What do we need to know to get V_{ub}?

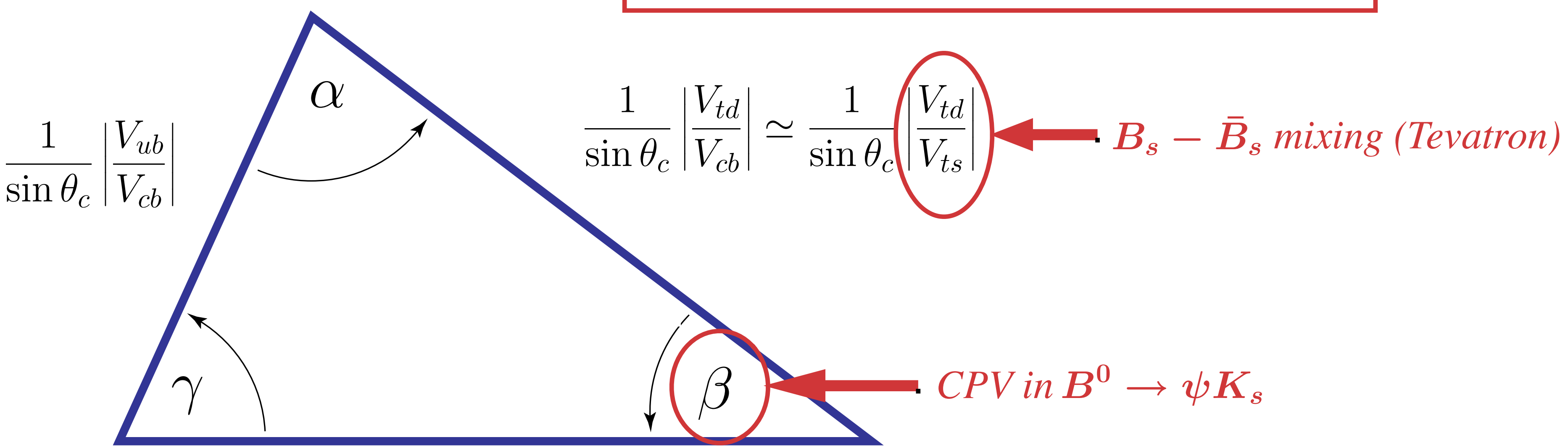
Michael Luke
Department of Physics
University of Toronto

Outline:

1. Introduction
2. Exclusive decays (brief!)
3. Inclusive decays - a guide to phase space and cuts
4. Uncertainties: perturbative, nonperturbative, higher twist
5. Summary

The unitarity triangle provides a simple way to visualize SM relations:

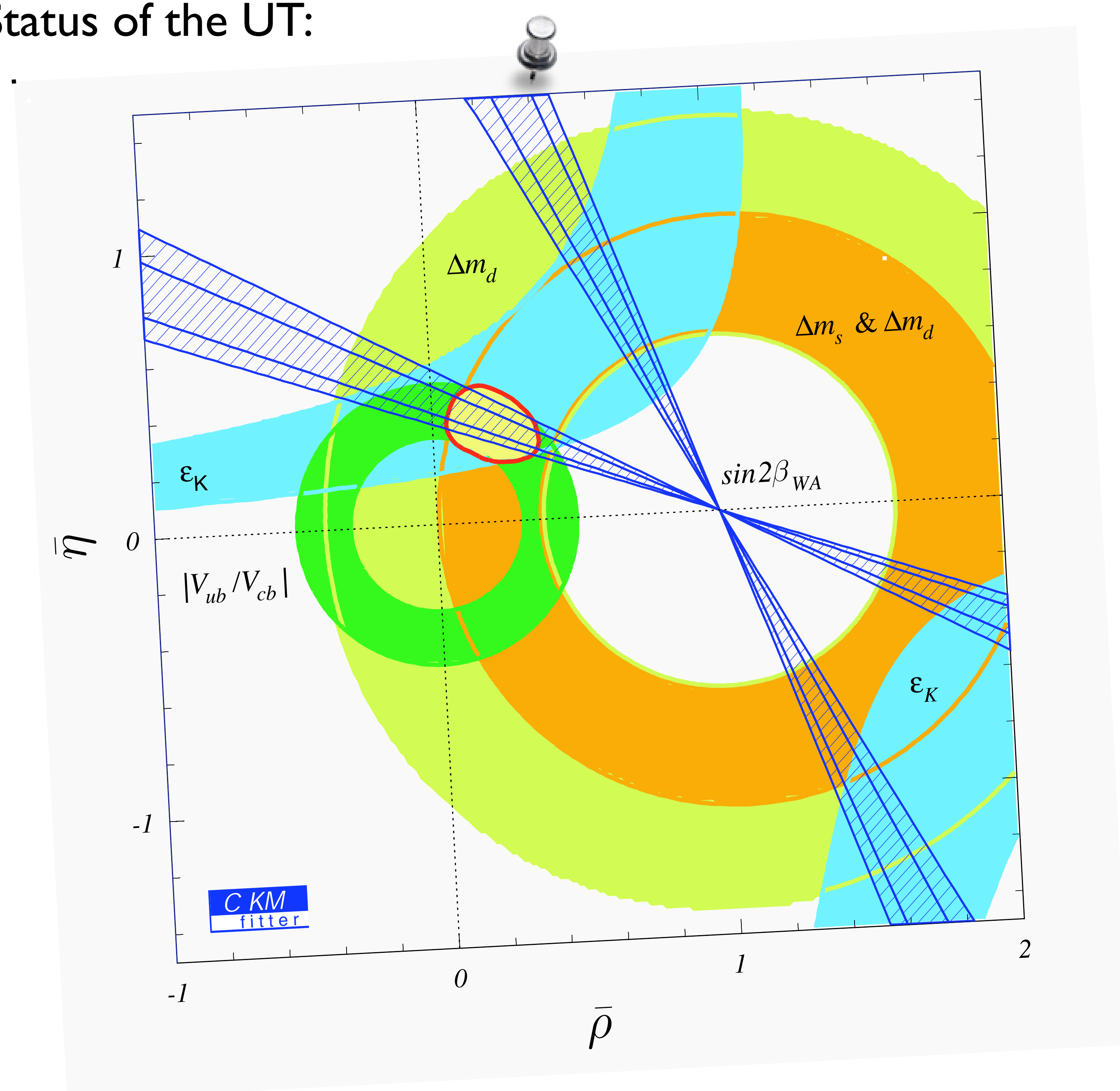
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



⇨ $V_{cb},\ \sin 2\beta,\ |V_{td}/V_{ts}|$: "easy" (theory and experiment both tractable)

⇨ $V_{ub},\ \alpha,\ \gamma$: HARD - our ability to test CKM depends on the precision with which these can be measured

Status of the UT:



World average '02:

$\sin 2\beta = 0.734 \pm 0.054$

- any deviation from SM will require precision measurements!

- theoretical errors must be fully under control (*cf.* *g*-2)

Determining V_{ub}:

(i) Exclusive Decays: $B \to \pi \ell \bar{\nu},\ B \to \rho \ell \bar{\nu}$

$$\langle \pi(p_\pi)|V^\mu|B(p_B)\rangle = f_+(E)\left[p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2}q^\mu\right] + f_0(E)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu$$

nonperturbative - need to model (QCD sum rules) or calculate on lattice

vanishes for m_ℓ=0

Sum rules: $f_+(0) = 0.26 \pm 0.06 \pm 0.05$

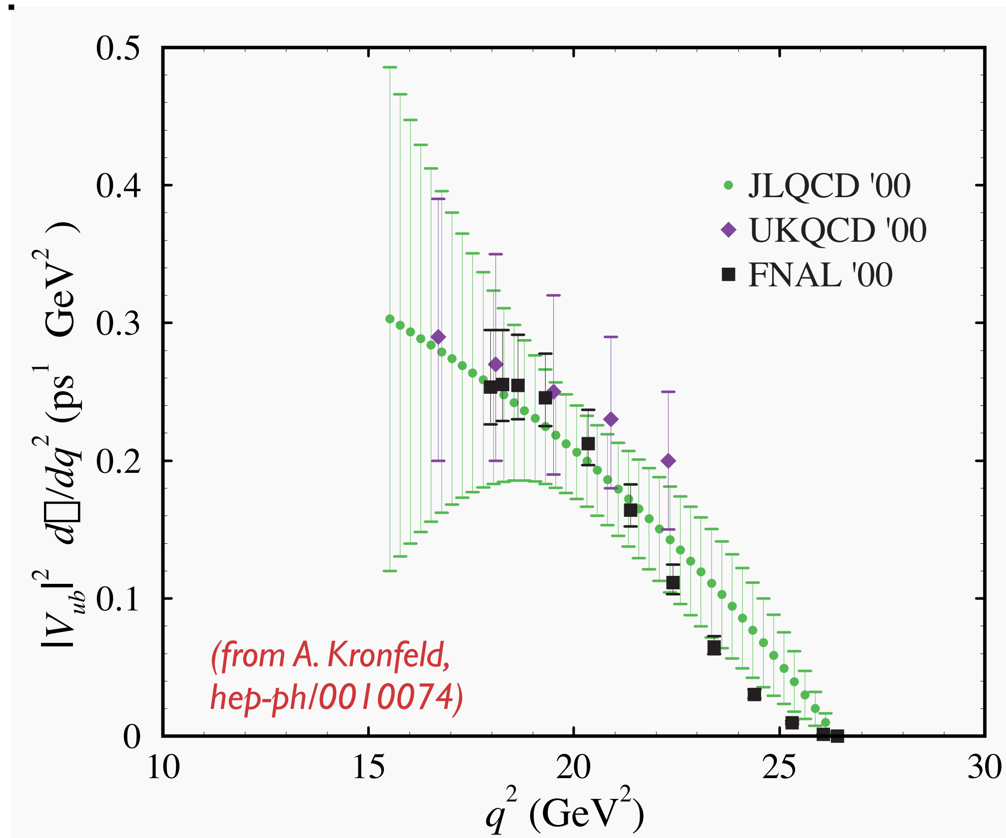
(Ball and Zwicky)

model dependence - hard to improve on (see P. Ball's talk)

Lattice: current theoretical error is δV_{ub} ≈15-18% + quenching error

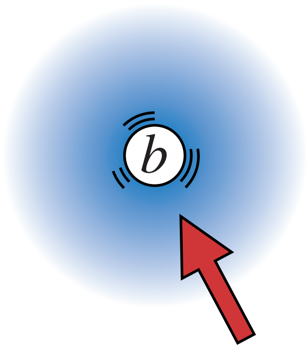
(El-Khadra, et. al., 2001)

- goal for future is unquenched, error of ~ few percent



(ii) Inclusive Decays: $\bar{B} \to X_u \ell \bar{\nu}_\ell$

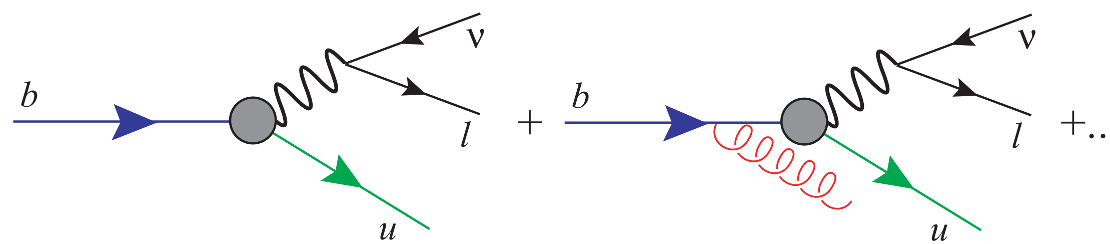
Inclusive decays are in principle model independent ...



"Fermi motion"

$k^\mu \sim \Lambda_{QCD}$

$$\frac{d\Gamma}{d(\mathrm{P.S.})} \sim \underbrace{\text{parton model}}_{} + \sum_n C_n \left(\frac{\Lambda_{QCD}}{m_b}\right)^n$$

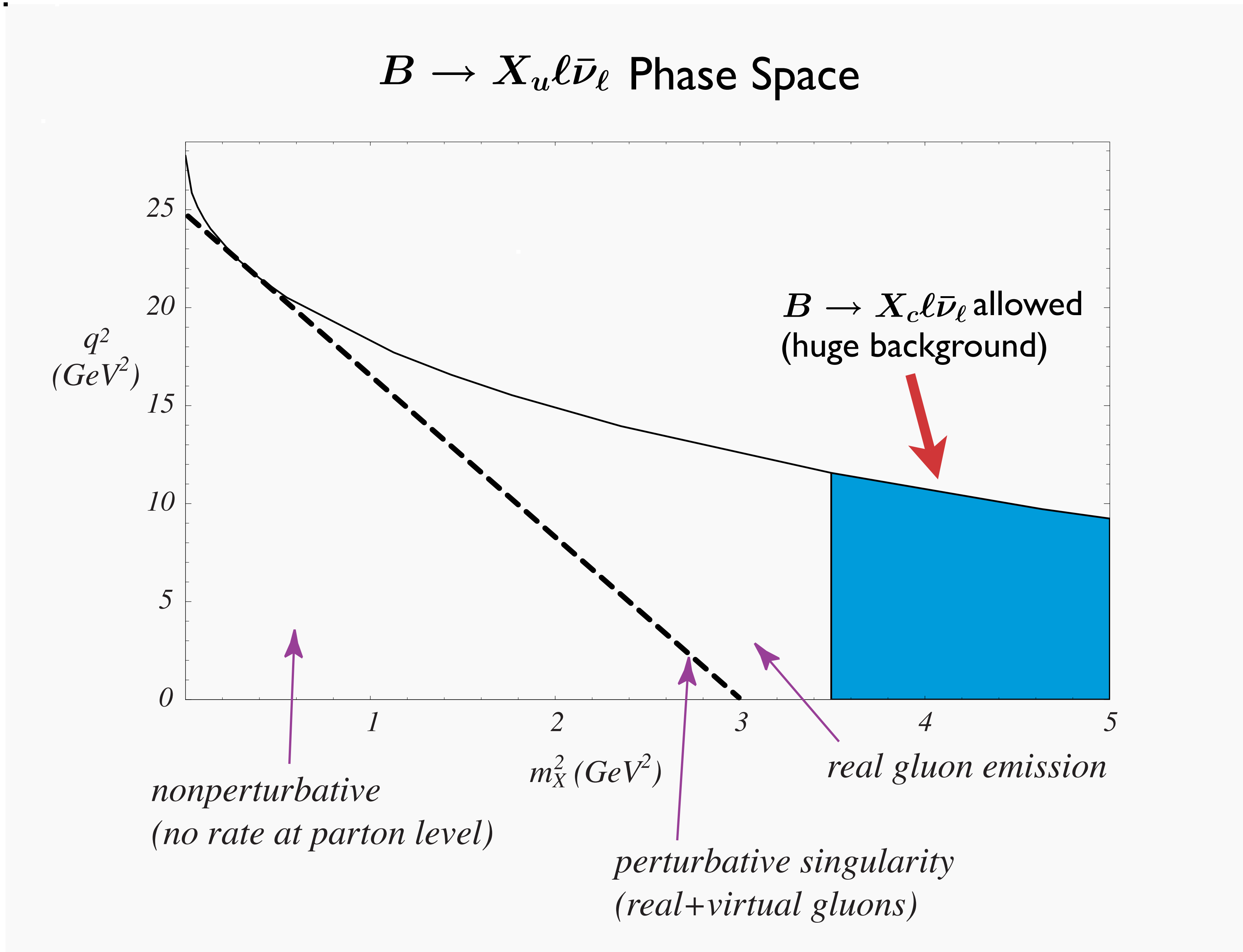


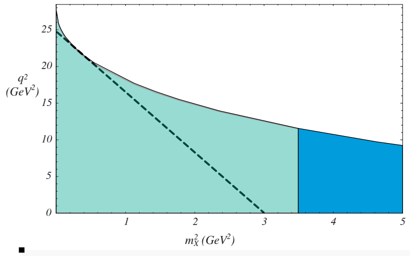
"Most" of the time, details of b quark wavefunction are unimportant - only averaged properties (i.e. $\langle k^2 \rangle$) matter

$$\Gamma(\bar{B} \to X_u \ell \bar{\nu}_\ell) = \frac{G_F^2 |V_{ub}|^2 m_b^5}{192\pi^3} \left(1 - 2.41\frac{\alpha_s}{\pi} - 21.3\left(\frac{\alpha_s}{\pi}\right)^2 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2} + O\left(\alpha_s^3, \frac{\Lambda_{QCD}^3}{m_b^3}\right)\right)$$

but ... near perturbative singularities, life gets more complicated:

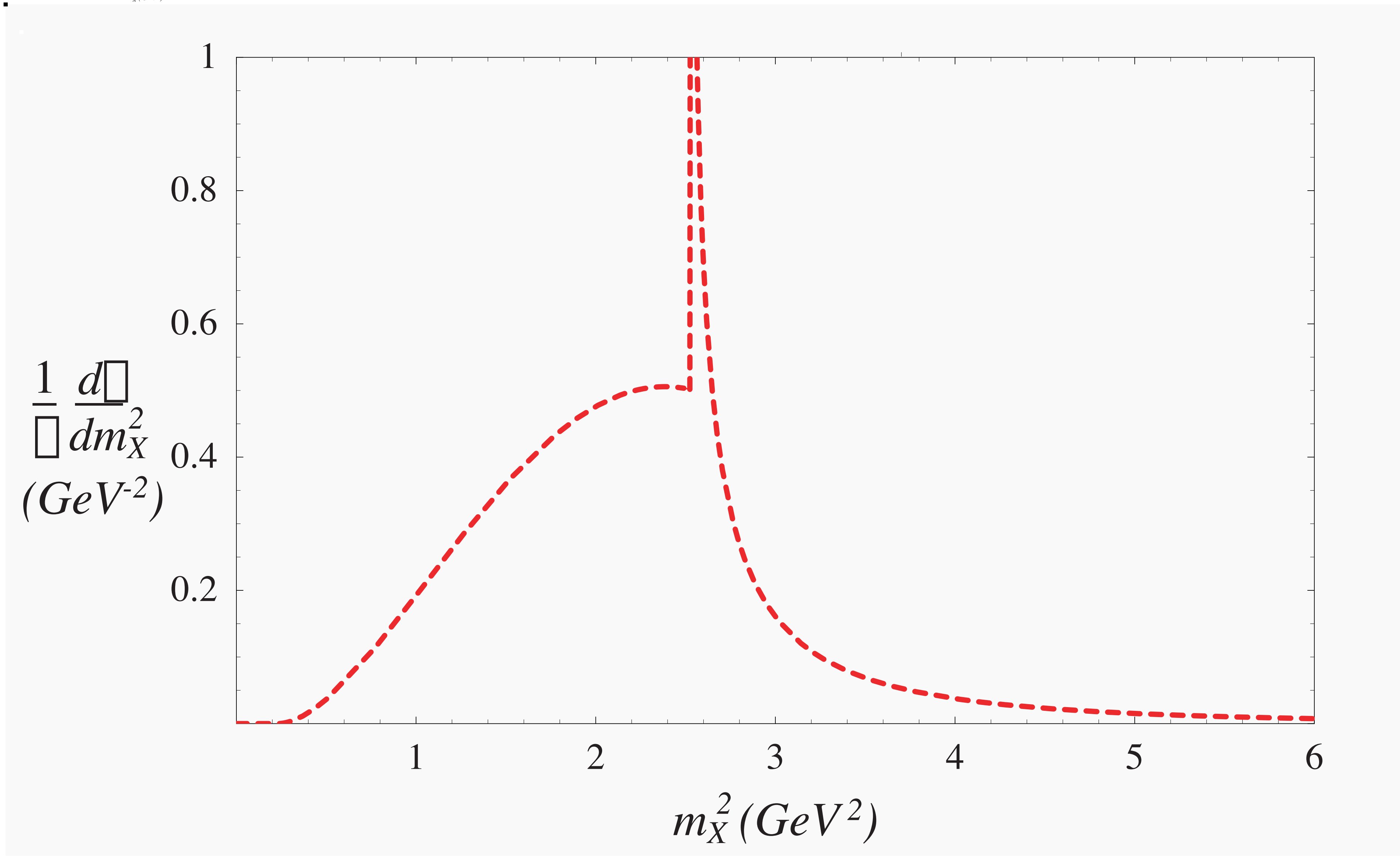
(Bigi, Shifman, Vainshtein, Uraltsev; Neubert)





Hadronic invariant mass spectrum:

(Falk, Ligeti, Wise; Dikeman, Uraltsev)



Hadronic invariant mass spectrum:

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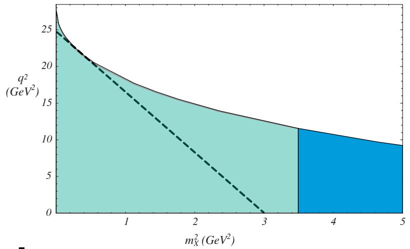
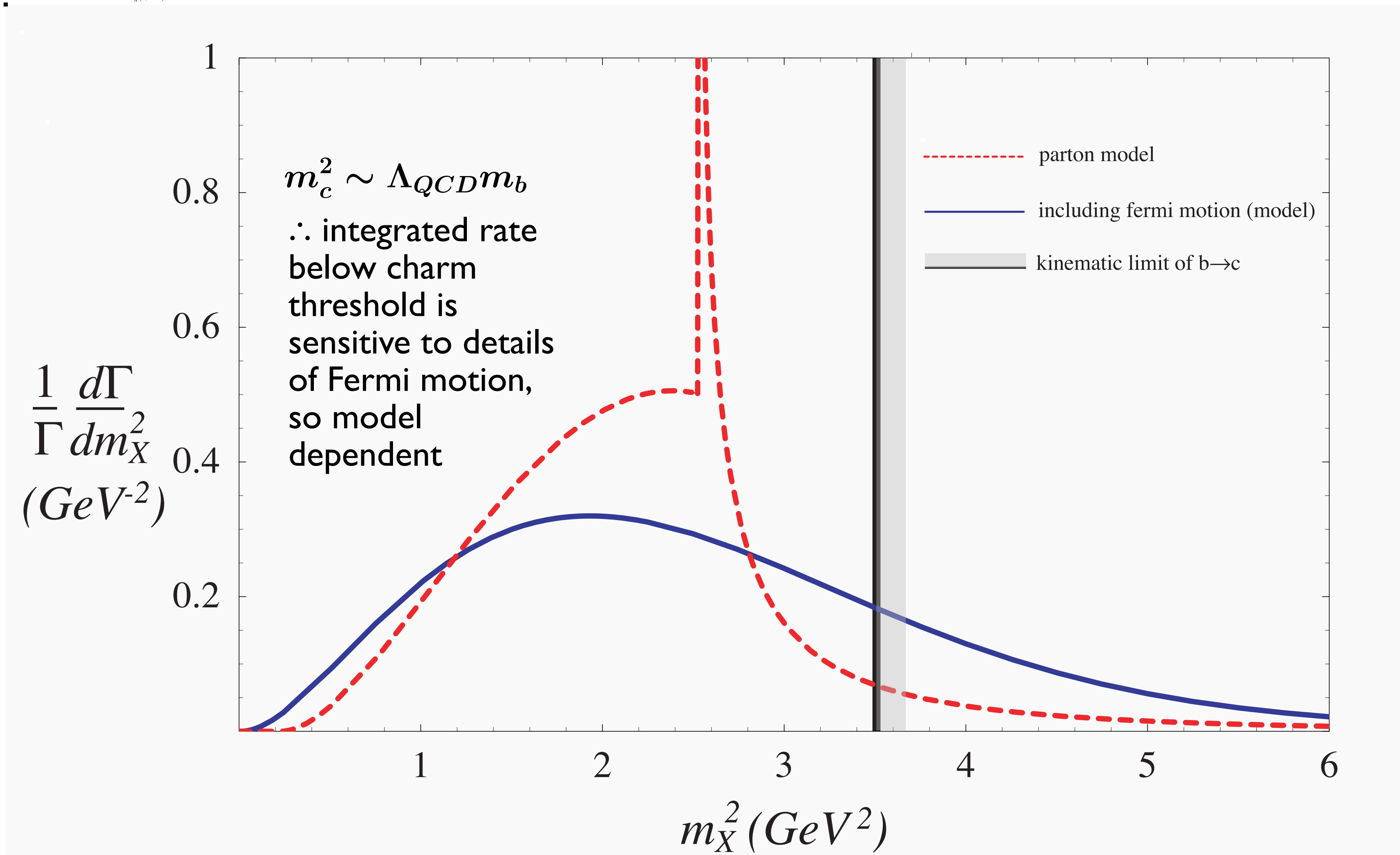
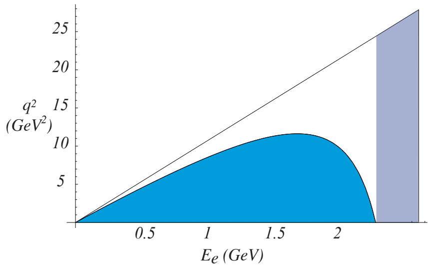


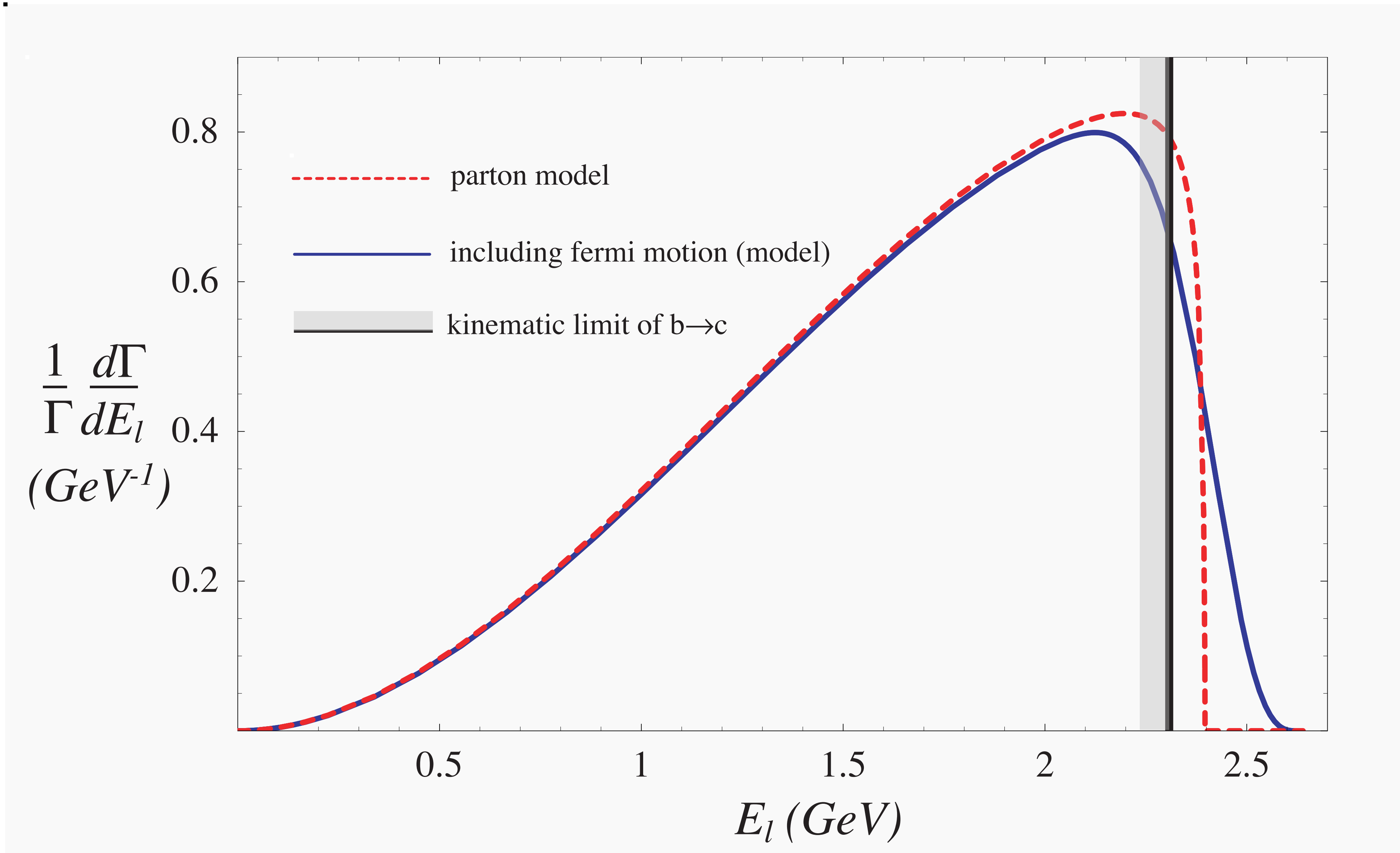


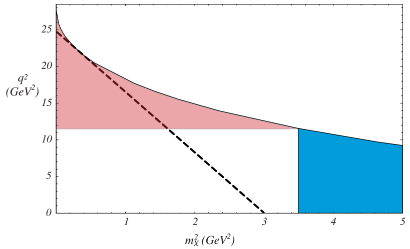
$m_c^2 \sim \Lambda_{QCD} m_b$

∴ integrated rate below charm threshold is sensitive to details of Fermi motion, so model dependent



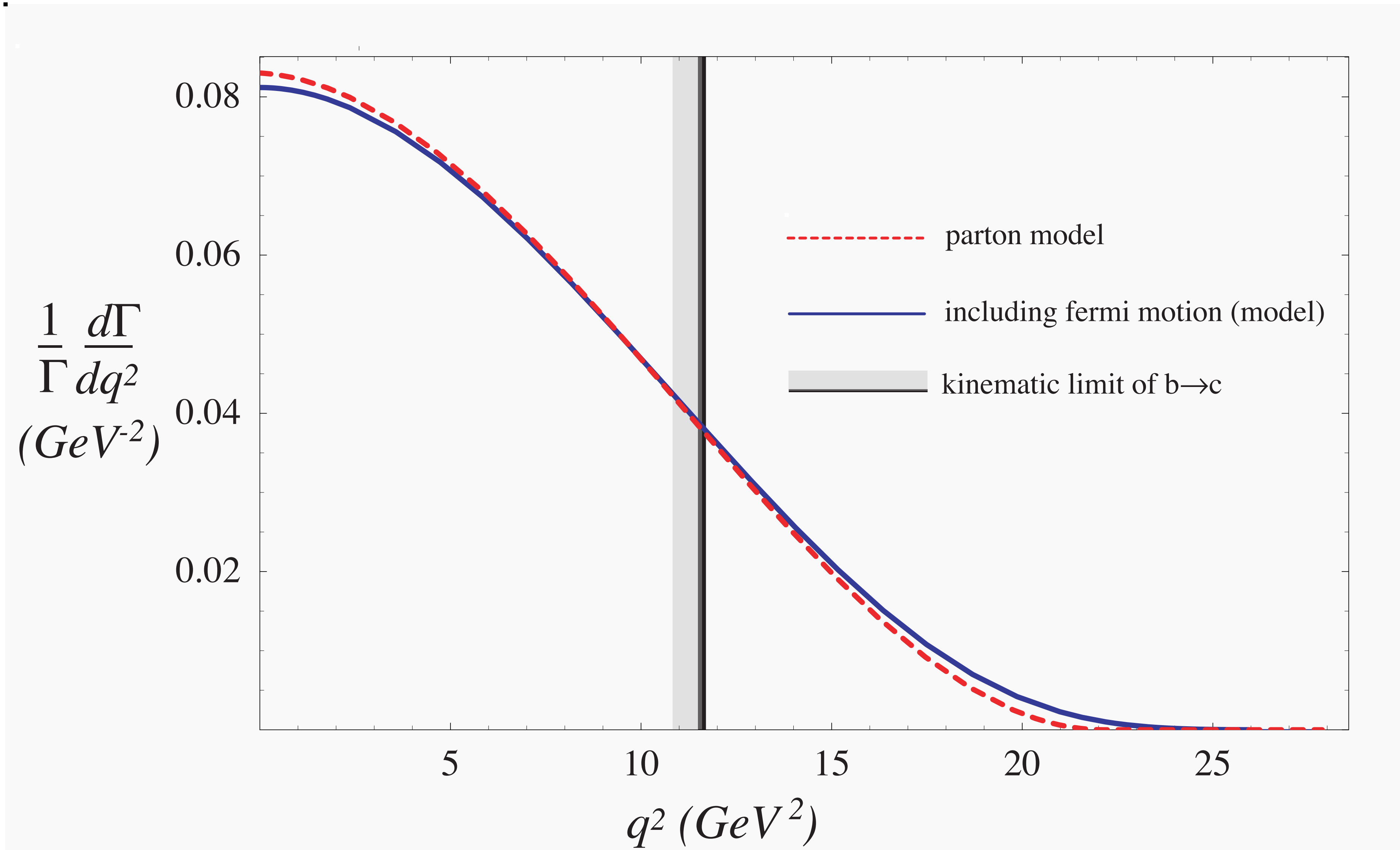
The same thing happens near the endpoint of the lepton energy spectrum:

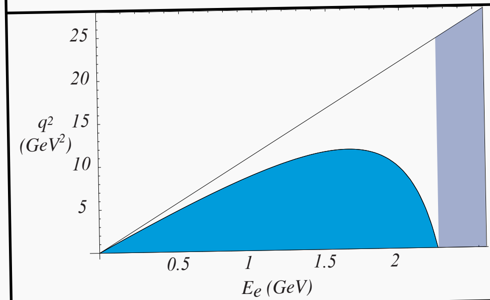
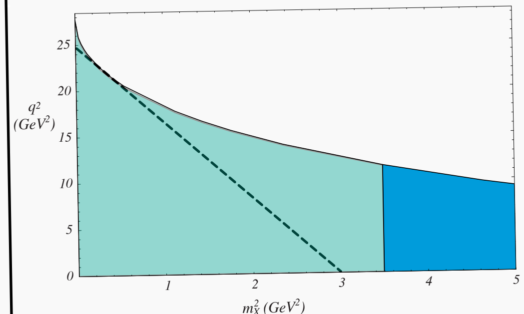
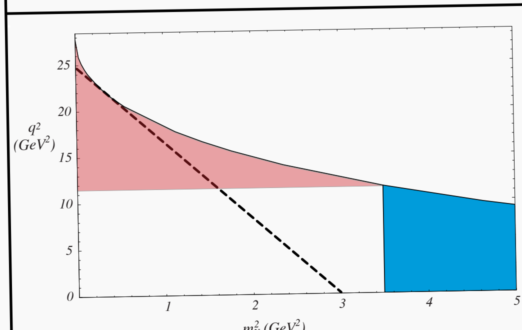
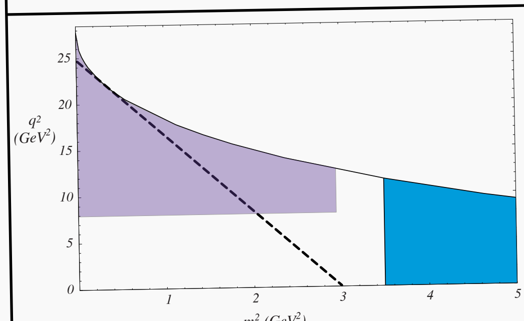




but not always ... i.e. leptonic q^2 spectrum:

(Bauer, Ligeti, ML)



cut		% of rate	good	bad
	$E_\ell > \frac{m_B^2 - m_D^2}{2m_B}$	~10%	don't need neutrino	
	$s_H < m_D^2$	~80%	lots of rate	
	$q^2 > (m_B - m_D)^2$	~20%	insensitive to $f(k^+)$	
	"Optimized cut"	~45%	- insensitive to $f(k^+)$ - lots of rate - can move cuts away from kinematic limits and still get small uncertainties	

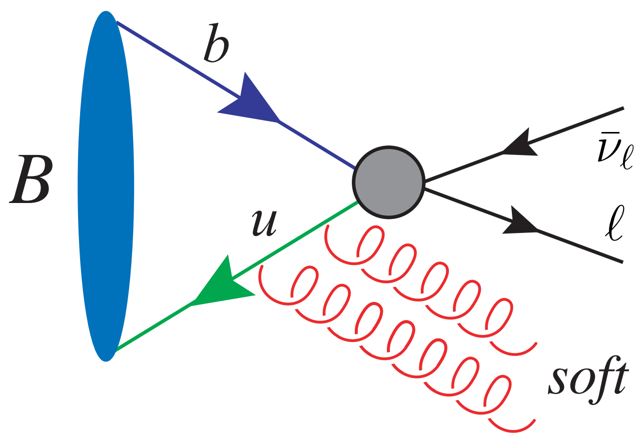
Theoretical Issues:

- Weak Annihilation (WA)
- Fermi motion - at leading and subleading order
- how well do we know m_b ?
- perturbative corrections - known (in most cases) to $O(\alpha_s^2\beta_0)$ - appear under control. When Fermi motion is important, leading and subleading Sudakov logarithms have been resummed.

Theoretical Issues:

- Weak annihilation

(Bigi & Uraltsev, Voloshin, Ligeti, Leibovich and Wise)



$$O\left(16\pi^2 \times \frac{\Lambda_{QCD}^3}{m_b^3} \times \text{ factorization violation}\right) \sim 0.03 \left(\frac{f_B}{0.2\,\text{GeV}}\right)\left(\frac{B_2 - B_1}{0.1}\right)$$

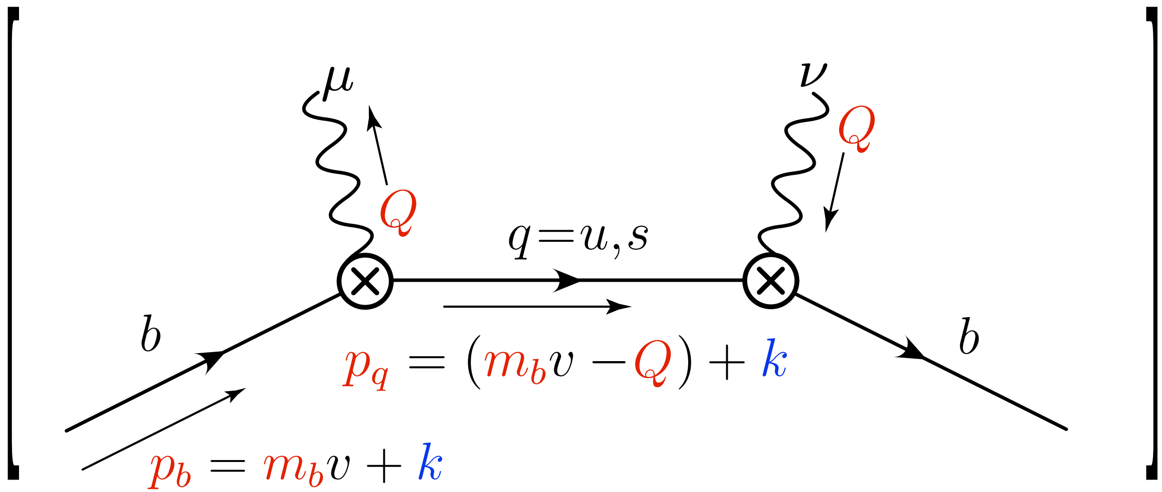
~3% (?? guess!) contribution to rate at $q^2 = m_b^2$

- an issue for all inclusive determinations
- relative size of effect gets worse the more severe the cut (lepton endpoint: ~10% of rate, so ~30% correction to rate at endpoint)
- no reliable estimate of size - can test by comparing charged and neutral B's

Theoretical Issues:

- Fermi motion - for s_H, E_ℓ cuts

$d\Gamma \sim \mathrm{Im}$



In the "shape function" region, p_q is almost lightlike:

$(n^\mu \equiv (1,0,0,1),\ k \cdot n \equiv k_+)$

$$p_q^\mu \simeq \frac{m_b}{2}(1,0,0,1) + \Delta^\mu \Rightarrow p_q^2 \sim O(m_b \Delta \cdot n) \sim O(m_b \Lambda_{QCD}) \ll m_b^2$$

definition of shape function region

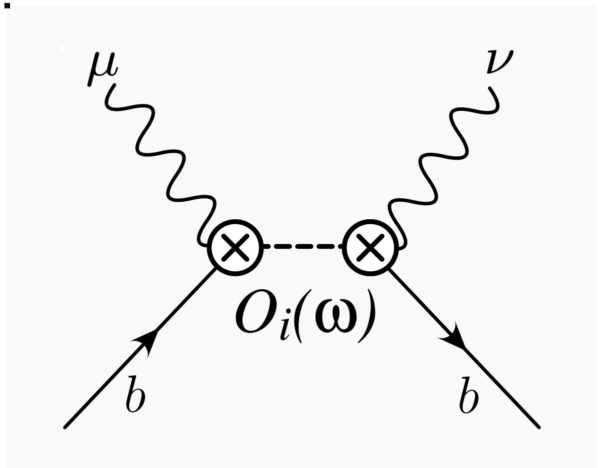
OPE: $$\frac{1}{(m_b v - Q + k)^2} = \frac{1}{m_b} \frac{1}{k \cdot n - \Delta \cdot n + i\epsilon} + \ldots$$

- imaginary part $\propto \delta(k \cdot n - \Delta \cdot n)$
- sensitive to functional form of k_+ distribution, not just moments

- the appropriate expansion is not about pointlike operators, but nonlocal operators where the two vertices are separated along the lightcone (cf twist expansion) so we can write the OPE in the shape function region as

$$d\Gamma \sim \int d\omega\ C_0(\omega)\langle O_0(\omega)\rangle + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

sum over operators becomes an integral



where at leading order the only operator is

$$O_0(\omega) = \bar{b}\ \delta(\omega + in\cdot D)\ b$$

which is the Fourier transform of the bilocal operator

(this is just DIS all over again)

$$\tilde{O}_0(t) = \bar{b}(0)P\exp\left(\frac{i}{m_b}\int_0^t n\cdot A(t')\ dt'\right)b(t)$$

points separated along the lightcone

$$b(t) \equiv b\left(\frac{tn^\mu}{m_b}\right)$$

The parton distribution function is therefore defined as

$$f(\omega) = \frac{1}{2m_B}\langle B|\bar{b}\ \delta(\omega + i\hat{D}\cdot n)b|B\rangle$$

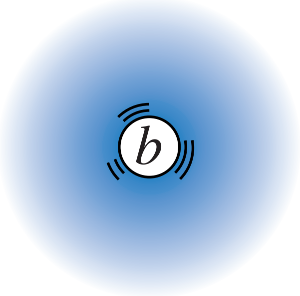
universal distribution function (applicable to all decays)

(NB the nonlocal OPE is equivalent at leading order in 1/m to smearing the partonic rate with the distribution function)

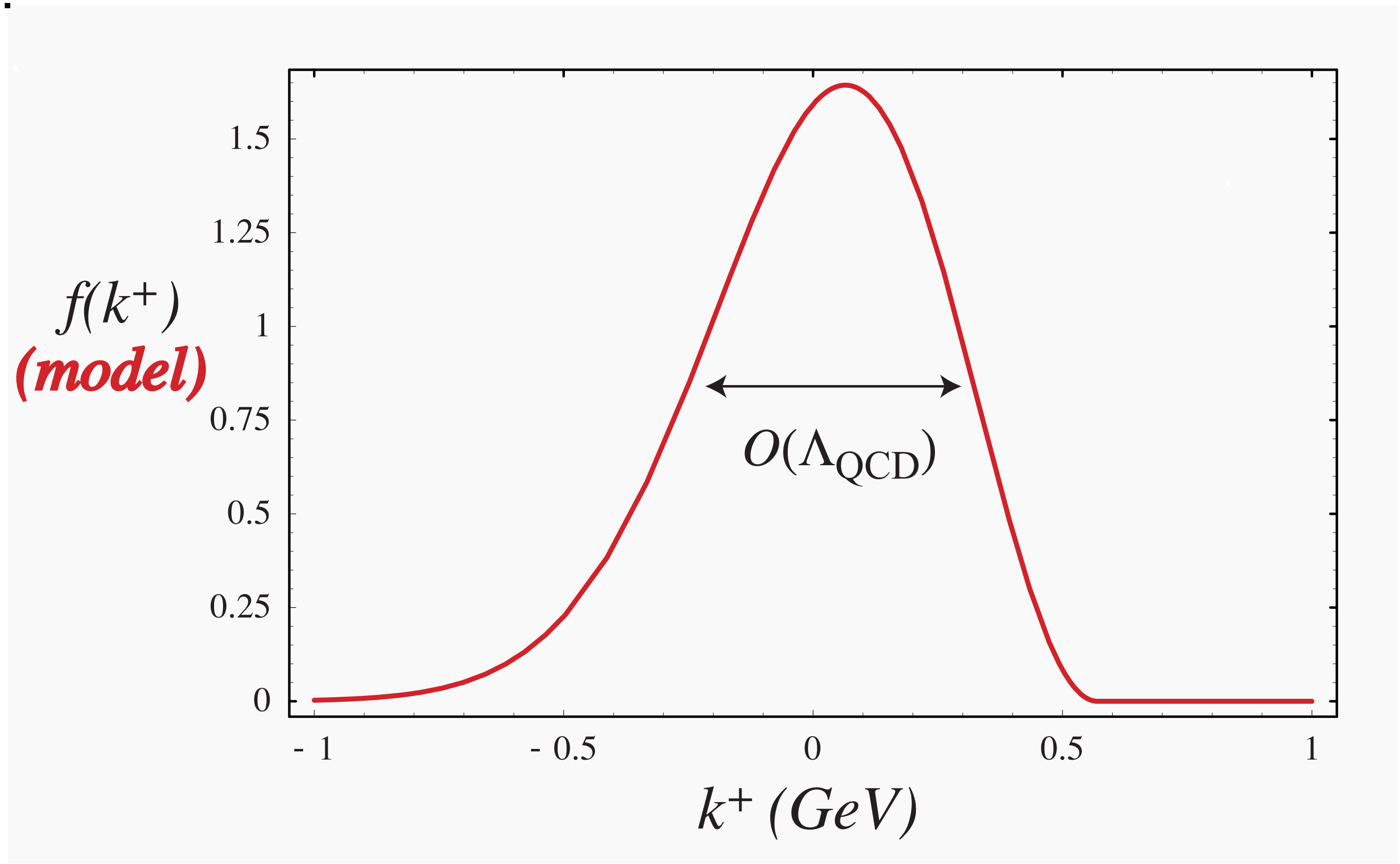
Options:

$$f(\omega) = \frac{1}{2m_B}\langle B|\bar{b}\,\delta(\omega + i\hat{D}\cdot n)b|B\rangle$$

universal distribution function



(i) model (cf Babar '03)



$$f(k^+) = N(1-x)^a e^{(1+a)x}$$

$$x \equiv \frac{k_+}{\bar{\Lambda}}$$

(de Fazio and Neubert)

a, N determined by $\bar{\Lambda}$, λ_1 (gets first two moments right .. but the uncertainty in $f(k^+)$ is not simply given by the uncertainties in $\bar{\Lambda}$, λ_1)

- lose model independence - harder to estimate uncertainties reliably
- sensitivity to functional form gets stronger as cut is moved away from kinematic boundary $s_H < m_D^2$, $E_\ell > (m_B^2 - m_D^2)/2m_B$

(ii) determine from experiment: $f(\omega)$ is universal:

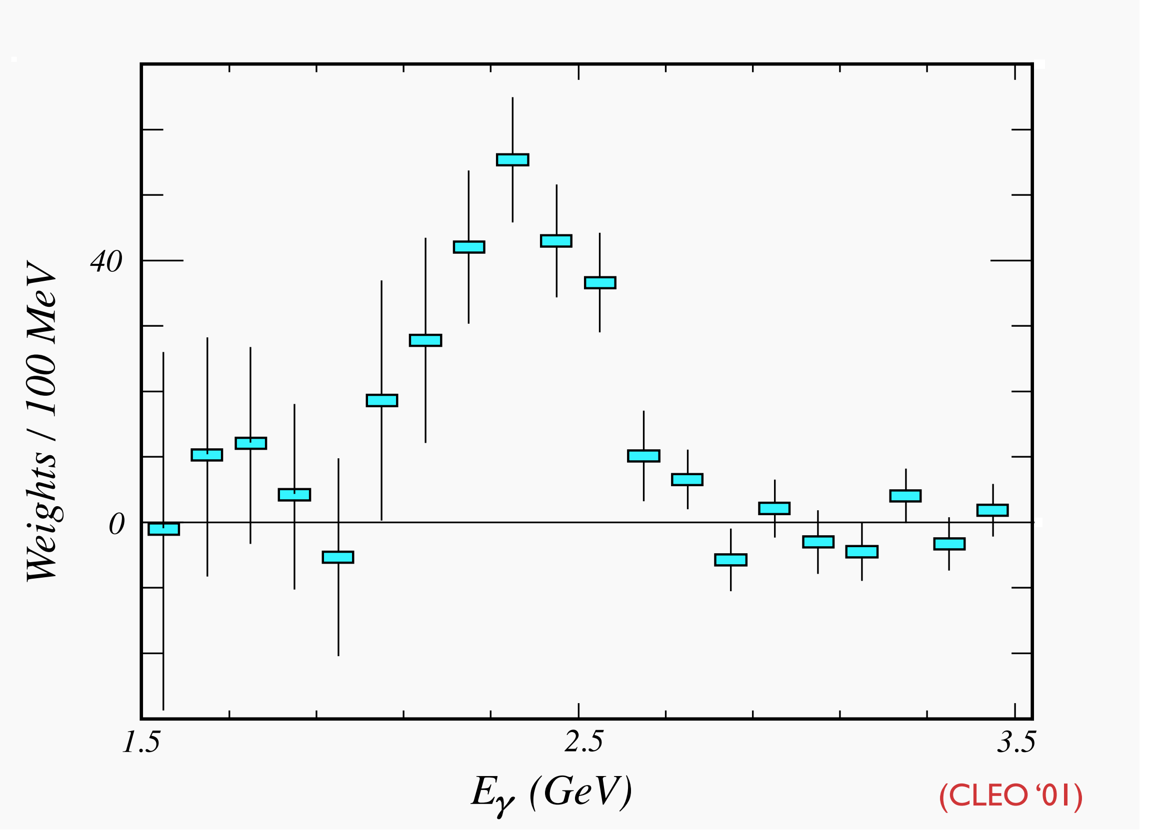
$$\frac{1}{\Gamma_0}\frac{d\Gamma}{d\hat{E}_\gamma}(B \to X_s\gamma) = \int d\omega\ \delta(1 - 2\hat{E}_\gamma - \omega) f(\omega) + \ldots$$

$$\frac{1}{2\Gamma_0}\frac{d\Gamma}{d\hat{E}_\ell}(B \to X_u \ell \bar{\nu}_\ell) = \int d\omega\ \theta(1 - 2\hat{E}_\ell - \omega) f(\omega) + \ldots$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{d\hat{s}_H}(B \to X_u \ell \bar{\nu}_\ell) = \int d\omega\ \frac{2\hat{s}_H^2(3\omega - 2\hat{s}_H)}{\omega^4}\theta(\omega - \hat{s}_H) f(\omega - \hat{\Lambda}) + \ldots$$

and so can be measured from the photon spectrum in $\bar{B} \to X_s\gamma$:

(NB must subtract off contributions of operators other than O_7)
(Neubert)



(CLEO '01)

NB: can relate integrated rates without assuming a functional form for $f(k^+)$:

(Neubert, Leibovich, Low, Rothstein)

$$\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2 = \frac{3\alpha}{\pi}|C_7^{eff}|^2\frac{\Gamma_u(E_c)}{\Gamma_s(E_c)} + O(\alpha_s) + O\left(\frac{\Lambda_{QCD}}{m_B}\right)$$

$$\Gamma_u(E_c) \equiv \int_{E_c}^{m_B/2} dE_\ell \frac{d\Gamma_u}{dE_\ell}$$

$$\Gamma_s(E_c) \equiv \frac{2}{m_b}\int_{E_c}^{m_B/2} dE_\gamma (E_\gamma - E_c)\frac{d\Gamma_s}{dE_\gamma}\,.$$

Including resummation of subleading Sudakov logs:

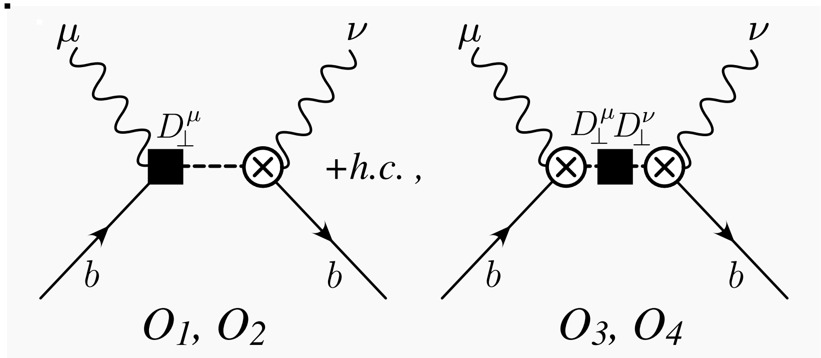
(Leibovich, Low, Rothstein)

$$\frac{|V_{ub}|^2}{|V_{ts}^* V_{tb}|^2} = \frac{3\,\alpha\,C_7(m_b)^2}{\pi}\int_{x_B^c}^1 dx_B \frac{d\Gamma}{dx_B}$$
$$\times\left\{\int_{x_B^c}^1 dx_B \int_{x_B}^1 du_B\; u_B^2\,\frac{d\Gamma^\gamma}{du_B}\,K\left[x_B;\frac{4}{3\pi\beta_0}\log(1-\alpha_s\beta_0\,l_{x_B/u_B})\right]\right\}^{-1},$$

BUT all of this only holds at leading order in Λ_{QCD}/m_b ...

$$f(\omega) \sim \langle B|\underbrace{\bar b\,\delta(\omega - i\hat D\cdot n)b}|B\rangle$$

universal distribution function
(applicable to all decays)



... at $O(\Lambda_{QCD}/m_b)$, there is more structure:

sensitive to $k_\perp$

$$g_2(\omega_1,\omega_2) \sim \langle B|\bar b\,\delta(\omega_2 + in\cdot\hat D)(iD_\perp)^2\,\delta(\omega_1 + in\cdot\hat D)b|B\rangle$$

breaks spin symmetry (distinguishes semileptonic from radiative decays)

$$h_1(\omega) \sim \langle B|\bar b\,[iD_\mu,\ \delta(\omega + in\cdot\hat D)]\,\gamma_\lambda\gamma_5 b|B\rangle\ \epsilon_\perp^{\mu\lambda}$$

$$h_2^\lambda(\omega_1,\omega_2) \sim \langle B|\bar b\,\delta(\omega_2 + in\cdot\hat D)G_{\mu\nu}\delta(\omega_1 + in\cdot\hat D)\gamma^\lambda\gamma_5 b|B\rangle\ \epsilon_\perp^{\mu\nu}$$

sensitive to soft gluons

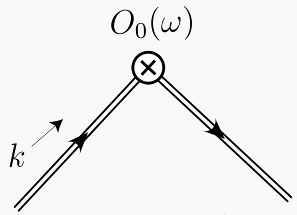
$$T(\omega) \sim \int e^{-i\omega t}\langle B|T(\bar b(0)b(t),\ O_{1/m}(y))|B\rangle$$

(Bauer, ML, Mannell)

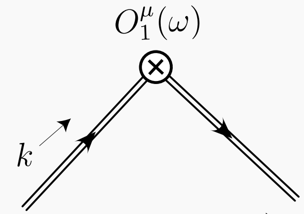
nonlocal T-product - only need to worry about if comparing with charm decay

(NB this is just DIS at subleading twist all over again)

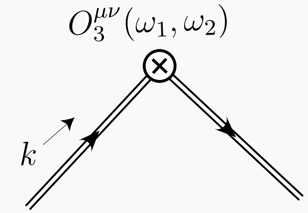
Feynman rules for subleading nonlocal operators:



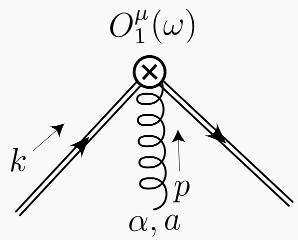
$\delta(\omega + n\cdot k)$



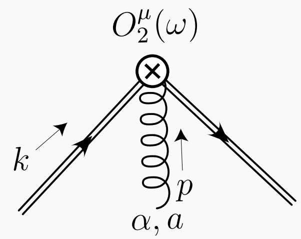
$2k^\mu \delta(\omega + n\cdot k)$



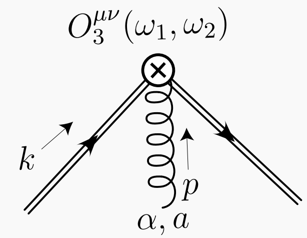
$2k_\perp^\mu k_\perp^\nu \delta(\omega_1 + n\cdot k)\delta(\omega_2 + n\cdot k)$



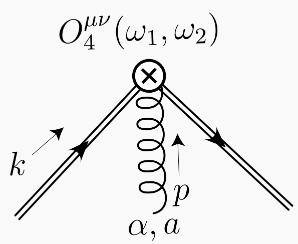
$$-gT^a g^{\mu\alpha}\left[\delta(\omega + n\cdot k) + \delta(\omega + n\cdot(k+p))\right]$$



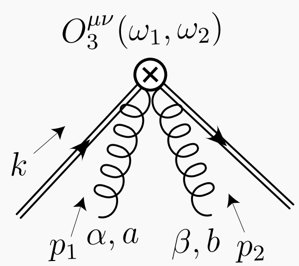
$$-igT^a g^{\mu\alpha}\left[\delta(\omega + n\cdot k) - \delta(\omega + n\cdot(k+p))\right]$$



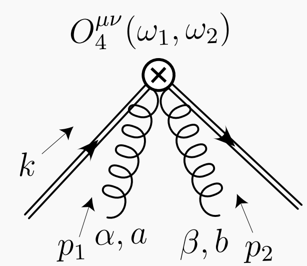
$$-gT^a\left[(p+2k)_\perp^\mu g_\perp^{\nu\alpha} + (p+2k)_\perp^\nu g_\perp^{\mu\alpha}\right]\times \delta(\omega_1 + n\cdot k)\delta(\omega_2 + n\cdot(p+k))$$



$$-igT^a\left(p_\perp^\mu g_\perp^{\nu\alpha} - p_\perp^\nu g_\perp^{\mu\alpha}\right)\delta(\omega_1 + n\cdot k) \times\delta(\omega_2 + n\cdot(p+k))$$



$$g^2\left(g_\perp^{\mu\alpha}g_\perp^{\nu\beta} + g_\perp^{\mu\beta}g_\perp^{\nu\alpha}\right)\{T^a,T^b\}\times \delta(\omega_1 + n\cdot k)\delta\left(\omega_2 + n\cdot(k+p_1+p_2)\right)$$



$$ig^2\left(g_\perp^{\mu\alpha}g_\perp^{\nu\beta} - g_\perp^{\mu\beta}g_\perp^{\nu\alpha}\right)[T^a,T^b]\times \delta(\omega_1 + n\cdot k)\delta\left(\omega_2 + n\cdot(k+p_1+p_2)\right)$$

The effect of these subleading "shape functions" can be surprisingly large in the lepton energy endpoint region ….

(Leibovich, Ligeti, Wise; Bauer, ML, Mannell)

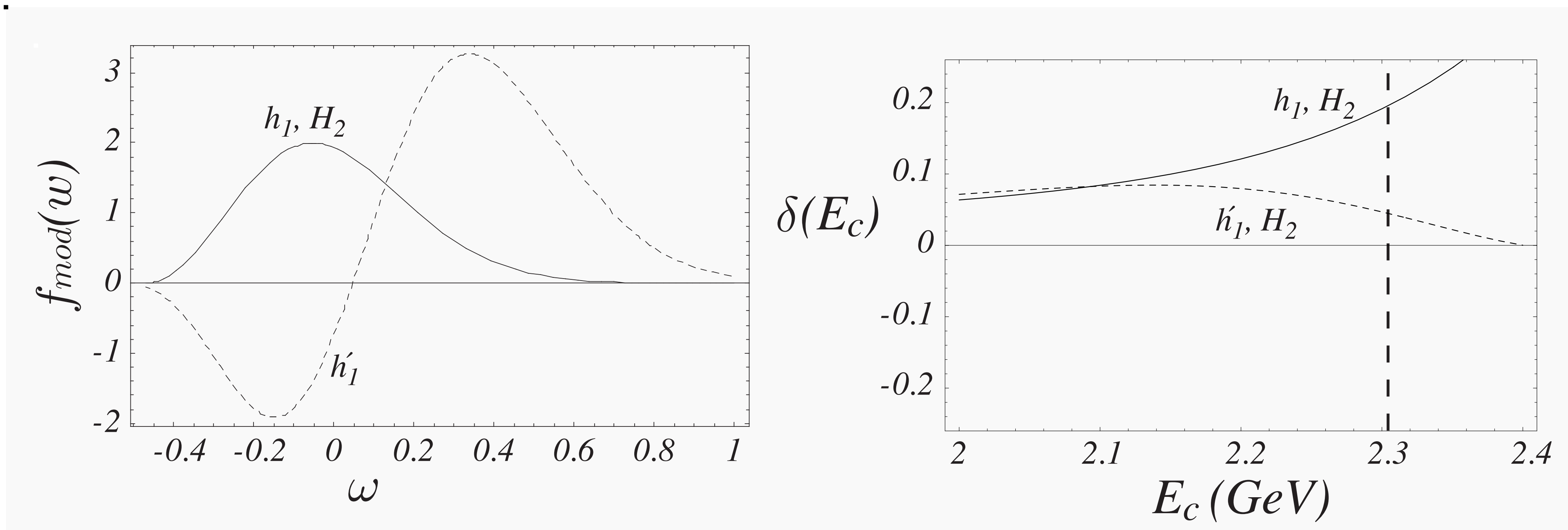
$$\frac{d\Gamma}{dy} \sim 2\theta(1-y) - \frac{\lambda_1}{3m_b^2}\delta'(1-y) - \frac{\rho_1}{9m_b^3}\delta''(1-y) + \dots$$

leading "twist" terms … sum to $f(k^+)$

$$-\frac{\lambda_1}{3m_b^2}\delta(1-y) - \frac{11\lambda_2}{m_b^2}\delta(1-y) + \dots$$

subleading "twist" terms … sum to new distribution functions

corresponding coefficient in B X_s is 3



2 different models for subleading shape functions...

... and the corresponding effect on the determination of $|V_{ub}|$

... but appear to be smaller for the invariant mass spectrum.

(Burrell, ML, Williamson)

Theoretical Issues:

- How well do we know m_b?

- rate is proportional to m_b^5 - 100 MeV error is a ~5% error in V_{ub}. But restricting phase space increases this sensitivity - with q^2 cut, scale as $\sim m_b^{10}$ (Neubert)

- "Optimized cuts":

$\Delta m_b = 30\,\mathrm{MeV} \Rightarrow$ 5% uncertainty in rate (half that in V_{ub})

$\Delta m_b = 80\,\mathrm{MeV} \Rightarrow$ 13% uncertainty in rate (half that in V_{ub})

(but this gets worse if the cuts are less optimal)

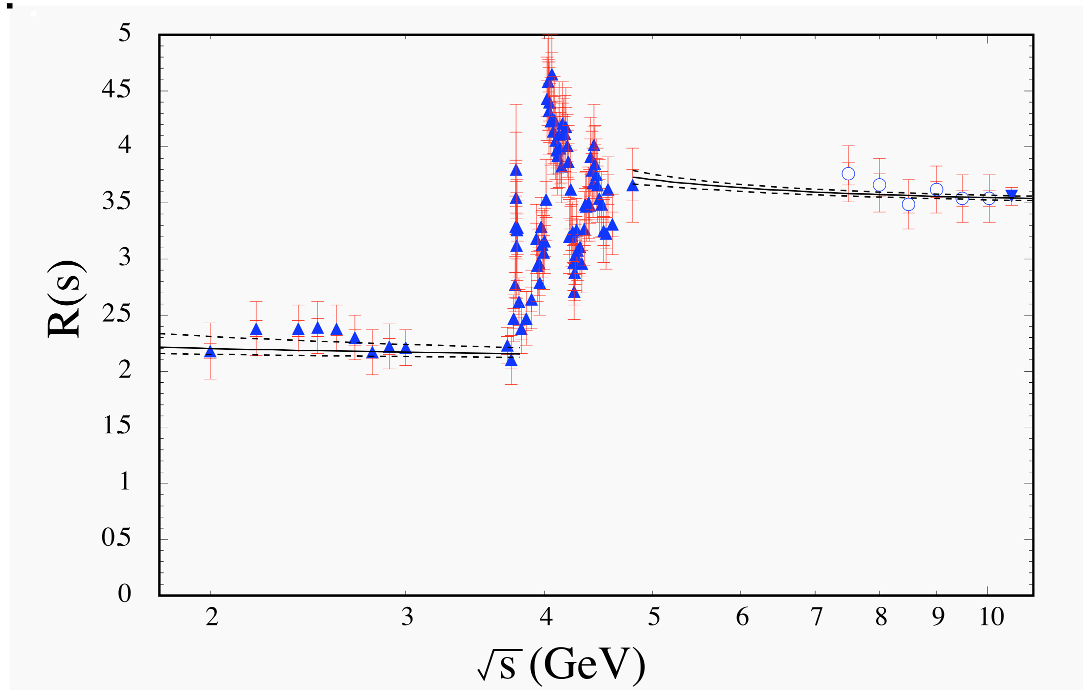
So what is the uncertainty in m_b?

Determination of m_b via Υ sum rules: (Voloshin, ...)

$$P_n = \int dE \frac{\sigma(e^+e^- \to \bar{b}bX)}{E^{2n+1}} \propto m_b^{-2n}$$

- for large *n*, moments are
(1) very sensitive to
(2) dominated by resonances
(experimentally well-measured)

(but $E \propto \frac{m_b}{n} > \Lambda_{QCD}$ so need $n \lesssim 10$
or nonperturbative effects become large)



Results:

State of the art: NNLO, or relative order 1/*n*

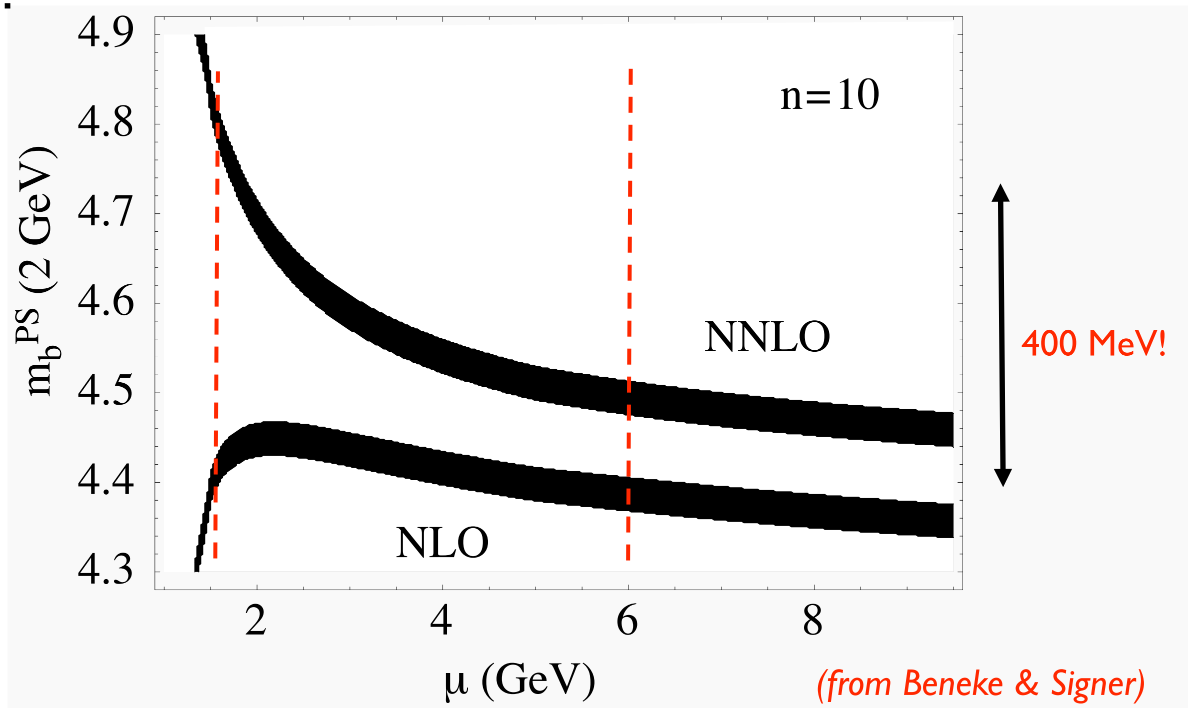
$$m_b^{1S} = 4.71 \pm 0.02_E \pm 0.02_T \,\mathrm{GeV} \Rightarrow \bar{m}_b(\bar{m}_b) = 4.17 \pm 0.05 \,\mathrm{GeV}$$ (Hoang)

$$m_b^{PS}(2\,\mathrm{GeV}) = 4.60 \pm 0.11 \,\mathrm{GeV} \Rightarrow \bar{m}_b(\bar{m}_b) = 4.26 \pm 0.10 \,\mathrm{GeV}$$ (Beneke, Signer)

$$m_b^{kin} = 4.56 \pm 0.06 \,\mathrm{GeV} \Rightarrow \bar{m}_b(\bar{m}_b) = 4.20 \pm 0.10 \,\mathrm{GeV}$$ (Melnikov, Yelkovsky)

Caveat: stability!!

• nonrelativistic expansion is not converging well: change in m_b from LO to NLO is about the same as NLO to NNLO *(big two-loop correction to heavy quark potential)*
• varying μ between 1.5 GeV and 6 GeV gives the (pessimistic?) error estimate $\Delta m_b \sim \pm 200$ MeV



(from Beneke & Signer)

• different groups handle this in different ways *(M&Y: conjecture alternating series, B&S: neglect large NLO/NNLO shift, Hoang: simultaneous fit to different moments has reduced scale dependence and NNLO shift)* to get theoretical error of ~100 MeV or better
• renormalization group improvement required? (worked for analogous problem in *tt* production …)

Determination of m_b via spectral moments:

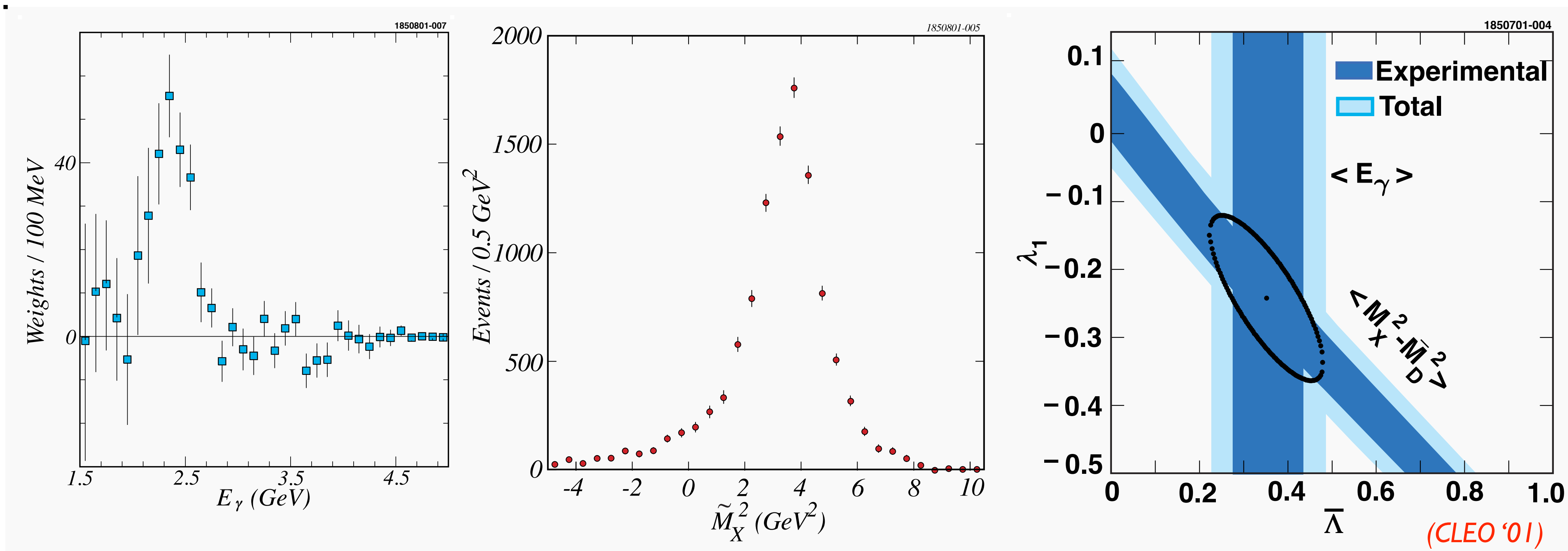
- like rate, moments of spectra can be calculated as a power series in $\alpha_s(m_b)$, Λ_{QCD}/m_b:

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b^2} + \ldots$$

$$\langle E_\gamma \rangle = \frac{m_B - \bar{\Lambda}}{2} + \ldots$$

$$\frac{1}{m_B^2}\langle s_H - \bar{m}_D^2 \rangle_{E_\ell > 1.5\,\mathrm{GeV}} = 0.21\frac{\bar{\Lambda}}{\bar{m}_B} + 0.26\frac{\bar{\Lambda}^2 + 3.8\lambda_1 - 1.2\lambda_2}{\bar{m}_B^2} + \ldots$$

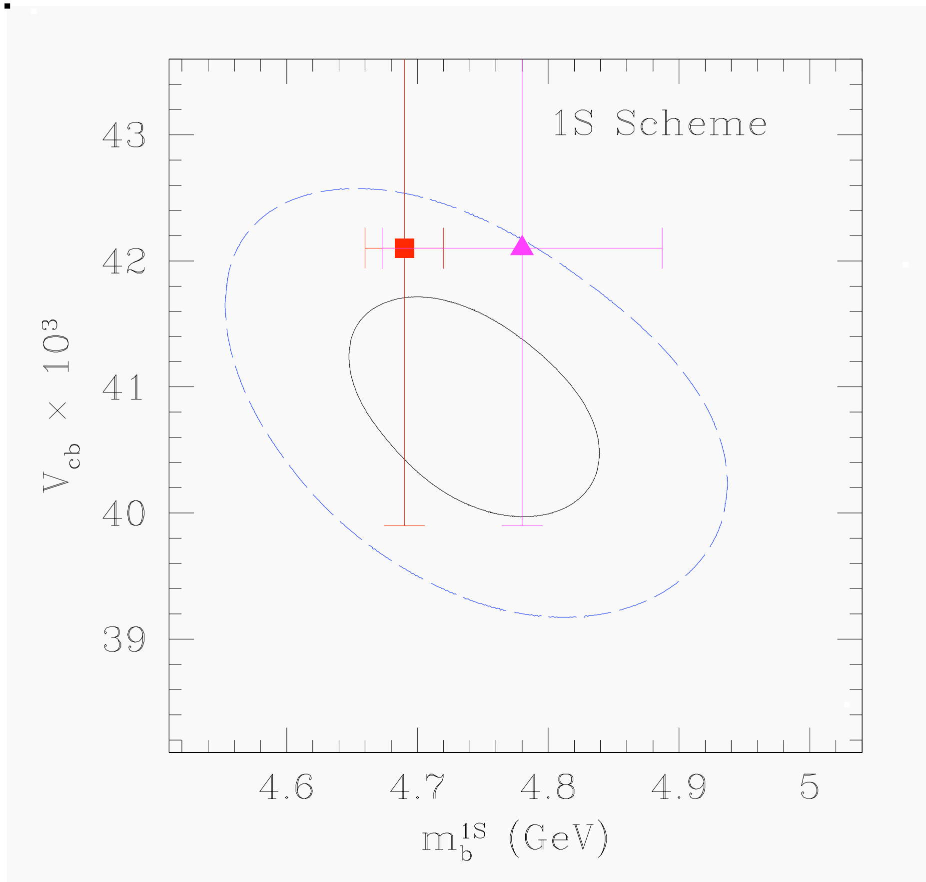
Constrain different linear combinations of $\bar{\Lambda}$, λ_1
⇨ fit m_b



(CLEO '01)

Global fits (summer '02):

(fit including $1/m^3$ effects)



- lepton energy and hadronic invariant mass moments $(\bar{B} \to X_c \ell \bar{\nu})$, photon energy spectrum moments $(\bar{B} \to X_s \gamma)$

- measured with varying cutoffs by DELPHI, CLEO and BaBar

$$m_b^{1S} = 4.74 \pm 0.10 \, \mathrm{GeV}$$

$$|V_{cb}| = (40.8 \pm 0.9) \times 10^{-3}$$

(Bauer, Ligeti, ML and Manohar, PRD67:054012, 2003 - BaBar s_H spectra not included in fit)

$$m_b(1\,\mathrm{GeV}) = 4.59 \pm 0.08 \, \mathrm{GeV} \Rightarrow m_b^{1S} = 4.69 \, \mathrm{GeV}$$

$$m_c(1\,\mathrm{GeV}) = 1.13 \pm 0.13 \, \mathrm{GeV}$$

$$|V_{cb}| = (41.9 \pm 1.1) \times 10^{-3}$$

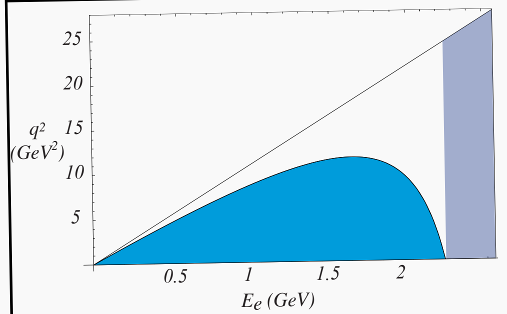
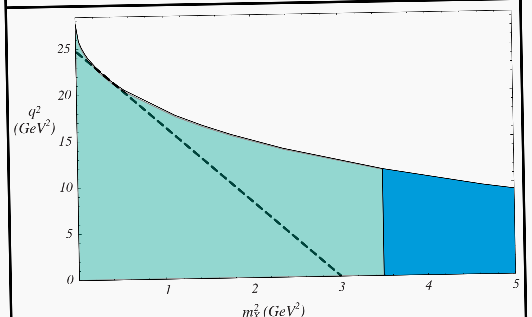
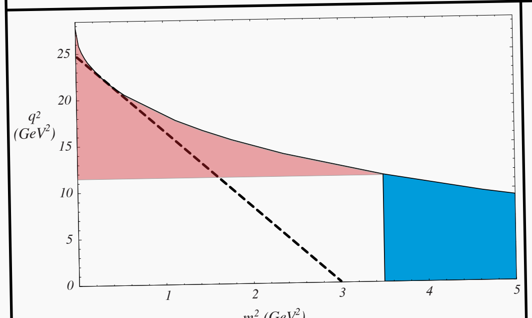
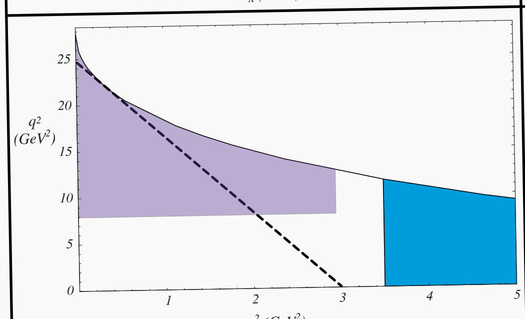
(Battaglia *et. al.*, PLB556:41, 2003, using DELPHI data)

■ Hoang
▲ Beneke

} exclusive V_{cb} extraction, b mass from $\bar{b}b$ sum rules

... and just for fun, setting all experimental errors to zero we find

$$\delta(|V_{cb}|) \times 10^3 = \pm 0.35, \;\; \delta(m_b) = \pm 35 \, \mathrm{MeV}$$

cut	% of rate	good	bad
	~10%	don't need neutrino	- depends on $f(k^+)$ (and subleading corrections) - WA corrections may be substantial - reduced phase space - duality issues?
	~80%	lots of rate	depends on $f(k^+)$ (and subleading corrections)
	~20%	insensitive to $f(k^+)$	- very sensitive to m_b - WA corrections may be substantial - effective expansion parameter is $1/m_c$
	~45%	- insensitive to $f(k^+)$ - lots of rate - can move cuts away from kinematic limits and still get small uncertainties	

Experimental measurements can help beat down the theoretical errors:

(a) better determination of m_b (moments of B decay distributions)

(b) test size of WA (weak annihilation) effects - compare D^0 & D_S S.L. widths, extract $|V_{ub}|$ from $B^{\pm}$ and B^0 separately

(c) improve measurement of $B \to X_s \gamma$ photon spectrum - get $f(k^+)$ - lowering cut reduces effects of subleading corrections, as well as sensitivity to details of $f(k^+)$

(d) (most important) measure $|V_{ub}|$ in as many CLEAN ways as possible - different techniques have different sources of uncertainty (*c.f.* inclusive and exclusive determinations of $|V_{cb}|$)

Summary:

- $1/m_Q$ expansion allows precise theoretical predictions for inclusive decays - uncertainties are at the $1/m^3$ level
- measuring $|V_{ub}|$ requires probing restricted regions of phase space - some (but not all!) regions are sensitive to nonperturbative structure function
- size of weak annihilation (formally $1/m^3$) and precision on m_b can be limiting factors
- a number of regions of phase space may be used to determine $|V_{ub}|$, with different sources of uncertainty