Inclusive determinations of sides of the unitarity triangle: theory

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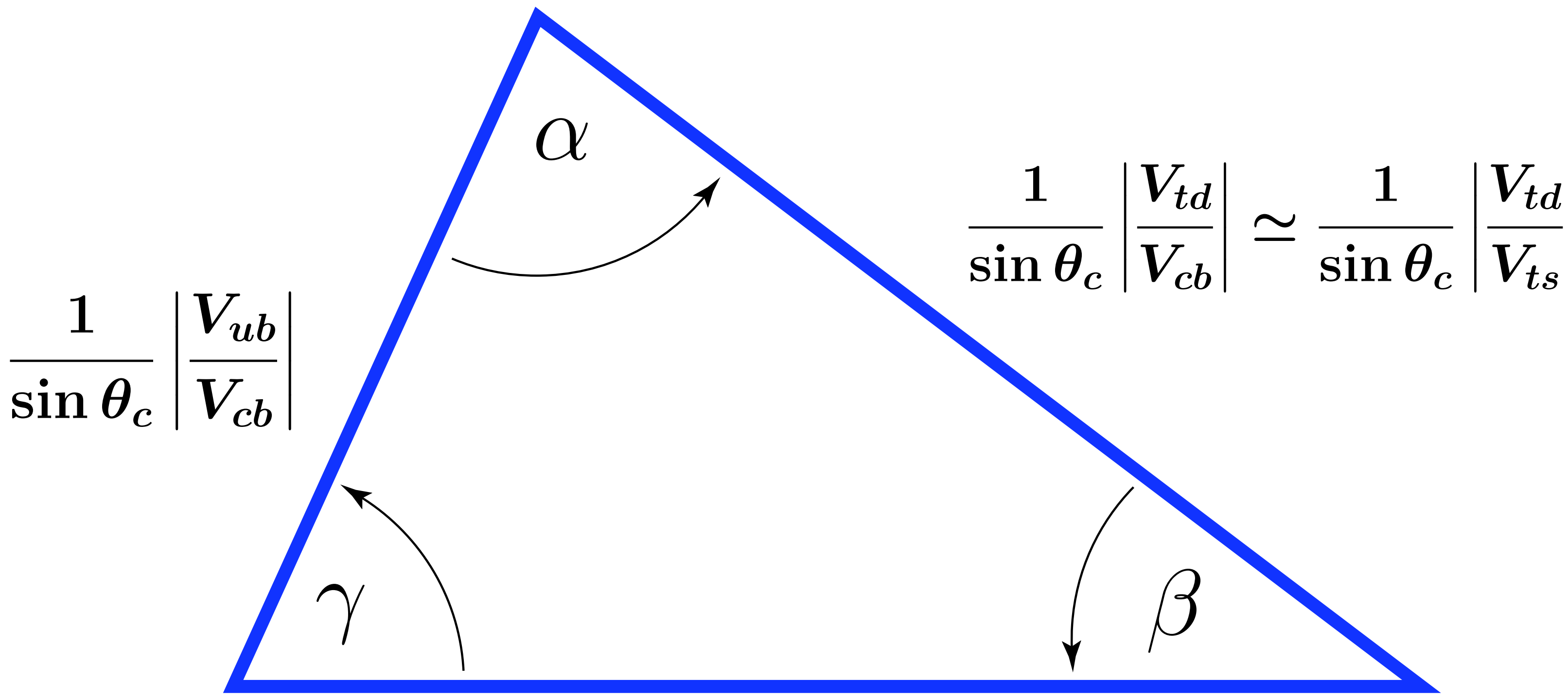
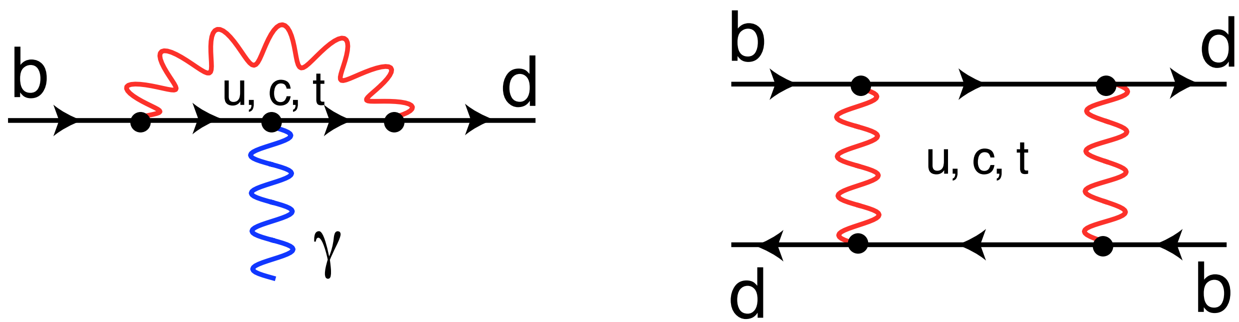
Outline:

1. Introduction
2. V_{ub} from $B \to X_u \ell \bar{\nu}$
3. V_{cb} from $B \to X_c \ell \bar{\nu}$
4. V_{td} from $B \to X_d \gamma$
5. Summary

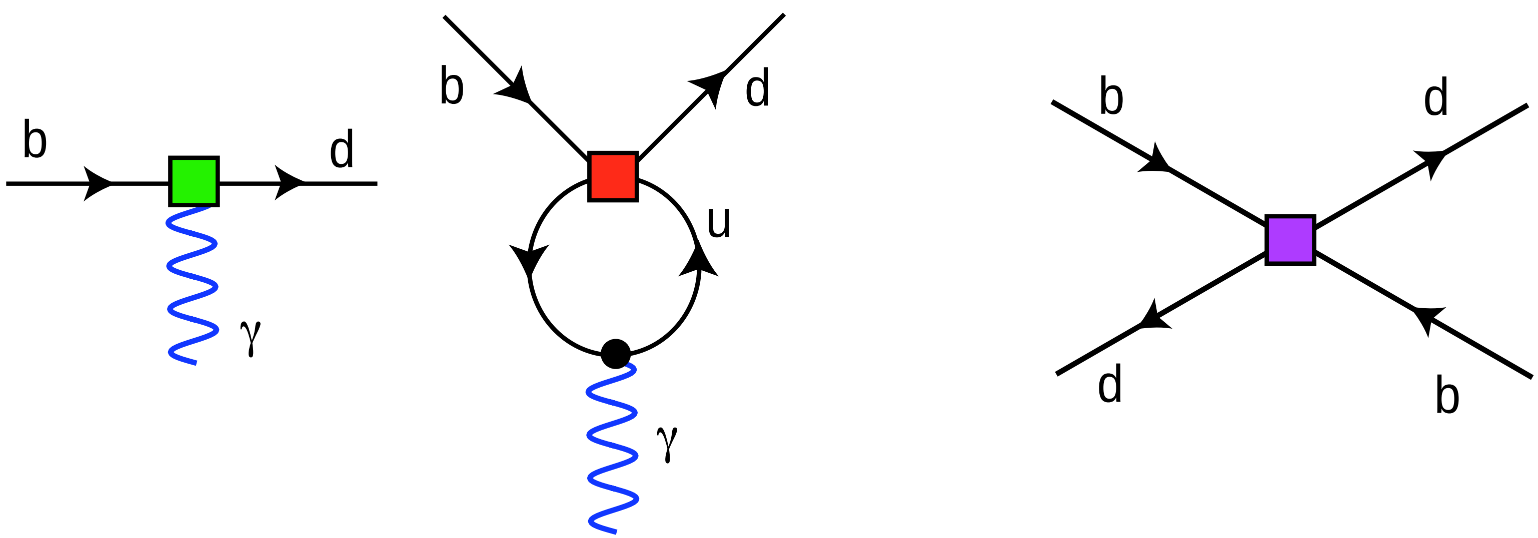
The Unitarity triangle provides a convenient way to visualize SM relations ....



... but we aren't interested in measuring the sides per se, but rather looking for New Physics/inconsistencies ... "redundant" measurements (in the SM) are important ex: B-$\bar{B}$ mixing and $b \to d\gamma$ are both determined by V_{td} in the SM:



BUT they are really measuring different physics - agreement is a nontrivial test of the validity of the SM



Why inclusive decays?

Exclusive decays are HARD - need to understand QCD at long distances to describe hadronizaton:

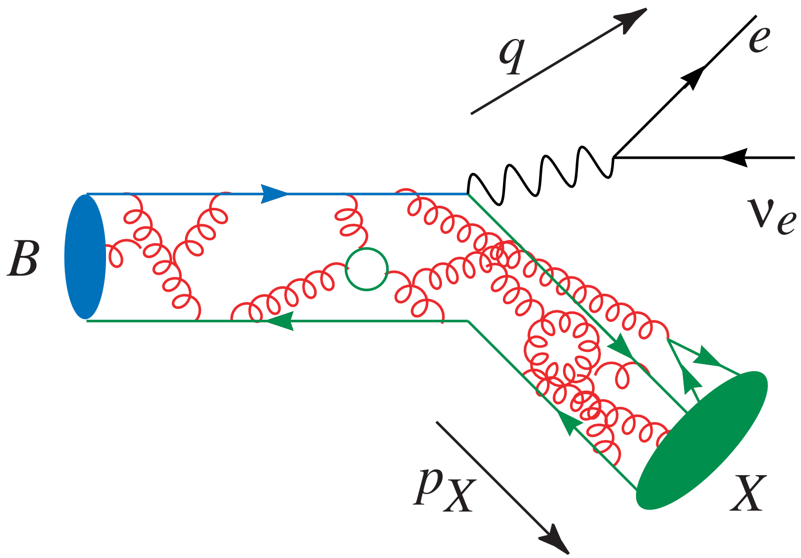
ex: $\bar{B} \longrightarrow \pi \ell \bar{\nu}$

$$\langle \pi(p_\pi) | V^\mu | B(p_B) \rangle = f_+(E) \left[p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right] + \cancel{f_0(E) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu}$$

vanishes for m_ℓ=0

nonperturbative - need to model (QCD sum rules) or calculate on lattice

Theorists love inclusive decays ...



Decay: short distance (calculable)
Hadronization: long distance (nonperturbative) - but at leading order, long and short distances are cleanly separated and probability to hadronize is unity

$$\frac{d\Gamma}{d(P.S.)} \sim \text{parton model} + \sum_n C_n \left(\frac{\Lambda_{QCD}}{m_b}\right)^n$$

"Most" of the time, details of b quark wavefunction are unimportant - only averaged properties (i.e. $\langle k^2 \rangle$) matter

"Fermi motion"
$k^\mu \sim \Lambda_{QCD}$

$$\Gamma(\bar{B} \to X_u \ell \bar{\nu}_\ell) = \frac{G_F^2 |V_{ub}|^2 m_b^5}{192\pi^3} \left(1 - 2.41\frac{\alpha_s}{\pi} - 21.3\left(\frac{\alpha_s}{\pi}\right)^2 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2} + O\left(\alpha_s^2, \frac{\Lambda_{QCD}^3}{m_b^3}\right)\right)$$

... but there are still plenty of issues

- phase space boundaries - cuts can mess up theory (V_{ub})
- nonperturbative parameters needed for high precision (V_{ub}, V_{cb})
- long-distance physics - fragmentation, light quark loops ($b \to (s/d)\gamma$)
- "quark-hadron duality" (all)
- + the usual suspects (perturbation theory, quark masses ...)

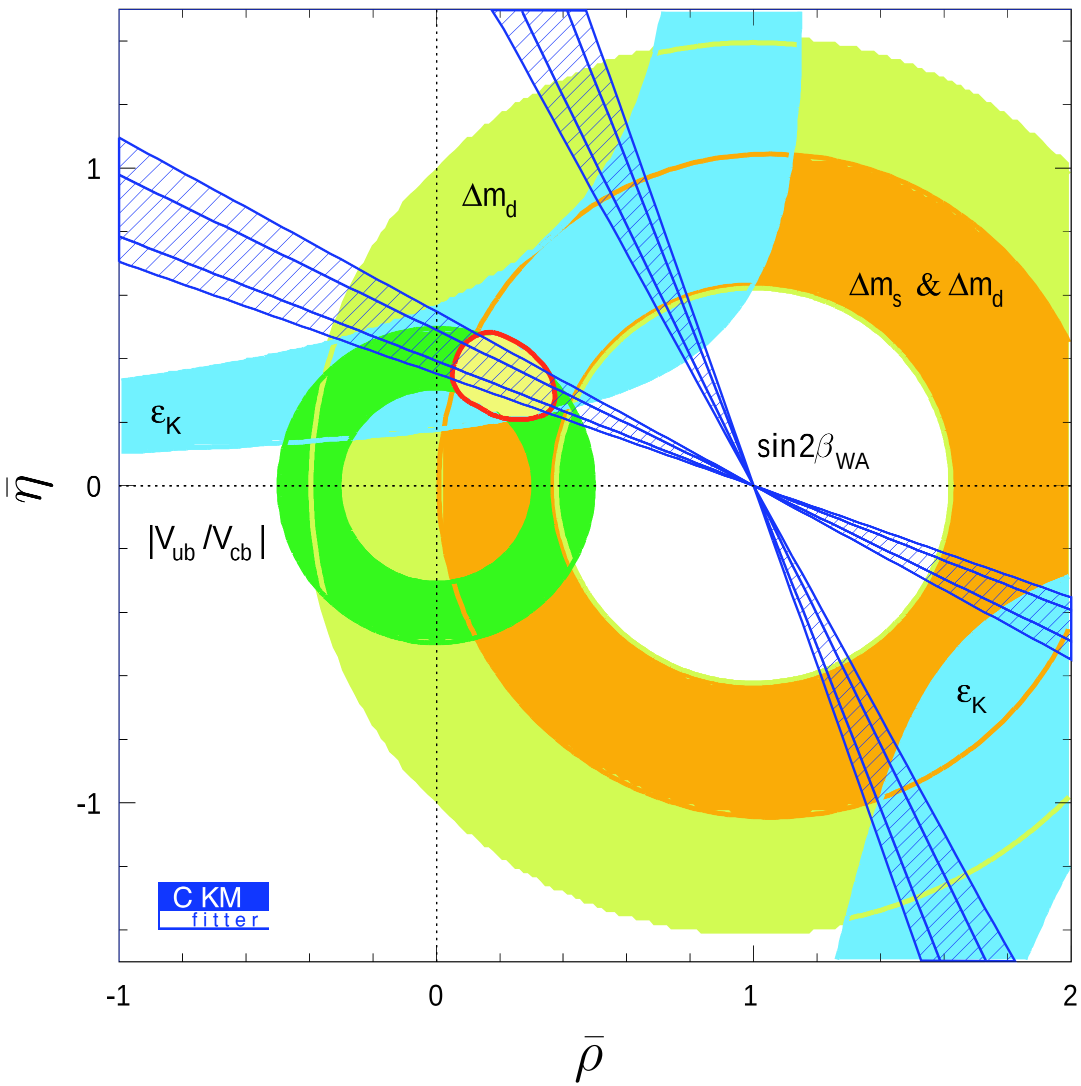
What can a 10^{36} machine do for us?

- more statistics - rare decays, spectra
- large sample of fully reconstructed events
  - reduce/eliminate backgrounds
  - allow phase space constraints to be relaxed

BUT ... the gains to be made in V_{ub} and V_{cb} are likely at the factor of ~2 improvement in the errors currently achievable

V_{ub}

V_{ub} and V_{cb} are both determined from tree level processes (SL decay) so unlikely to contain NP (unlike V_{td}, which is measured in loops)



World average '02:

$$\sin 2\beta = 0.734 \pm 0.054$$

- any further improvement in sin 2β won't tell us anything more about consistency without a better determination of V_{ub}

Best option: measure total inclusive semileptonic rate ...

$$|V_{ub}| = (3.06 \pm 0.08 \pm 0.08) \times 10^{-3} \left(\frac{B(B \to X_u \ell \bar{\nu})}{0.001} \frac{1.6\,\mathrm{ps}}{\tau_B} \right)^{1/2}$$

50 MeV uncertainty on $m_b(1S)$

perturbative uncertainty

(Hoang, Ligeti, Manohar)

combine to a ~5% error

- very clean theoretically: greatest uncertainty is b quark mass ... nonperturbative effects are small (caveat: WA)

... but this requires cutting out ~100 times larger background from charm (could this be done??)

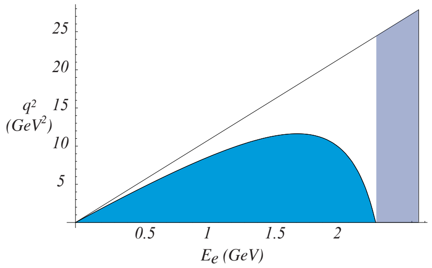
... if that doesn't work, need to impose phase space cuts

- life gets more complicated because

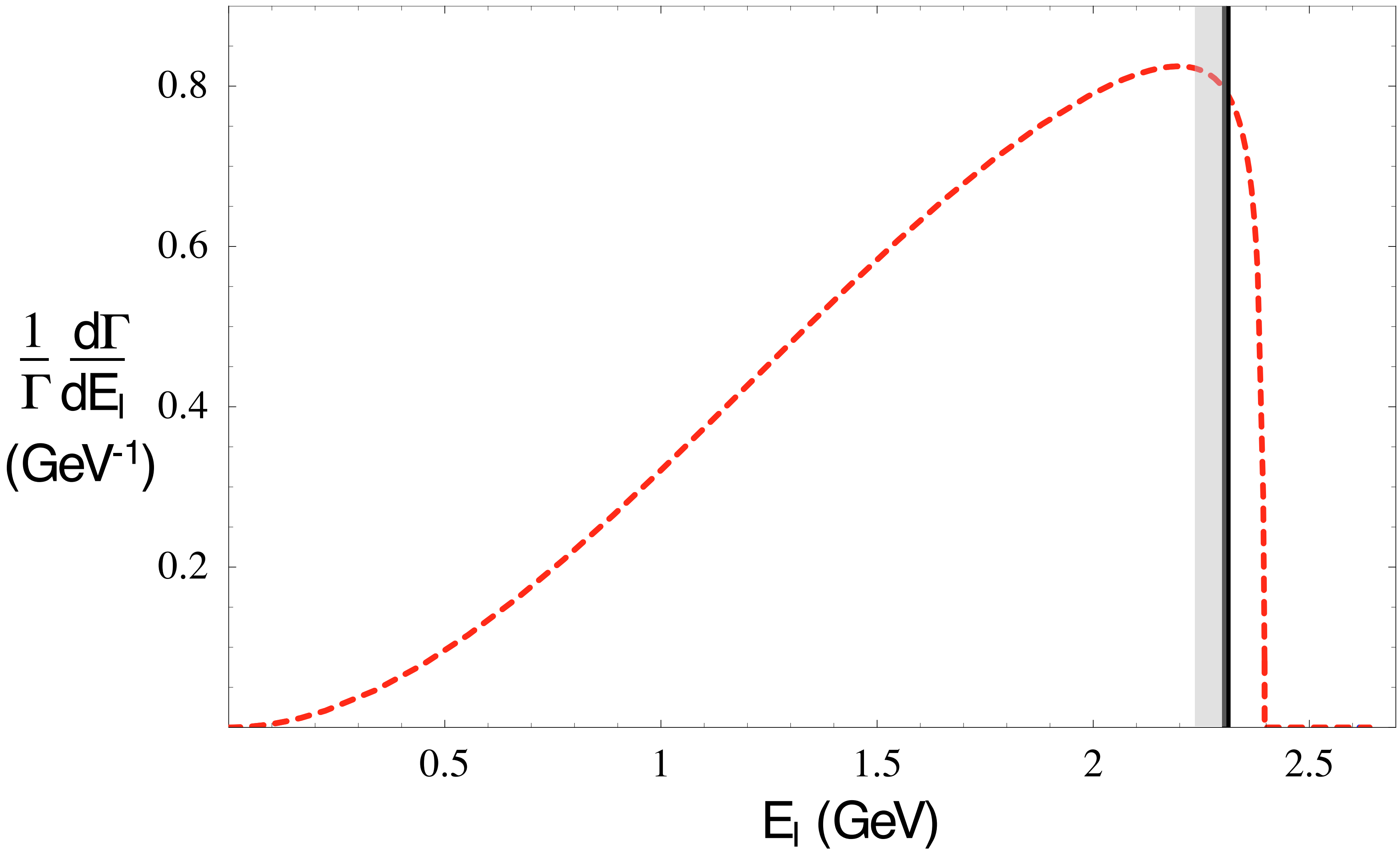
(1) smaller momentum transfer increases size of perturbative, nonperturbative corrections

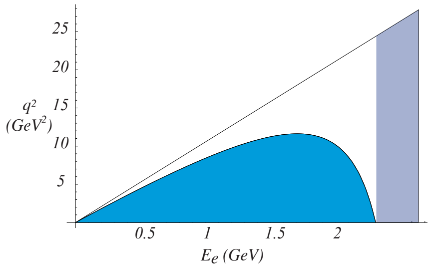
(2) cuts near perturbative singularies enhance certain nonperturbative (and perturbative) effects

(Bigi, Shifman, Vainshtein, Uraltsev; Neubert)



The Classic Method: cut on the endpoint of the charged lepton spectrum





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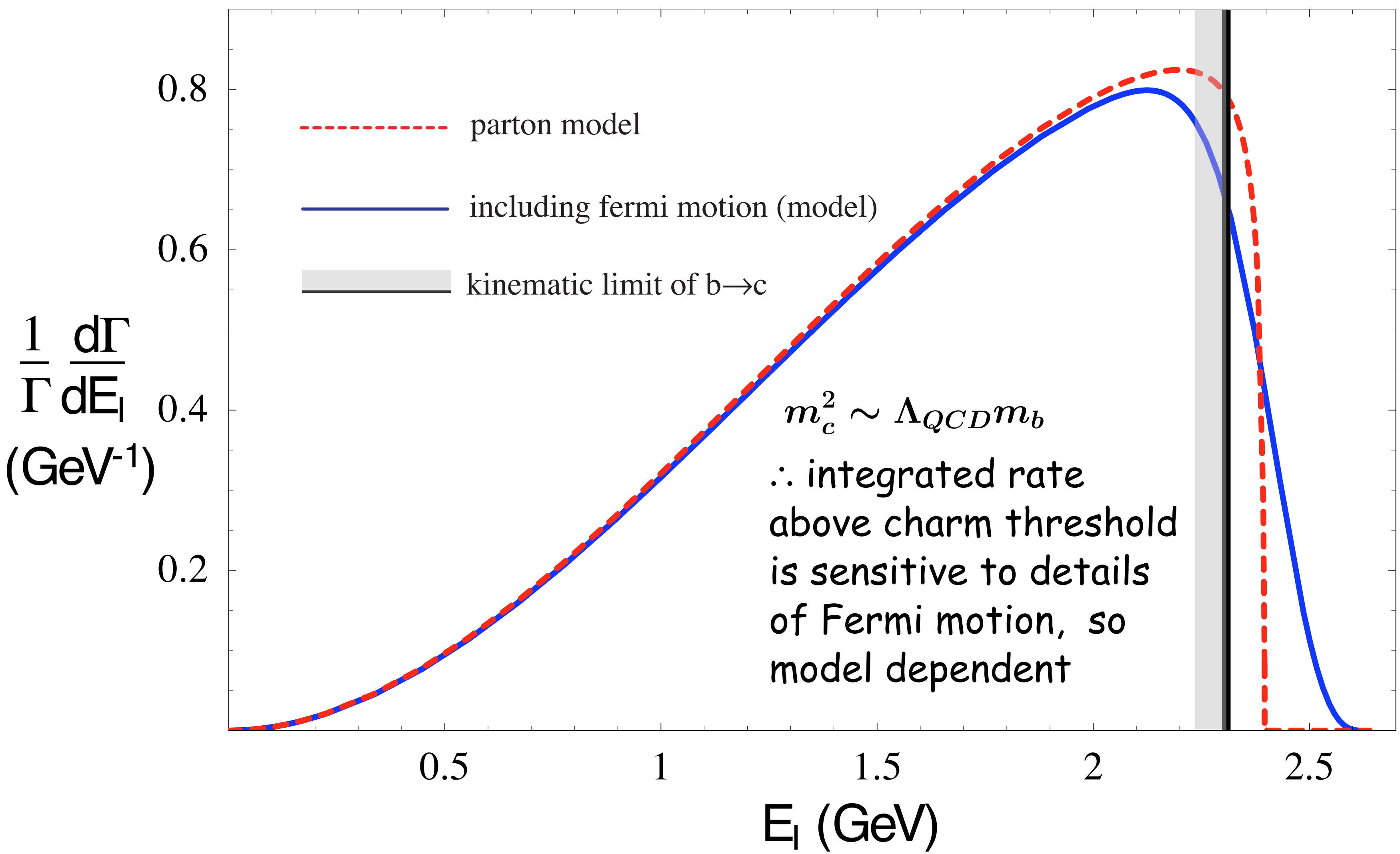
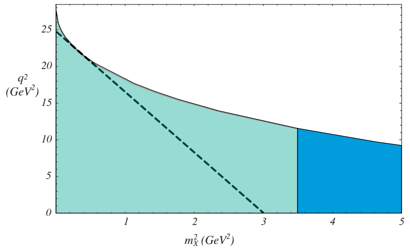
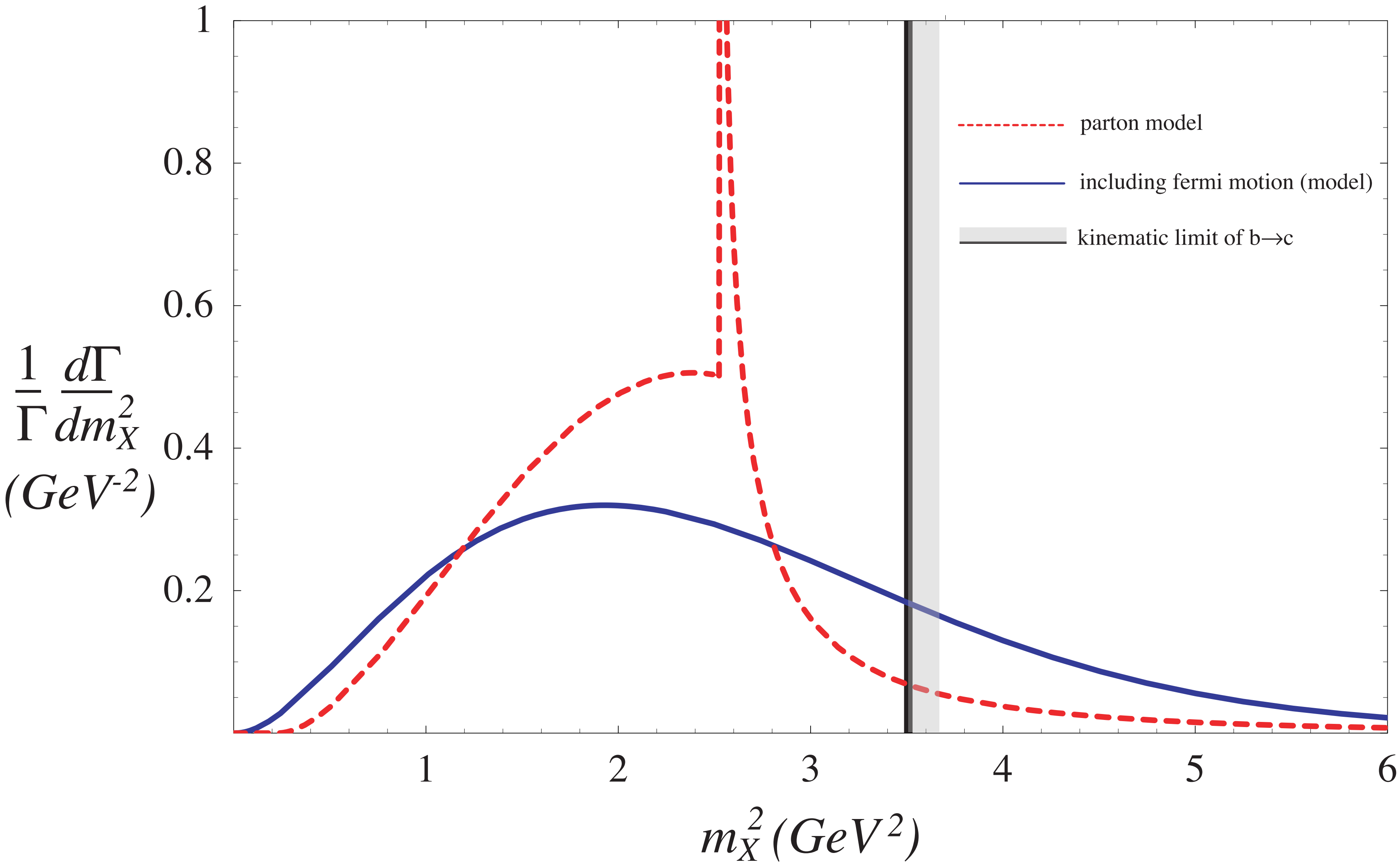
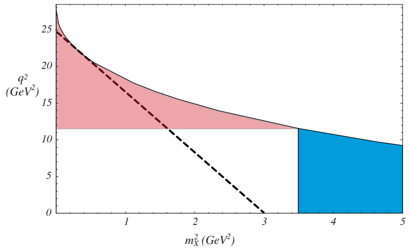




Cutting on the hadronic invariant mass spectrum gives more rate, but has the same problem with Fermi motion:

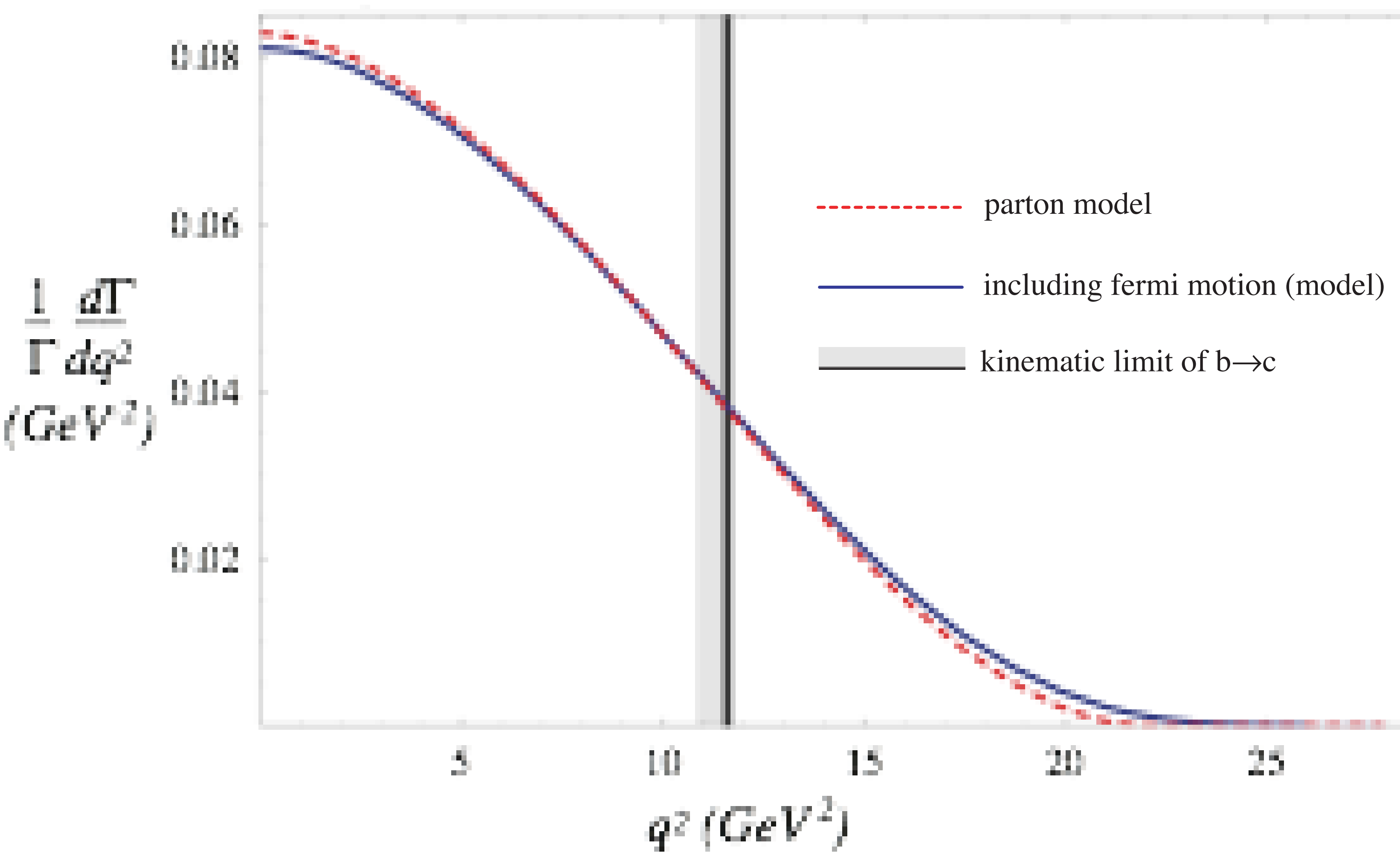
(Falk, Ligeti, Wise; Dikeman, Uraltsev)

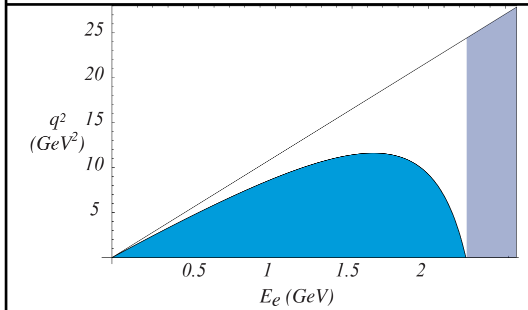
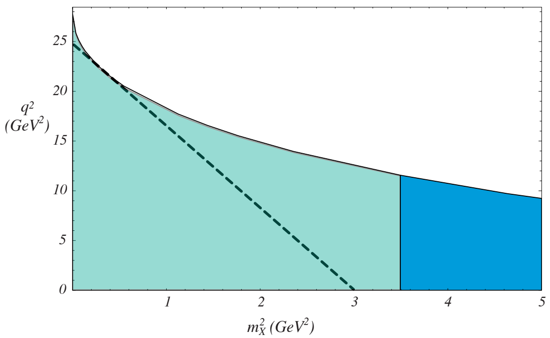
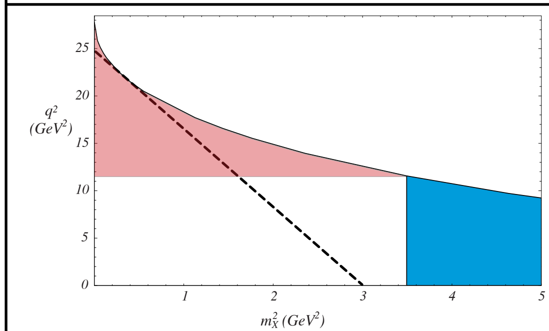
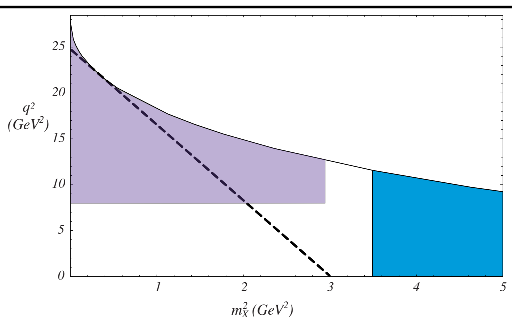




But this doesn't always happen (depends on proximity of cut to perturbative singularities)
... i.e. leptonic q^2 spectrum:

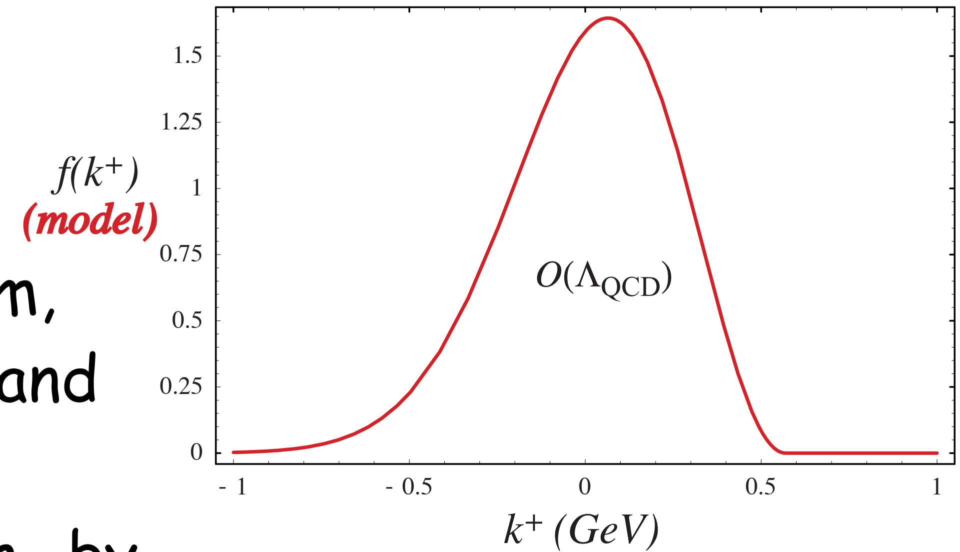
(Bauer, Ligeti, ML)



cut		% of rate	good	bad
	$E_\ell > \frac{m_B^2 - m_D^2}{2m_B}$	~10%	don't need neutrino	- depends on $f(k^+)$ (and subleading corrections) - WA corrections may be substantial - reduced phase space - duality issues?
	$s_H < m_D^2$	~80%	lots of rate	- depends on $f(k^+)$ (and subleading corrections)
	$q^2 > (m_B - m_D)^2$	~20%	insensitive to $f(k^+)$	- very sensitive to m_b - WA corrections may be substantial - effective expansion parameter is $1/m_c$
	"Optimized cut"	~45%	- insensitive to $f(k^+)$ - lots of rate - can move cuts away from kinematic limits and still get small uncertainties	- less rate than pure m_X cut - gets worse as cuts are loosened

Theoretical Issues:

(i) Fermi motion:



Shapes of charged lepton spectrum, hadronic invariant mass spectrum and photon energy spectrum are ALL determined at leading order in $1/m_b$ by a UNIVERSAL parton distribution function

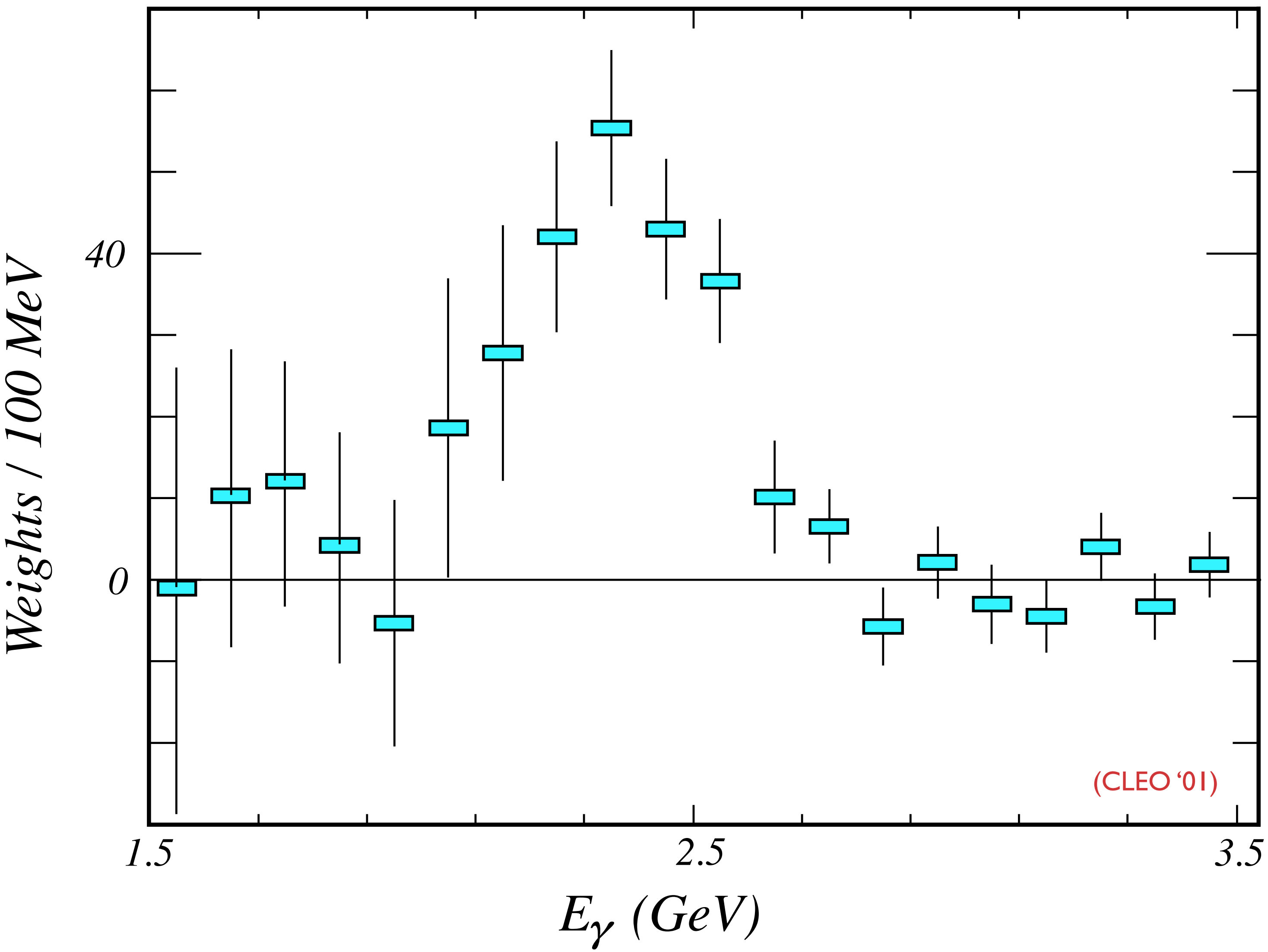
$$\boxed{f(\omega) = \frac{1}{2m_B}\langle B|\bar{b}\;\delta(\omega + i\hat{D}\cdot n)b|B\rangle}$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{d\hat{E}_\gamma}(B \to X_s\gamma) = \int d\omega\;\delta(1 - 2\hat{E}_\gamma - \omega)f(\omega) + \ldots$$

$$\frac{1}{2\Gamma_0}\frac{d\Gamma}{d\hat{E}_\ell}(B \to X_u\ell\bar{\nu}_\ell) = \int d\omega\;\theta(1 - 2\hat{E}_\ell - \omega)f(\omega) + \ldots$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{d\hat{s}_H}(B \to X_u\ell\bar{\nu}_\ell) = \int d\omega\;\frac{2\hat{s}_H^2(3\omega - 2\hat{s}_H)}{\omega^4}\theta(\omega - \hat{s}_H)f(\omega - \hat{\Lambda}) + \ldots$$

$f(\omega)$ is universal, and so can be measured in the photon spectrum in $\bar{B} \to X_s\gamma$, and then used to predict the charged lepton and hadronic invariant mass spectrum in $\bar{B} \to X_u \ell \bar{\nu}$:

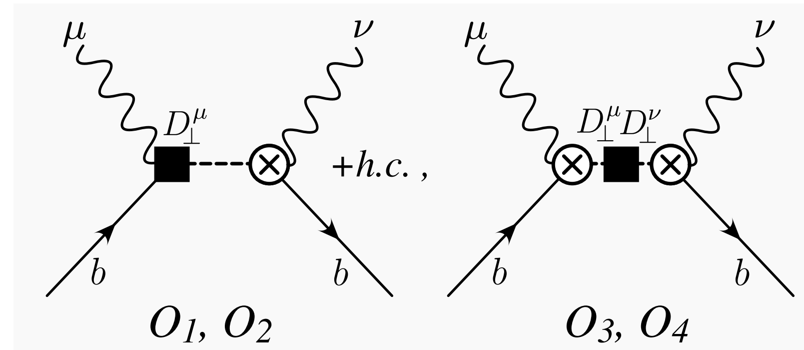


(NB must subtract off contributions of operators other than O_7) (Neubert)

This universality only holds at leading order in Λ_{QCD}/m_b ...

$$f(\omega) \sim \langle B|\underbrace{\bar{b}\,\delta(\omega - i\hat{D}\cdot n)b}|B\rangle$$

universal distribution function
(applicable to all decays)



... at $O(\Lambda_{QCD}/m_b)$ there is more structure:

sensitive to $k_\perp$

$$g_2(\omega_1,\omega_2) \sim \langle B|\bar{b}\,\delta(\omega_2 + in\cdot\hat{D})(iD_\perp)^2\,\delta(\omega_1 + in\cdot\hat{D})b|B\rangle$$

$$h_1(\omega) \sim \langle B|\bar{b}\,[iD_\mu,\,\delta(\omega + in\cdot\hat{D})]\,\gamma_\lambda\gamma_5 b|B\rangle\,\epsilon_\perp^{\mu\lambda}$$

breaks spin symmetry
(distinguishes semileptonic from
radiative decays)

$$h_2^\lambda(\omega_1,\omega_2) \sim \langle B|\bar{b}\,\delta(\omega_2 + in\cdot\hat{D})G_{\mu\nu}\delta(\omega_1 + in\cdot\hat{D})\gamma^\lambda\gamma_5 b|B\rangle\,\epsilon_\perp^{\mu\nu}$$

sensitive to soft gluons

$$T(\omega) \sim \int e^{-i\omega t}\langle B|T(\bar{b}(0)b(t),\, O_{1/m}(y))|B\rangle$$

(Bauer, ML, Mannell)

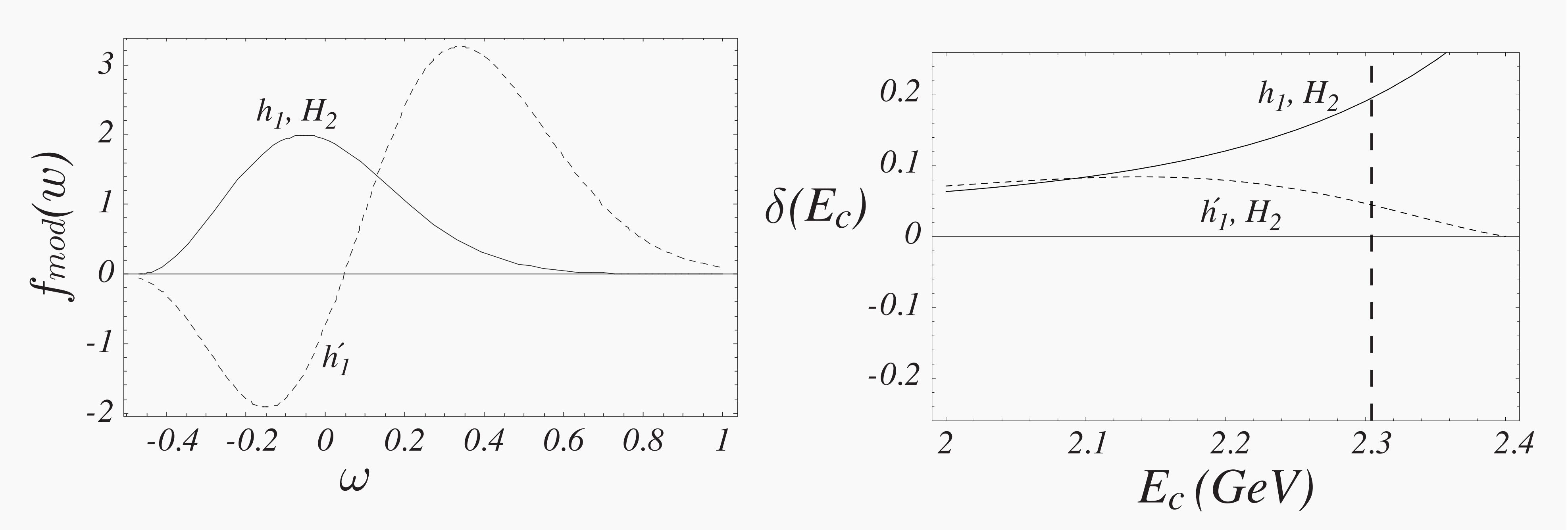
nonlocal T-product - only need to worry about
if comparing with charm decay

(NB this is just DIS at subleading twist all over again)

The effect of subleading "shape functions" can be surprisingly large in the lepton energy endpoint region ....

(Leibovich, Ligeti, Wise; Bauer, ML, Mannell)

but the uncertainty gets smaller as the lepton cut is lowered:

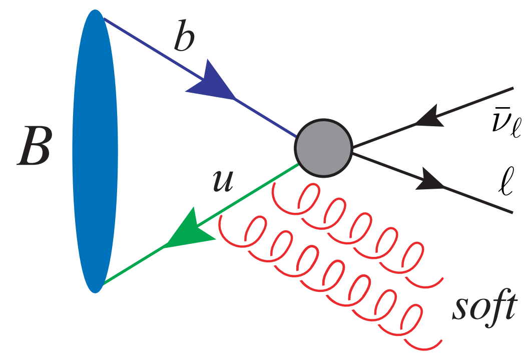


2 different models for subleading shape functions...

... and the corresponding effect on the determination of $|V_{ub}|$

- so want to make the lepton energy cut (and corresponding photon cut) as low as possible ... fully reconstructed events?

Weak annihilation is bad news, particularly for the charged lepton spectrum:



(Bigi & Uraltsev, Voloshin, Ligeti, Leibovich and Wise)

$$O\left(16\pi^2 \times \frac{\Lambda^3_{QCD}}{m_b^3} \times \text{ factorization violation}\right) \sim 0.03 \left(\frac{f_B}{0.2\,\mathrm{GeV}}\right)\left(\frac{B_2 - B_1}{0.1}\right)$$

- naively a ~3% contribution to rate at $q^2 = m_b^2$, but there is a huge uncertainty on this estimate

*** particularly damaging to the lepton endpoint determination - ~10% of rate, so ~30% correction to rate at endpoint - for precise determination of Vub, forced to rely on one of the other methods (and therefore need to reconstruct the neutrino) ***

- no reliable estimate of size - can test by comparing charged and neutral B's - lattice calculations?

Other sources of uncertainty:

- m_b: rate is proportional to m_b^5 - 100 MeV error is a ~5% error in V_{ub}. But restricting phase space increases this sensitivity - with q^2 cut, scale as ~m_b^{10} (Neubert)
- perturbative corrections - known (in most cases) to $O(\alpha_s^2 \beta_0)$ - appear under control. When Fermi motion is important, leading and subleading Sudakov logarithms have been resummed.
(Leibovich, Low, Rothstein)

Experimental measurements that can reduce the theoretical uncertainty:

(a) push experimental cuts as close to charm region as possible - increases rate, decreases theoretical uncertainty. Measure V_{ub} as a function of the cuts to check for consistency.

(b) improve measurement of $B \to X_s \gamma$ photon spectrum - get $f(k^+)$ - lowering cut reduces effects of subleading corrections, as well as sensitivity to details of $f(k^+)$

(c) test size of WA (weak annihilation) effects - compare D^0 & D_S S.L. widths, extract $|V_{ub}|$ from $B^\pm$ and B^0 separately

(d) better determination of m_b (moments of B decay distributions)

Summary for V_{ub}:

- high precision determination will require reconstructing neutrino, measuring m_X, q^2 (or some combination of these) spectra
- likely limit of theoretical uncertainty is at the 5% level
- if the TOTAL inclusive rate could be measured (no cuts) many of the theoretical issues would go away/be much improved

V_{cb}

V_{cb} is theoretically (and experimentally) much simpler to extract from inclusive decays than V_{ub}:

- local OPE is valid (convergence is best in a physical mass scheme)
- current theoretical uncertainties are set by
  1. $O(1/m^3)$ terms (4 free parameters)
  2. precision of $O(1/m, 1/m^2)$ terms (2 free parameters)
  3. radiative corrections - need full two loop corrections for spectral moments

$$\Gamma(B \to X_c \ell \bar{\nu}) = \frac{G_F^2 \, |V_{cb}|^2}{192\pi^3} (0.534) \left(\frac{m_\Upsilon}{2}\right)^5 \times$$

$$\Bigg[1 - 0.22 \left(\frac{\Lambda_{1S}}{500\ \mathrm{MeV}}\right) - 0.011 \left(\frac{\Lambda_{1S}}{500\ \mathrm{MeV}}\right)^2 - 0.052 \left(\frac{\lambda_1}{(500\ \mathrm{MeV})^2}\right) - 0.071 \left(\frac{\lambda_2}{(500\ \mathrm{MeV})^2}\right)$$
$$- 0.006 \left(\frac{\lambda_1 \Lambda}{(500\ \mathrm{MeV})^3}\right) + 0.011 \left(\frac{\lambda_2 \Lambda}{(500\ \mathrm{MeV})^3}\right) - 0.006 \left(\frac{\rho_1}{(500\ \mathrm{MeV})^3}\right) + 0.008 \left(\frac{\rho_2}{(500\ \mathrm{MeV})^3}\right)$$
$$+ 0.011 \left(\frac{T_1}{(500\ \mathrm{MeV})^3}\right) + 0.002 \left(\frac{T_2}{(500\ \mathrm{MeV})^3}\right) - 0.017 \left(\frac{T_3}{(500\ \mathrm{MeV})^3}\right) - 0.008 \left(\frac{T_4}{(500\ \mathrm{MeV})^3}\right)$$
$$- 0.096\, \epsilon - 0.030\, \epsilon_{BLM}^2 + 0.015\, \epsilon \left(\frac{\Lambda_{1S}}{500\ \mathrm{MeV}}\right) + \dots \Bigg]$$

$O(\Lambda_{QCD}/m_b)$: ~20% correction

$O(\Lambda_{QCD}^3/m_b^3)$: ~1-2% correction

$O(\Lambda_{QCD}^2/m_b^2)$: ~5-10% correction

Perturbative: ~few %

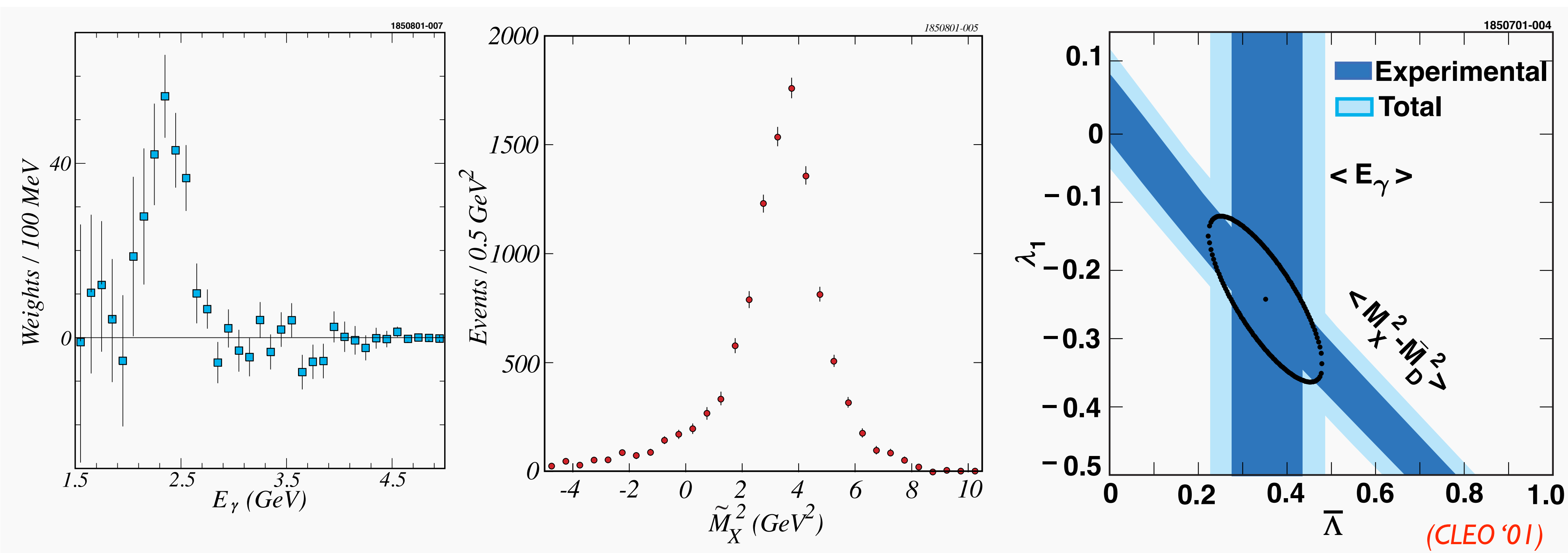
Hadronic matrix elements can be determined by measuring other observables (spectral moments):

- like rate, moments of spectra can be calculated as a power series in $\alpha_s(m_b)$, Λ_{QCD}/m_b :

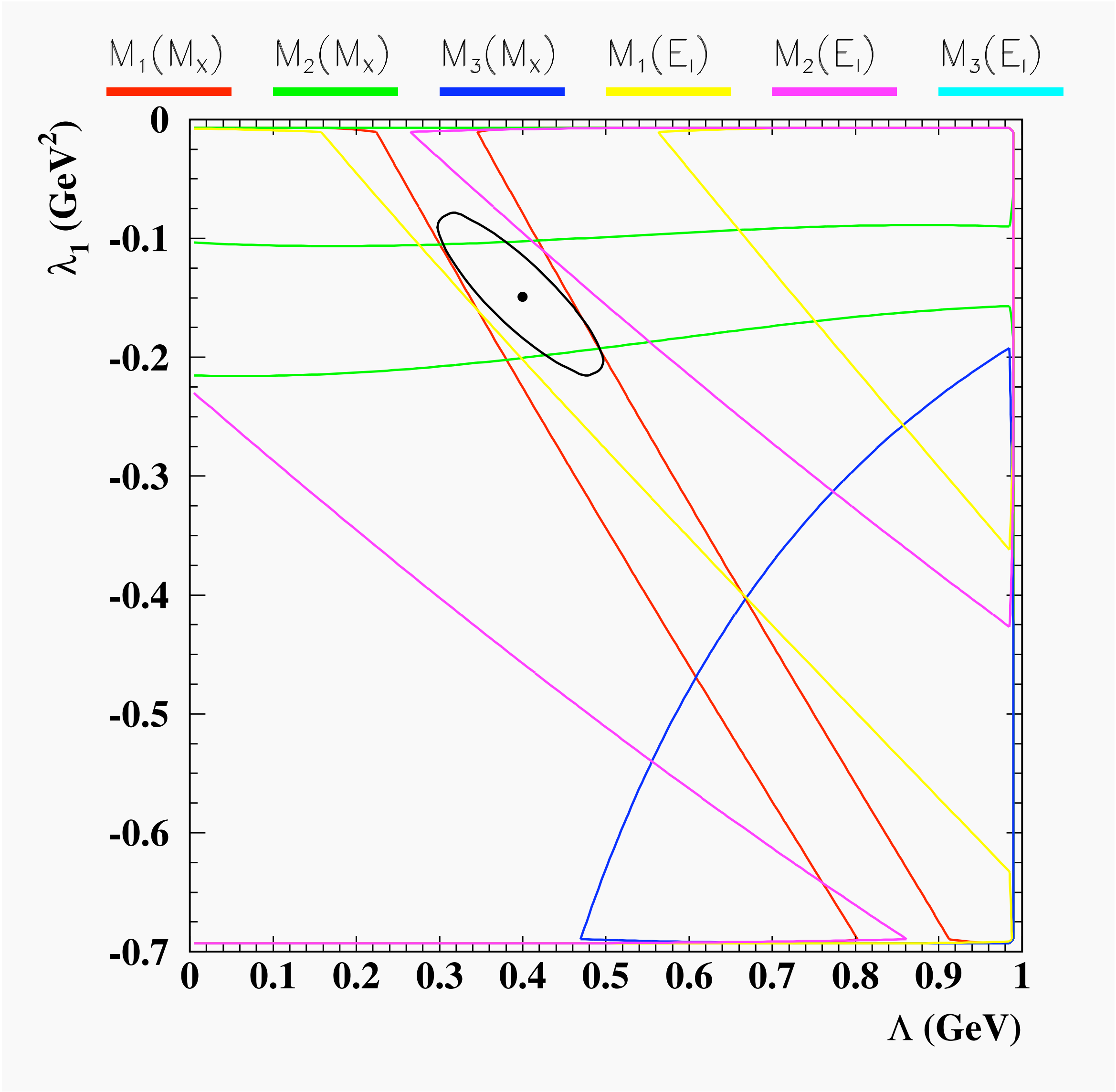
$$\langle E_\gamma \rangle = \frac{m_B - \bar{\Lambda}}{2} + \ldots$$

$$\frac{1}{m_B^2}\langle s_H - \bar{m}_D^2 \rangle_{E_\ell > 1.5\,\mathrm{GeV}} = 0.21\frac{\bar{\Lambda}}{\bar{m}_B} + 0.26\frac{\bar{\Lambda}^2 + 3.8\lambda_1 - 1.2\lambda_2}{\bar{m}_B^2} + \ldots$$

Constrain different linear combinations of Λ, λ_1



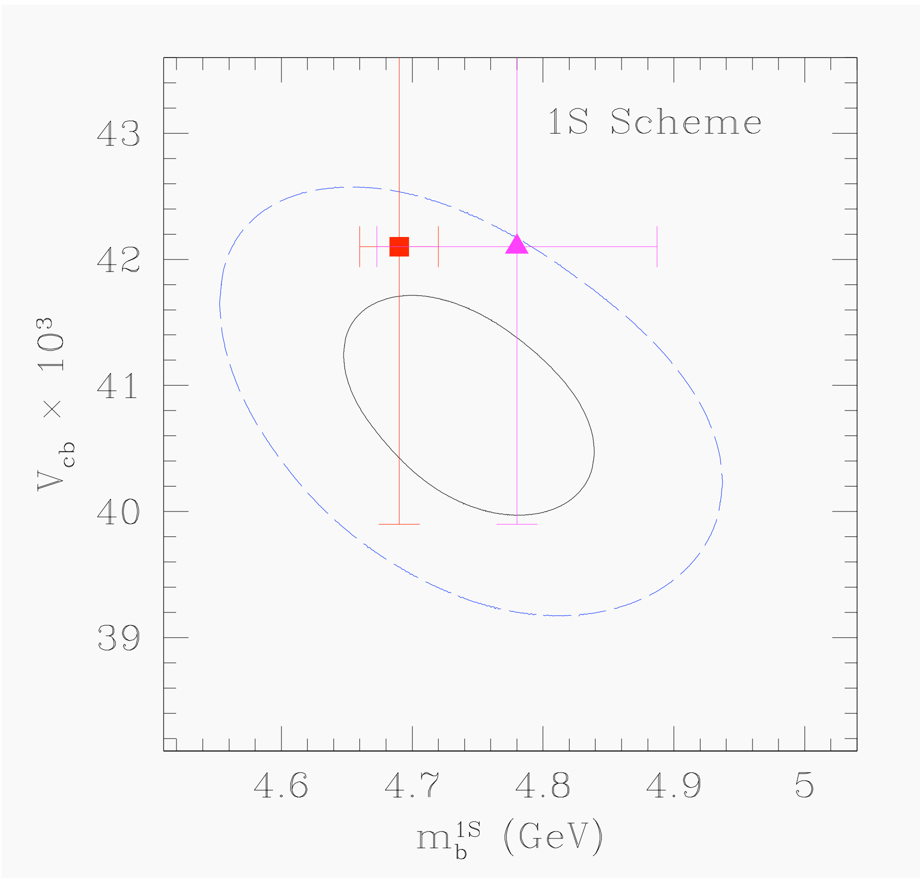
Many moments have now been measured, (i) allowing precision extractions of HQET matrix elements (and mb), and (ii) testing validity of the whole approach:



(Battaglia *et. al.*, PLB556:41, 2003, using DELPHI data)

Global fits (summer '02):

(fit including $1/m^3$ effects)



■ Hoang
▲ Beneke

exclusive V_{cb} extraction, b mass from $b\bar{b}$ sum rules

$$R_0(E_0, E_1) = \frac{\int_{E_1} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{E_0} \frac{d\Gamma}{dE_\ell}\, dE_\ell}, \qquad R_n(E_0) = \frac{\int_{E_0} E_\ell^n \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{E_0} \frac{d\Gamma}{dE_\ell}\, dE_\ell}, \; n = 1,\, 2$$

$$S_1(E_0) = \langle m_X^2 - \bar{m}_D^2 \rangle \Big|_{E_\ell > E_0}, \qquad S_2(E_0) = \Big\langle (m_X^2 - \langle m_X^2 \rangle)^2 \Big\rangle \Big|_{E_\ell > E_0}$$

$$T_1(E_0) = \langle E_\gamma \rangle \Big|_{E_\gamma > E_0}, \qquad T_2(E_0) = \Big\langle (E_\gamma - \langle E_\gamma \rangle)^2 \Big\rangle \Big|_{E_\gamma > E_0}$$

- lepton energy and hadronic invariant mass moments ($\bar{B} \to X_c \ell \bar{\nu}$), photon energy spectrum moments ($\bar{B} \to X_s \gamma$)
- measured with varying cutoffs by DELPHI, CLEO and BaBar
- simultaneously fit for hadronic matrix elements, m_b, V_{cb}

$$m_b^{1S} = 4.74 \pm 0.10\, \mathrm{GeV}$$

$$|V_{cb}| = (40.8 \pm 0.9) \times 10^{-3}$$

(Bauer, Ligeti, ML and Manohar, PRD67:054012, 2003 - BaBar s_H spectra not included in fit)

$$m_b(1\,\mathrm{GeV}) = 4.59 \pm 0.08\,\mathrm{GeV} \Rightarrow m_b^{1S} = 4.69\,\mathrm{GeV}$$

$$m_c(1\,\mathrm{GeV}) = 1.13 \pm 0.13\,\mathrm{GeV}$$

$$|V_{cb}| = (41.9 \pm 1.1) \times 10^{-3}$$

(Battaglia *et. al.*, PLB556:41, 2003, using DELPHI data)

The fit also allows us to make precise predictions of other moments as a cross-check (test of duality):

$$\boxed{\begin{aligned} D_3 &\equiv \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{0.7} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{1.5} \frac{d\Gamma}{dE_\ell}\, dE_\ell} = \begin{cases} 0.5190 \pm 0.0007 & \text{(theory)} \\ 0.5193 \pm 0.0008 & \text{(experiment)} \end{cases} \\ D_4 &\equiv \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{2.3} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{2.9} \frac{d\Gamma}{dE_\ell}\, dE_\ell} = \begin{cases} 0.6034 \pm 0.0008 & \text{(theory)} \\ 0.6036 \pm 0.0006 & \text{(experiment)} \end{cases} \end{aligned}}$$

(some fractional moments of lepton spectrum are very insensitive to $O(1/m^3)$ effects, and so can be predicted very accurately)

(C. Bauer and M. Trott)

... and just for fun, setting all experimental errors to zero we find

$$\delta(|V_{cb}|) \times 10^3 = \pm 0.35, \quad \delta(m_b) = \pm 35\,\mathrm{MeV}$$

<1%!

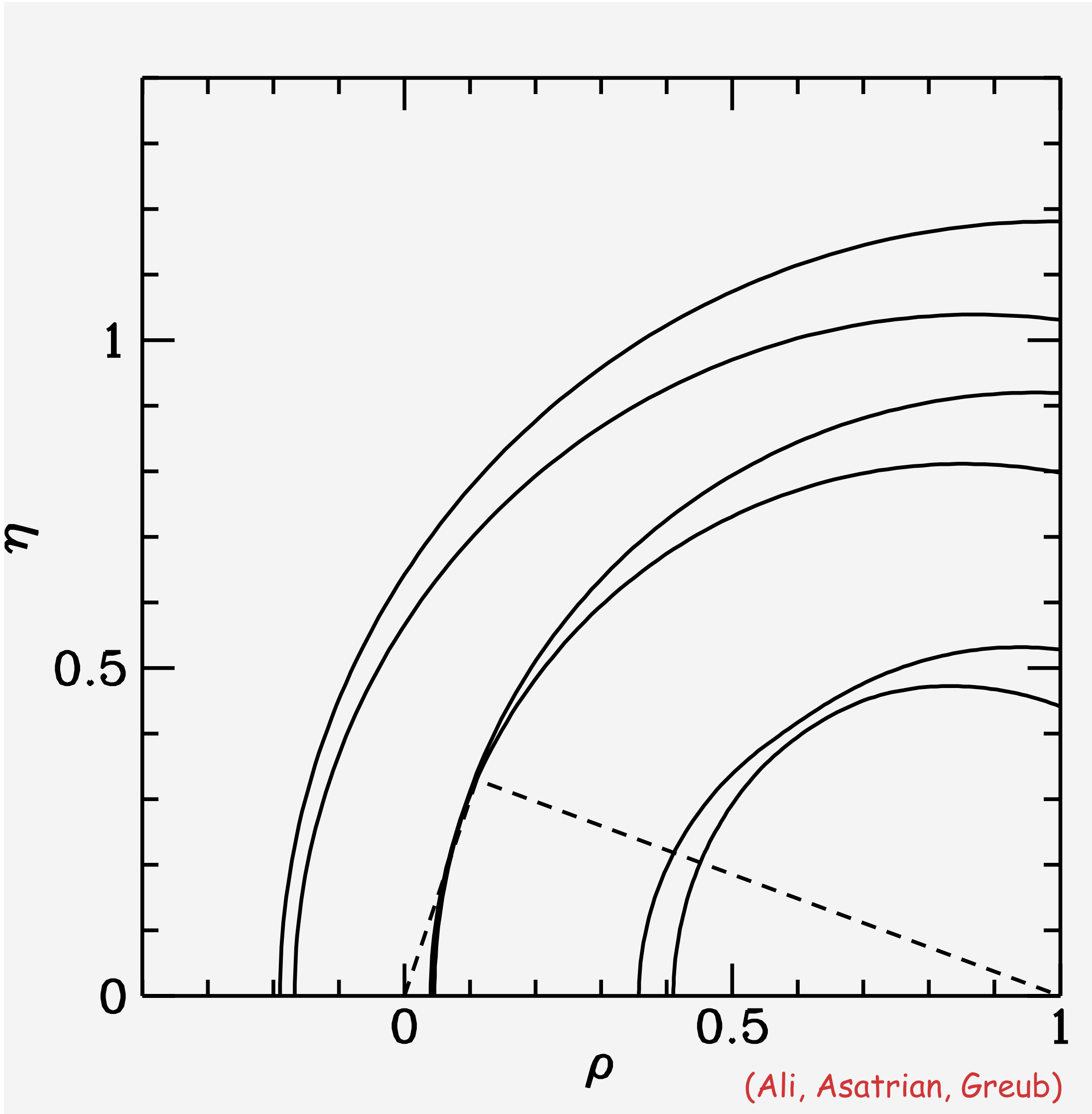
Summary for V_{cb}:

- current precision is already at few % level - are there sources of uncertainty we have neglected which become important at the % level?
- limiting factors from theory are precision of matrix elements - uncertainties are currently at the $1/m^3$ and α_s^2 level
- "duality" is very hard to quantify - cross-checks are important!

V_{td}

b->dγ (see Ali, Jessop talks)

(I won't discuss the weak Hamiltonian here)



$$R(d\gamma/s\gamma) \equiv \frac{\Gamma(B \to X_d\gamma)}{\Gamma(B \to X_s\gamma)}$$

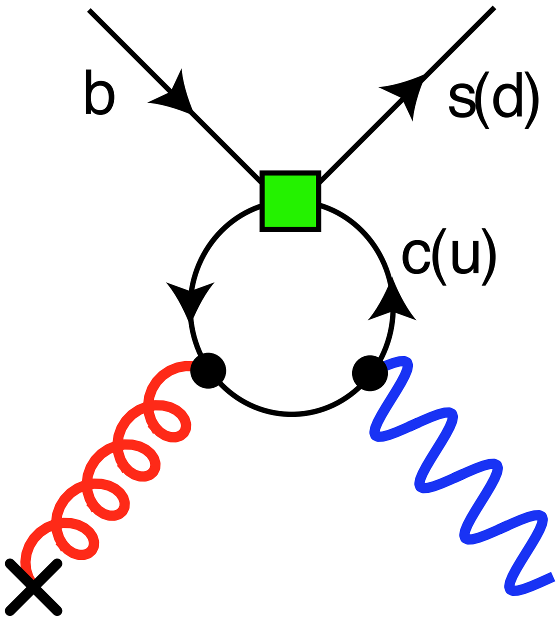
is sensitive to $\left|\frac{V_{td}}{V_{ts}}\right|$ (+ small corrections) in the SM

- many uncertainties drop out of the ratio R
- expected branching fraction in SM is

$$B(B \to X_d\gamma) \simeq 1.3 \times 10^{-5}$$

- difficulty is in picking it out from the B->X_sγ background!

Unlike semileptonic decays, radiative decays are NOT entirely determined by short-distance ($\mu > m_b$) physics



- light quark loop is long-distance - can't perform an OPE
- for c quark (b->sγ), , can expand in powers of $\Lambda_{QCD} m_b / m_c^2$ ... ~3% correction to rate

(Voloshin, ...)

- u quark loops are not well understood, but they have been argued to be small:
  - VMD and LCSR suggest ~10-15% effect in B->ργ
  - Hurth argues (by studying Feynman diagrams) that they are parametrically suppressed by Λ_{QCD}/m_b (dominant NP effect! not described by a local operator)

Other issues:

- background from $b \to s\gamma$ is about a factor of 20 - can this be handled? Does $s\bar{s}$ production in $b \to d\gamma$ from vacuum mess up kaon veto? How big an effect is this?
- background from $b \to u\bar{u}d$ fragmentation is large at low photon energies - how large a cut is required in E_γ?

Summary for V_{td}:

- b->dγ measures different physics than mixing - important measurement
- theoretically and experimentally challenging to get a precision measurement
  - b->sγ is a huge background ("Yesterday's news is today's calibration, and tomorrow's background.")
  - long-distance physics poorly understood, limits theoretical precision

Conclusions:

- Inclusive decays are in principle very clean theoretically, but can get complicated by experimental cuts and long-distance contributions
  - Progress in V_{ub} requires high precision spectra, neutrino reconstruction OR ability to measure over entire kinematic range
  - V_{cb} is in good shape - spectral moments can give m_b and reduce theoretical errors
- the theoretical walls for V_{ub} and V_{cb} from inclusive decays are probably at the ~5% and ~1% level
- V_{td} via $b \to d\gamma$ is challenging but important