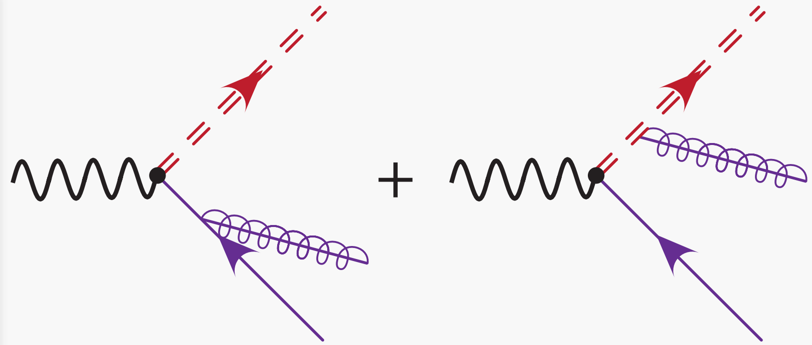
Ex: $q\bar{q}$ production current:

(2) SCET

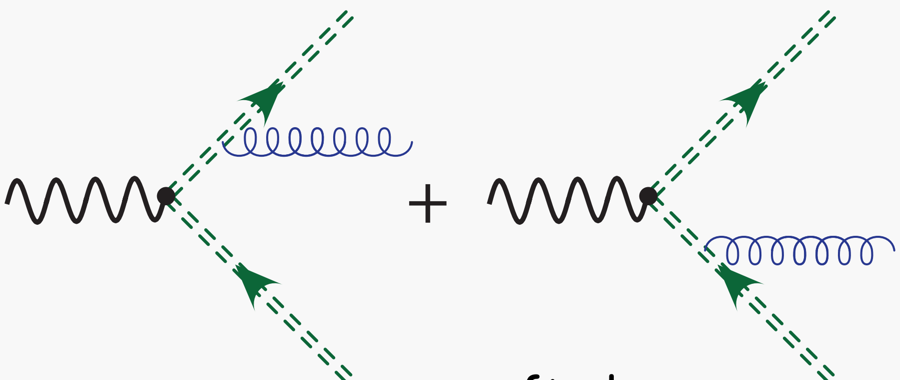
Wilson Line



n-collinear gluon

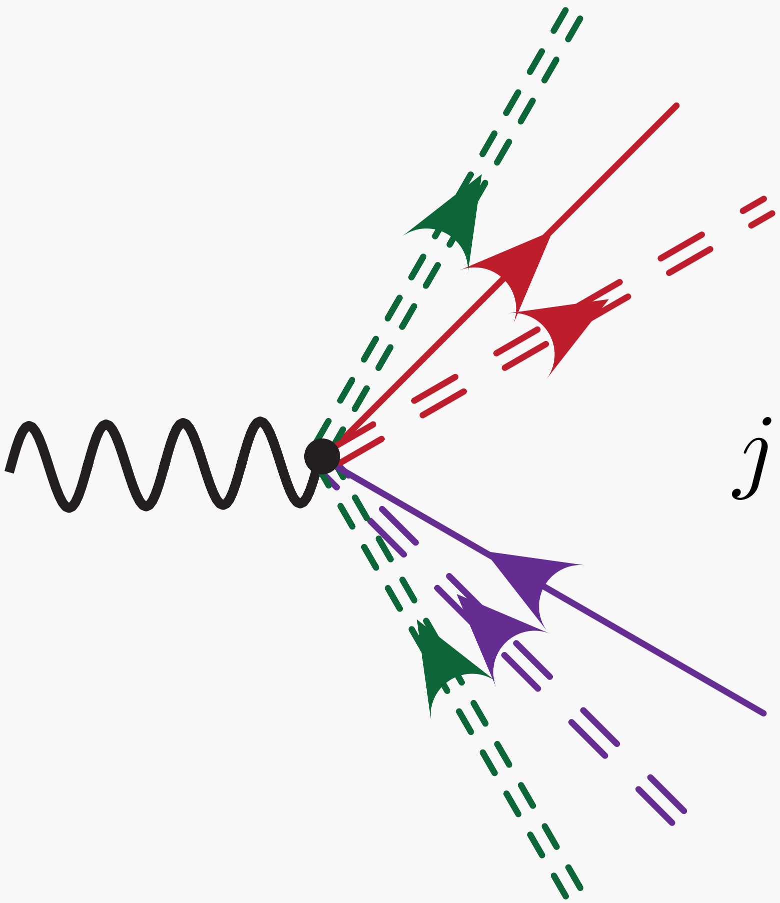


$\bar{n}$-collinear gluon



soft gluon

NB for processes with multiple collinear directions (i.e. multi-jet), there are separate collinear fields for each direction



n̄-collinear Wilson line

n-collinear Wilson line

$$j^\mu = \bar{\xi}_{\bar{n}} Y_{\bar{n}} W_{\bar{n}} \Gamma^\mu W_n^\dagger Y_n^\dagger \xi_n$$

soft Wilson line

quark field

antiquark field

The resulting SCET vertex is correspondingly complicated ...

SCET - what you get

Factorization formulas - more complex than before: discrete sum over operators becomes a convolution

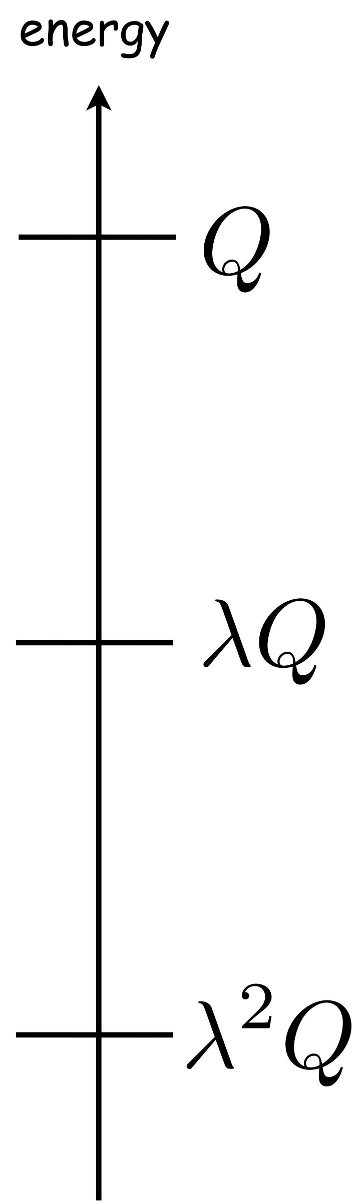
"hard" function

$$\sum_i C_i \mathcal{O}_i \rightarrow H \int J(x) S(x) dx$$

short distance

long distance

"jet" function "soft" function



(this form of factorization has been known since the 1980's, but now it is at the level of the Lagrangian of the EFT)

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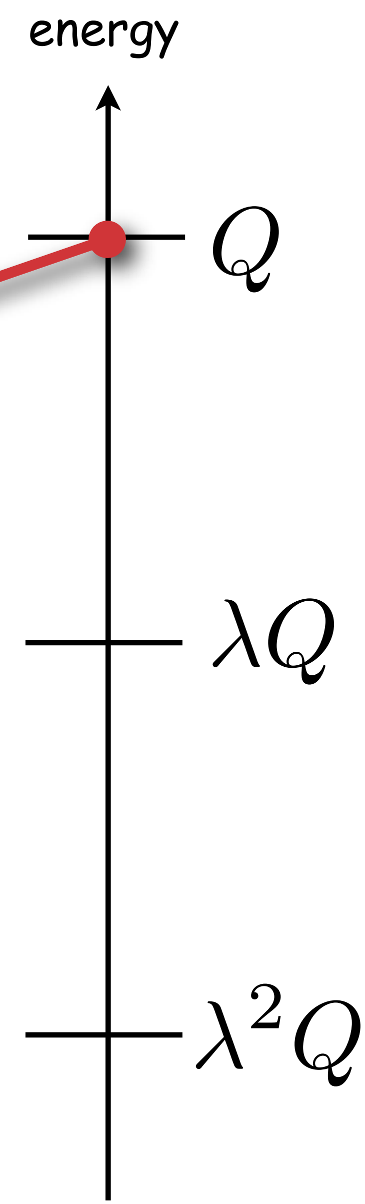

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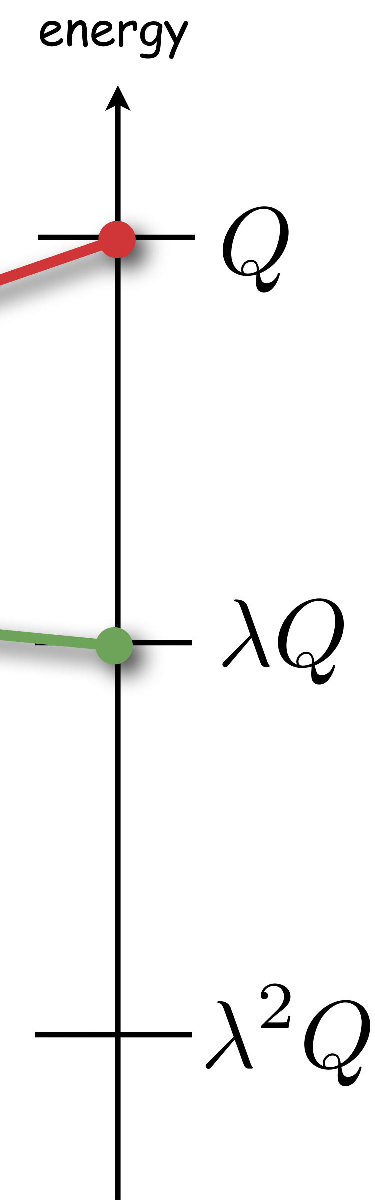

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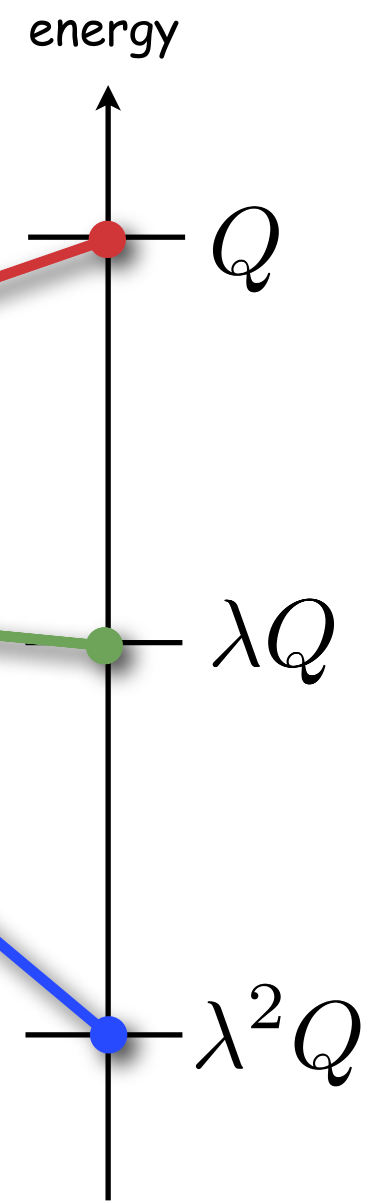


(this form of factorization has been known since the 1980's, but now it is manifest in the EFT, systematically improvable)

SCET: what you get out of it

Lots of applications:

(1) B decays .. grew out of HQET in regions of phase space where final state was restricted to be jet-like

(2) jets and collider physics - we come full circle. No “killer app” yet, but lots of directions - ex: top production, event shape distributions, jets, etc. ...

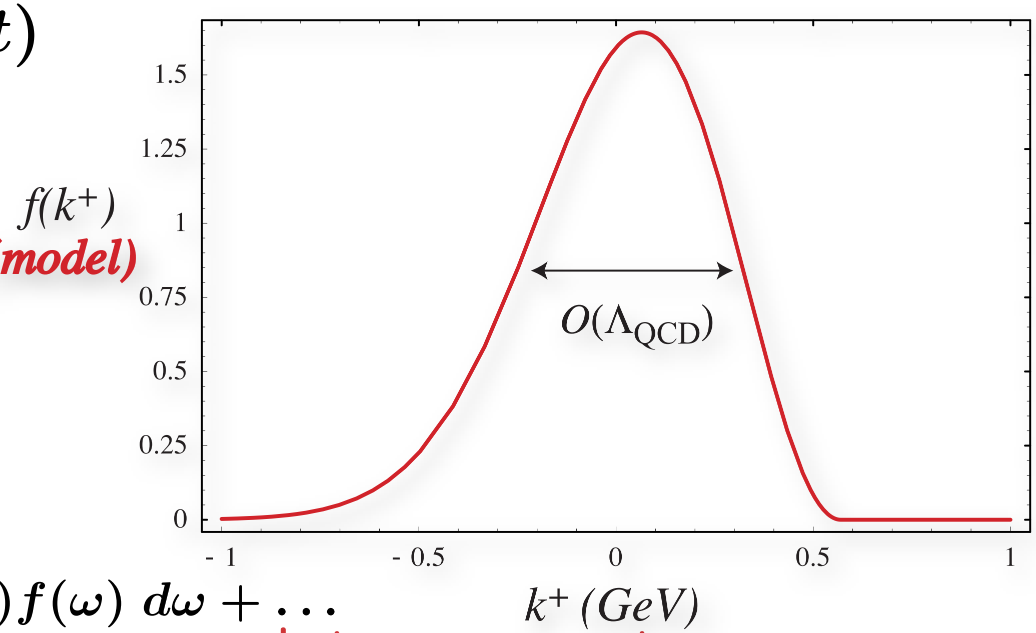
The "shape function" (parton distribution function for b quark in a meson)

$$\tilde{O}(t) \equiv \bar{b}(0) P e^{\frac{i}{m_b}\int_0^t n\cdot A(t')dt'} b(t)$$

nonlocal operator: quarks separated along light cone

$$f(\omega) \equiv \langle B|O(\omega)|B\rangle$$

universal distribution function



$$\frac{1}{\Gamma}\frac{d\Gamma}{d\hat{E}_\ell}(\bar{B}\to X_u\ell\bar{\nu}_\ell) = 4\int \theta(1-2\hat{E}_\ell-\omega)f(\omega)\,d\omega + \ldots$$

electron energy spectrum

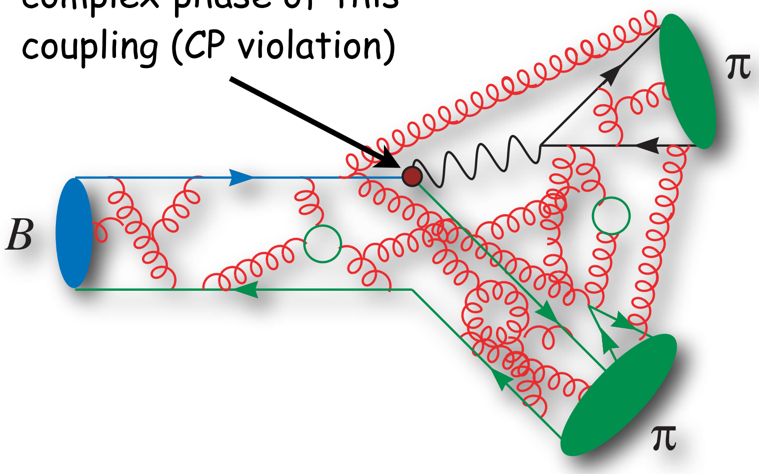
$$\frac{1}{\Gamma}\frac{d\Gamma}{d\hat{s}_H}(\bar{B}\to X_u\ell\bar{\nu}_\ell) = \int \frac{2\hat{s}_H^2(3\omega-2\hat{s}_H)}{\omega^4}\theta(\omega-\hat{s}_H)f(\omega-\hat{\Delta})\,d\omega + \ldots$$

hadronic invariant mass spectrum

in these corners of phase space, spectra are given by convolutions of short-distance functions with parton distributions

Exclusive B decays - i.e. $B \to \pi\pi$

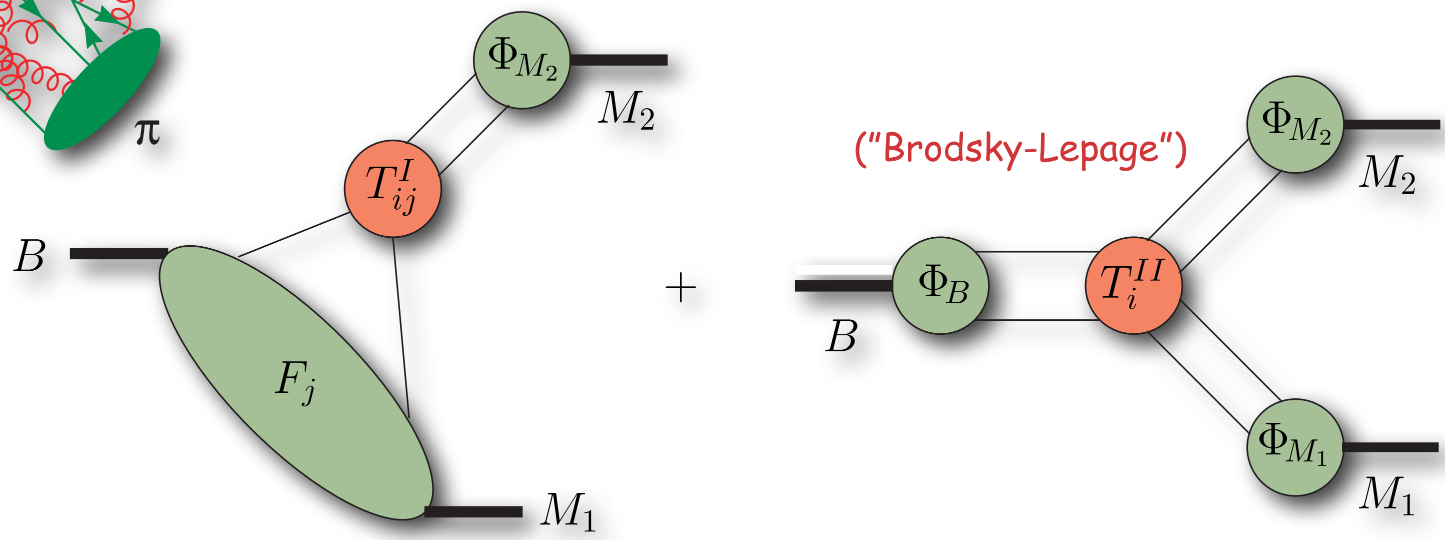
want to measure complex phase of this coupling (CP violation)



(Beneke, Buchalla, Neubert, Sachrajda, Bauer, Pirjol, Rothstein, Stewart)

=

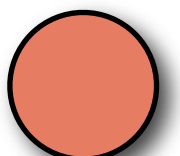
("Brodsky-Lepage")



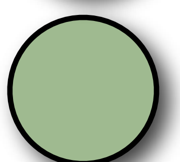
complicated convolutions (cf. parton model)

$+O(\Lambda_{QCD}/m_b)$

subprocesses:



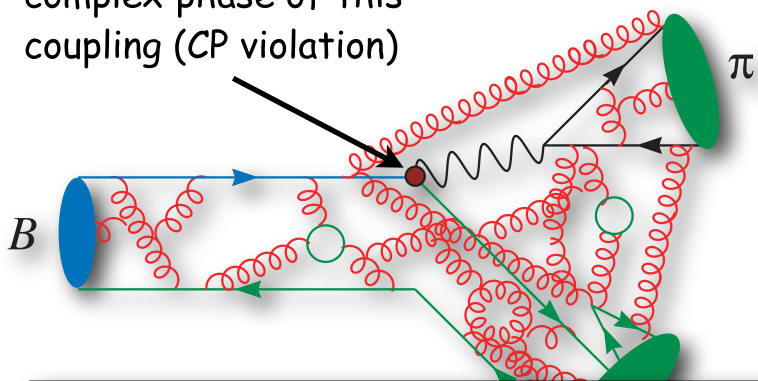
Short-distance QCD



Long-distance form factor/wave function

Exclusive B decays - i.e. $B \to \pi\pi$

want to measure
complex phase of this
coupling (CP violation)



(Beneke, Buchalla, Neubert, Sachrajda, Bauer, Pirjol, Rothstein, Stewart)

=

$$
\begin{aligned}
A(\bar{B} \to M_1 M_2) \;&=\; \lambda_c^{(f)} A_{c\bar{c}}^{M_1 M_2} + \frac{G_F m_B^2}{\sqrt{2}} \Bigg\{ f_{M_2}\, \zeta^{BM_1} \int_0^1 du\, T_{2\zeta}(u)\, \phi^{M_2}(u) \\
&+ f_{M_1}\, \zeta^{BM_2} \int_0^1 du\, T_{1\zeta}(u)\, \phi^{M_1}(u) + \frac{f_B f_{M_1} f_{M_2}}{m_b} \int_0^1 du \int_0^1 dx \int_0^1 dz \int_0^\infty dk_+\, J(z,x,k_+) \\
&\times \Big[T_{2J}(u,z) \phi^{M_1}(x) \phi^{M_2}(u) + T_{1J}(u,z) \phi^{M_2}(x) \phi^{M_1}(u) \Big] \phi_B^+(k_+) \Bigg\} + O\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)
\end{aligned}
$$

complicated convolutions (cf. parton model)

$+O(\Lambda_{QCD}/m_b)$

subprocesses:

Short-distance QCD

Long-distance form factor/wave function

Angularity Distributions in Jet production

(Lee, Hornig, Ovanesyan, 2009)

$$\tau_a(X) = \frac{1}{Q}\sum_{i\in X}|\mathbf{p}_i^T|e^{-|\eta_i|(1-a)}$$

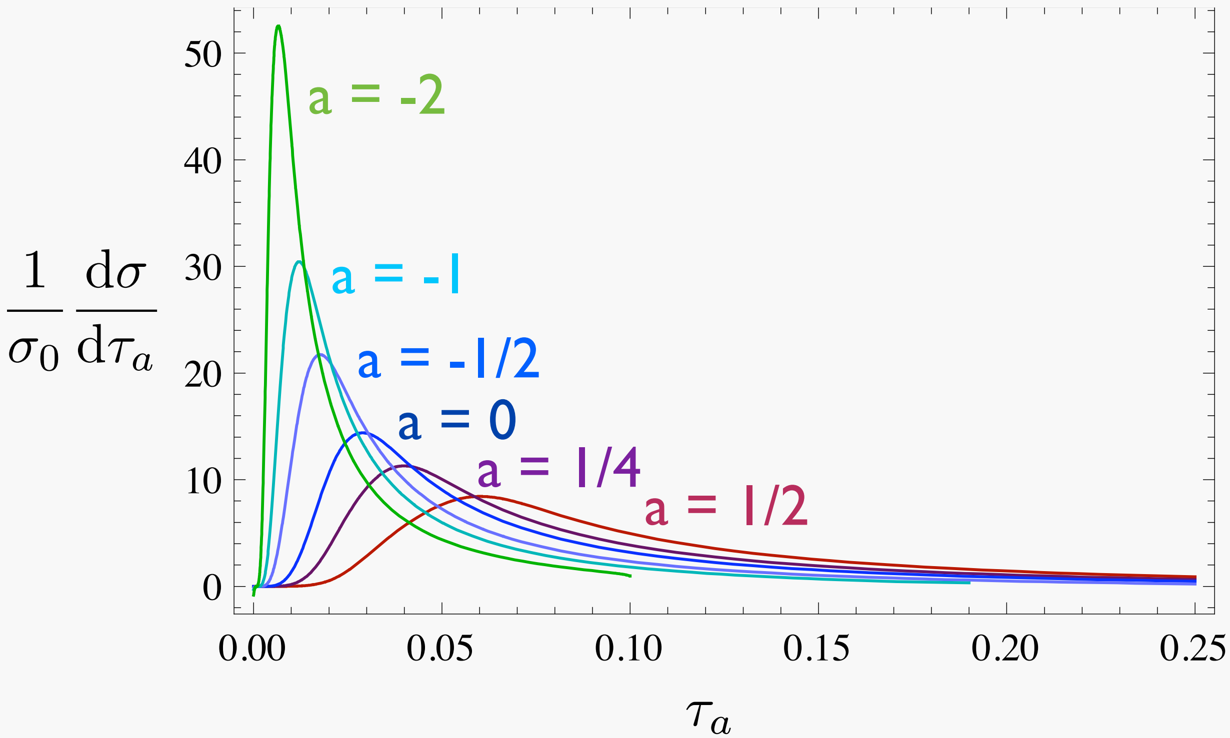


Figure 1: Angularity distributions for $-2 < a < \frac{1}{2}$ at $Q = 100$ GeV, with $\mathcal{O}(\alpha_s)$ hard, jet, and soft functions, NLL resummation, and gapped model soft function.

t-t̄ production - soft radiation and precision extraction of the top quark mass

(Fleming, Hoang, Mantry, Stewart, 2008)

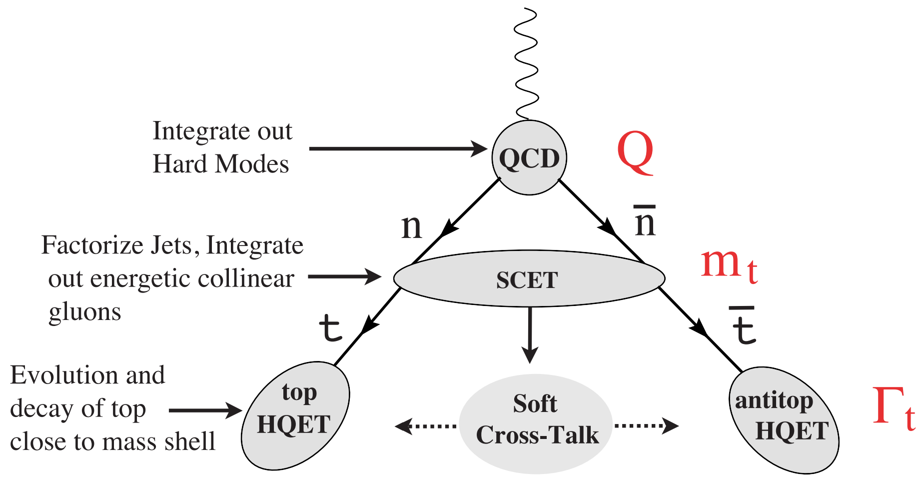


FIG. 1: Sequence of effective field theories used to compute the invariant mass distribution.

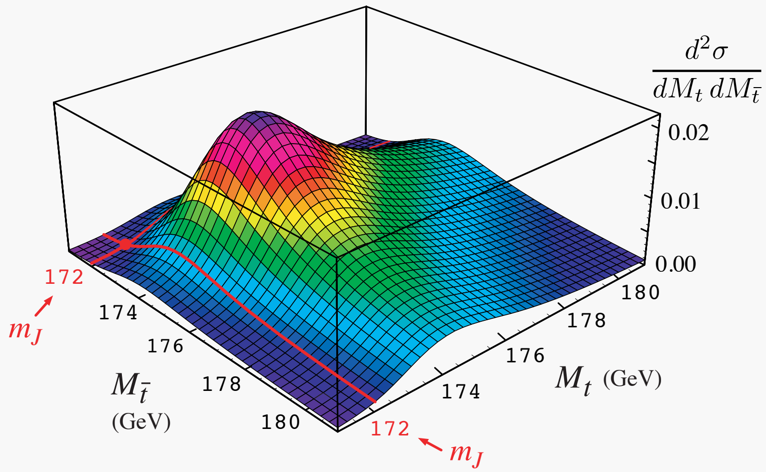


FIG. 15: F($M_t, M_{\bar t}$), the differential cross-section in units of σ_0/Γ_t^2, versus M_t and $M_{\bar t}$. The result is shown at NLL order.

Factorization for jet production

(Cheung, Freedman, ML, Zuberi, in progress)

- UV divergent phase space integrals in SCET treated consistently

- factorization studied for different jet definitions (SW, k_T, JADE)

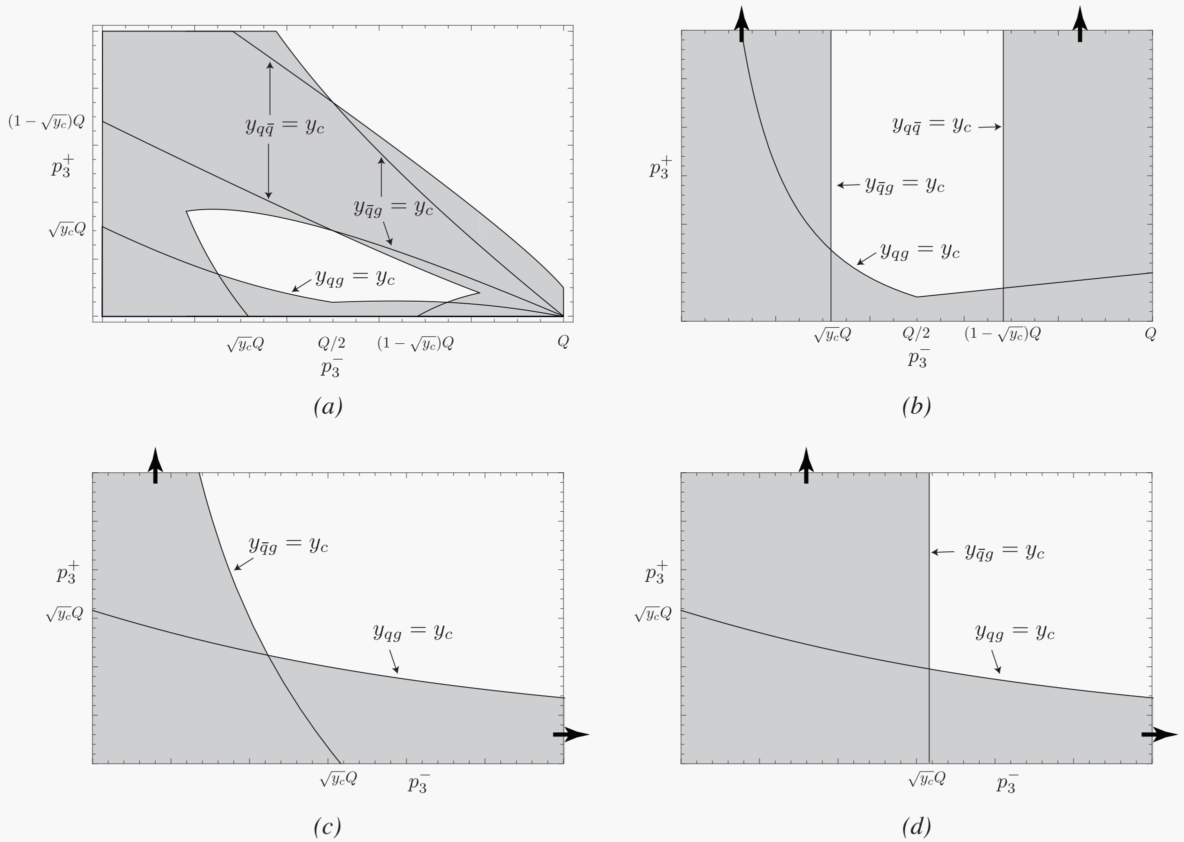
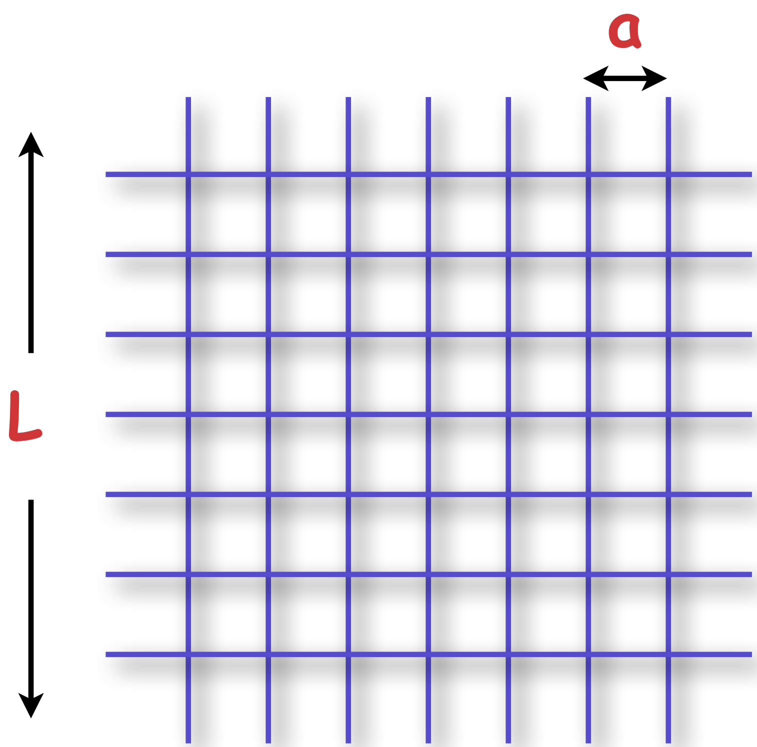


FIG. 3: Phase space corresponding to two-jet events using the $k_\perp$ algorithm in (a) QCD, (b) the n-collinear gluon sector, (c) the soft gluon sector, and (d) the zero-bin sector. As before, the arrows indicate integrations to infinity.

Final Comment

This is always going to be with us ... need to factorize problems for nonperturbative lattice QCD calculations as well!



- need L>1 fm to simulate proton
- need a<1/Q to simulate short-distance physics w/momentum Q

- extremely inefficient to simulate short-distance (perturbative) physics on the lattice!

Factorization -> do short-distance physics analytically, long-distance physics numerically with lattice spacing a>>1/Q

Summary:

- factorization allows us to separate short-distance (interesting) physics from long-distance QCD in a model-independent way - required to make rigorous predictions

- factorization takes many forms, from the relatively simple (inclusive B decays), to the more complicated (hard QCD processes, some B decays) - the form of factorization, and its generalizations to higher orders, can be determined using effective field theory

- lots of applications ...