Inclusive determinations of V_{ub} and V_{cb} - a theoretical perspective

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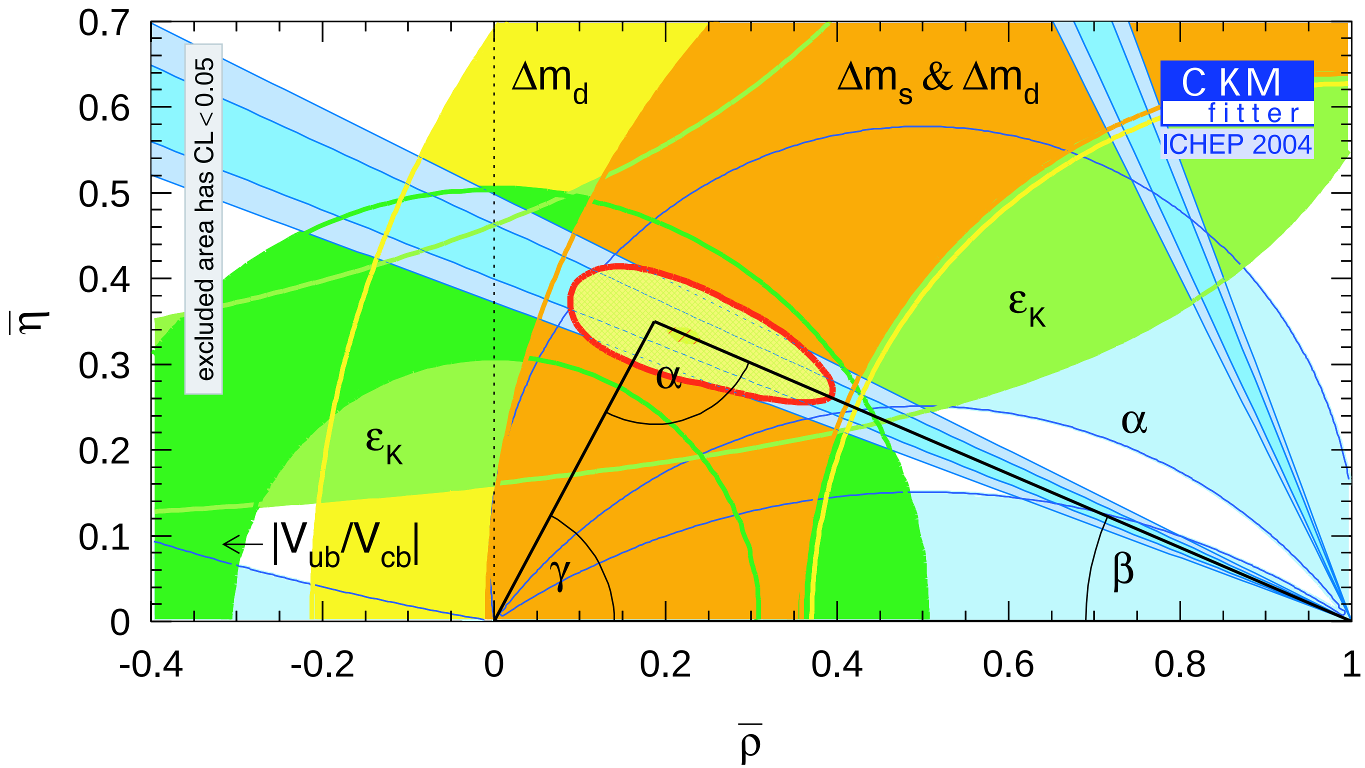
University of Toronto

Global fit, summer '04:

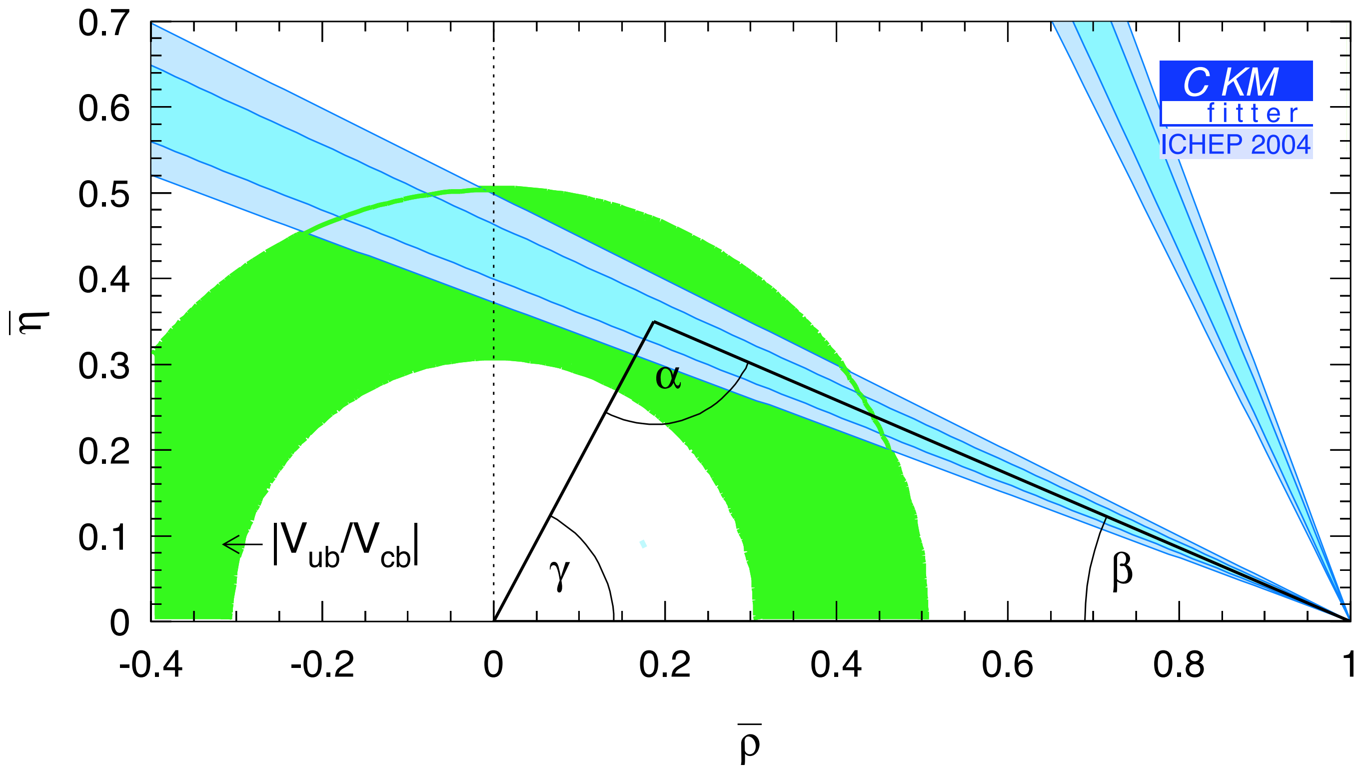


CKMfitter inputs:

$$\sin 2\beta = 0.726 \pm 0.037$$

$$V_{ub} = (3.90 \pm 0.08 \pm 0.68) \times 10^{-3}$$ (~20% uncertainty)

Global fit, summer '04:



- need a better determination of V_{ub} to check for consistency with sin 2β

CKMfitter inputs:

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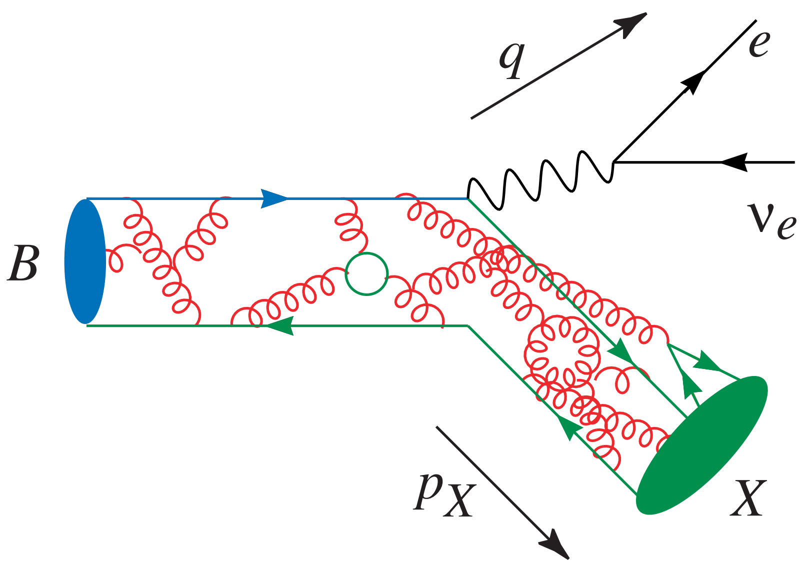
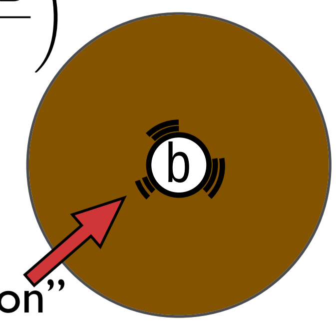
(~20% uncertainty)

Theorists love inclusive decays ...



Decay: short distance (calculable)
Hadronization: long distance (nonperturbative) - but at leading order, long and short distances are cleanly separated and probability to hadronize is unity

$$\frac{d\Gamma}{d(P.S.)} \sim \text{parton model} + \sum_n C_n \left(\frac{\Lambda_{QCD}}{m_b}\right)^n$$



"Most" of the time, details of b quark wavefunction are unimportant - only averaged properties (i.e. $\langle k^2 \rangle$) matter

"Fermi motion"

$k^\mu \sim \Lambda_{QCD}$

$$\Gamma(\bar{B} \to X_u \ell \bar{\nu}_\ell) = \frac{G_F^2 |V_{ub}|^2 m_b^5}{192\pi^3} \left(1 - 2.41\frac{\alpha_s}{\pi} - 21.3\left(\frac{\alpha_s}{\pi}\right)^2 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2} + O\left(\alpha_s^2, \frac{\Lambda_{QCD}^3}{m_b^3}\right)\right)$$

... the basic theoretical tools are more than a decade old

What progress has been made (a) in the past decade?

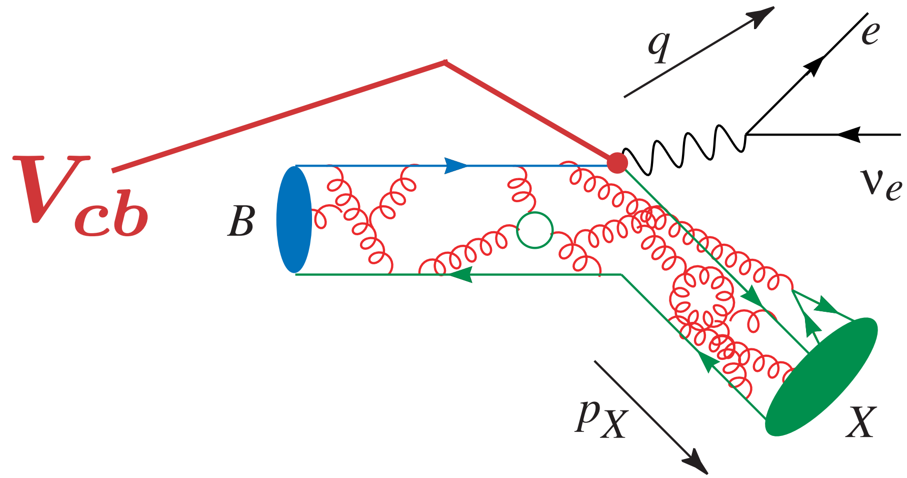
- V_{cb}: PRECISION
  - moment fits to determine nonperturbative matrix elements
  - extensive tests of consistency (limits possible duality violations)
  - data have improved to the level that theory is required to $(\Lambda_{QCD}/m_b)^3$
- V_{ub}: MODEL INDEPENDENCE
  - moved beyond lepton endpoint to theoretically cleaner cuts (hadronic invariant mass, lepton invariant mass, combined cuts, P_+, ...)
  - SCET et. al.: unravels scales relevant for cut spectra, generalizes shape function analysis beyond leading order, sums Sudakov logs ... theoretical errors now much better understood

What progress has been made (b) since CKM '03?

- V_{cb}:
  - Moment fits are better, inconsistencies have gone away with new data, error in V_{cb} down slightly.
- V_{ub}:
  - Further development of SCET/subleading theory
    - ➡ Perturbative and nonperturbative corrections & uncertainties are better understood.
  - New (possibly large) subleading effects discovered
  - P_+ cut on spectrum added to list - some useful features

V_{cb}

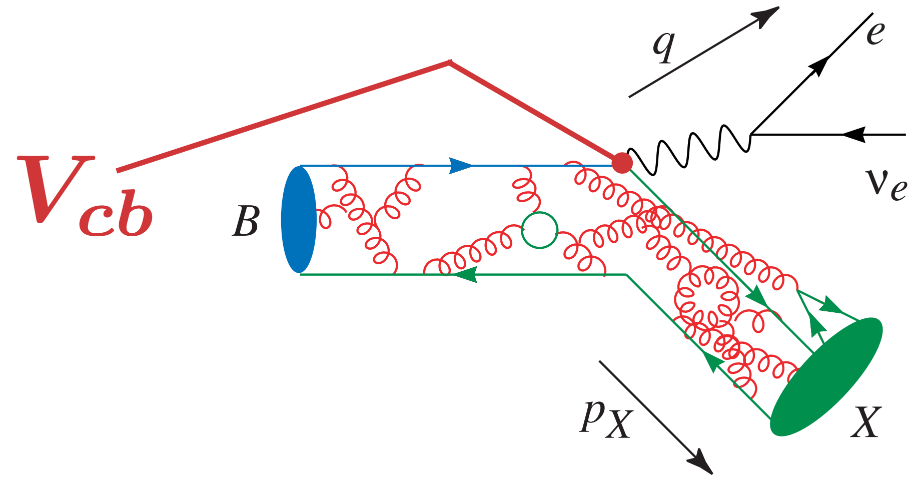
Inclusive semileptonic b→c decay:



$$\Gamma(B \to X_c \ell \bar{\nu}) = \frac{G_F^2 \, |V_{cb}|^2}{192\pi^3}(0.534)\left(\frac{m_\Upsilon}{2}\right)^5 \times$$

$$\Big[1$$

Inclusive semileptonic b→c decay:



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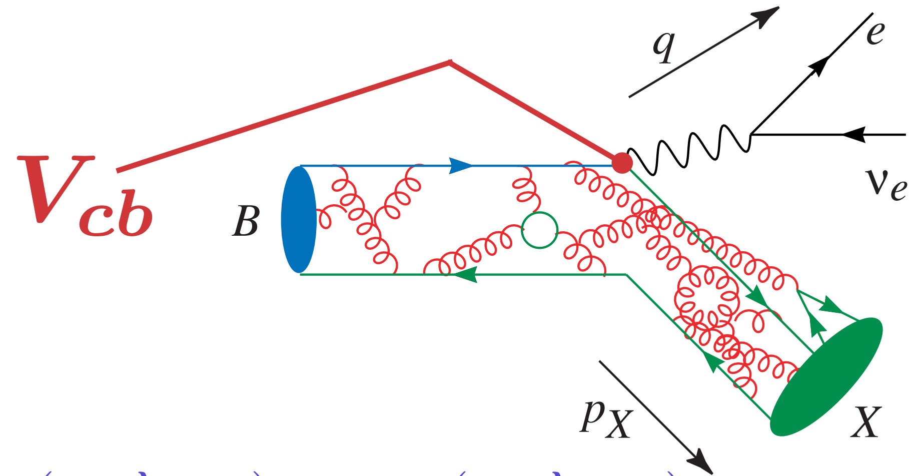
$$\left[1 - 0.22\left(\frac{\Lambda_{1S}}{500\,\mathrm{MeV}}\right)\right.$$

$O(\Lambda_{QCD}/m_b)$: ~20% correction

Inclusive semileptonic b→c decay:



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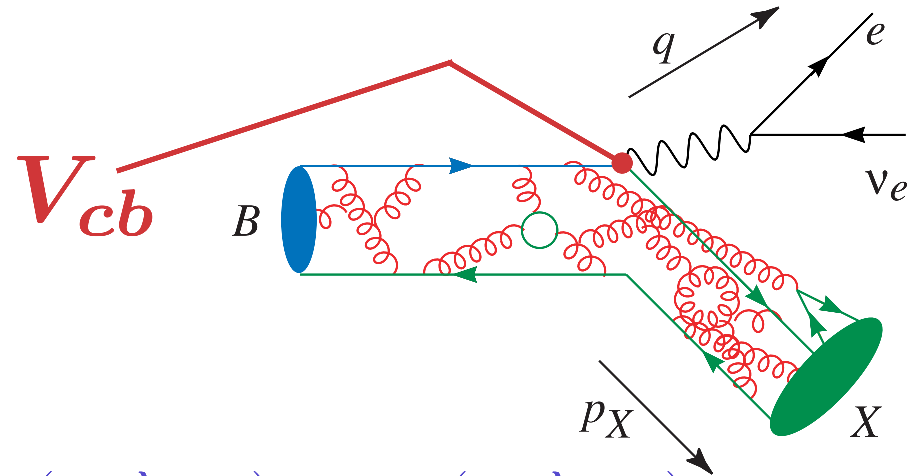
$$\left[1 - 0.22\left(\frac{\Lambda_{1S}}{500\ \mathrm{MeV}}\right) - 0.011\left(\frac{\Lambda_{1S}}{500\ \mathrm{MeV}}\right)^2 - 0.052\left(\frac{\lambda_1}{(500\ \mathrm{MeV})^2}\right) - 0.071\left(\frac{\lambda_2}{(500\ \mathrm{MeV})^2}\right)\right.$$

$O(\Lambda_{QCD}/m_b)$: ~20% correction

$O(\Lambda_{QCD}^2/m_b^2)$: ~5-10% correction

Inclusive semileptonic b→c decay:



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$$-0.006\left(\frac{\lambda_1\Lambda}{(500\ \mathrm{MeV})^3}\right) + 0.011\left(\frac{\lambda_2\Lambda}{(500\ \mathrm{MeV})^3}\right) - 0.006\left(\frac{\rho_1}{(500\ \mathrm{MeV})^3}\right) + 0.008\left(\frac{\rho_2}{(500\ \mathrm{MeV})^3}\right)$$

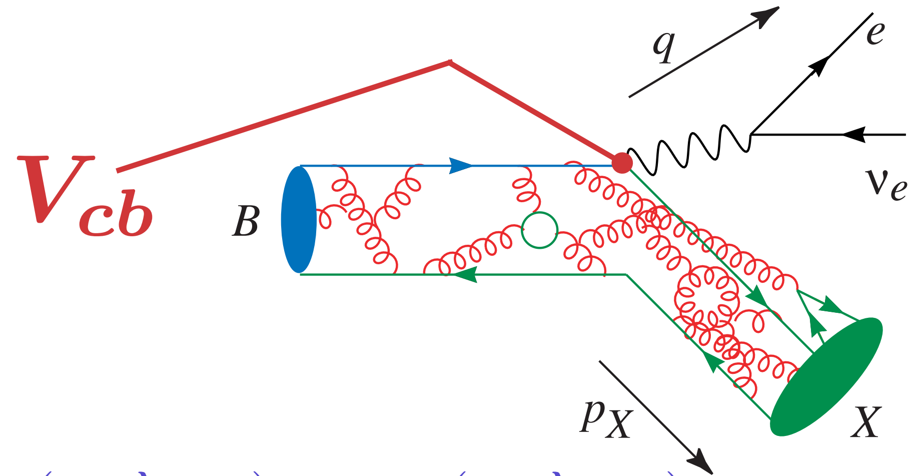
$$+0.011\left(\frac{T_1}{(500\ \mathrm{MeV})^3}\right) + 0.002\left(\frac{T_2}{(500\ \mathrm{MeV})^3}\right) - 0.017\left(\frac{T_3}{(500\ \mathrm{MeV})^3}\right) - 0.008\left(\frac{T_4}{(500\ \mathrm{MeV})^3}\right)$$

$O(\Lambda_{QCD}/m_b)$: ~20% correction

$O(\Lambda^3_{QCD}/m^3_b)$: ~1-2% correction

$O(\Lambda^2_{QCD}/m^2_b)$: ~5-10% correction

Inclusive semileptonic b→c decay:



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$$-0.096\,\epsilon - 0.030\,\epsilon^2_{BLM} + 0.015\,\epsilon\left(\frac{\Lambda_{1S}}{500\ \mathrm{MeV}}\right) + \ldots\Bigg]$$

$O(\Lambda_{QCD}/m_b)$: ~20% correction

$O(\Lambda^3_{QCD}/m^3_b)$: ~1-2% correction

$O(\Lambda^2_{QCD}/m^2_b)$: ~5-10% correction

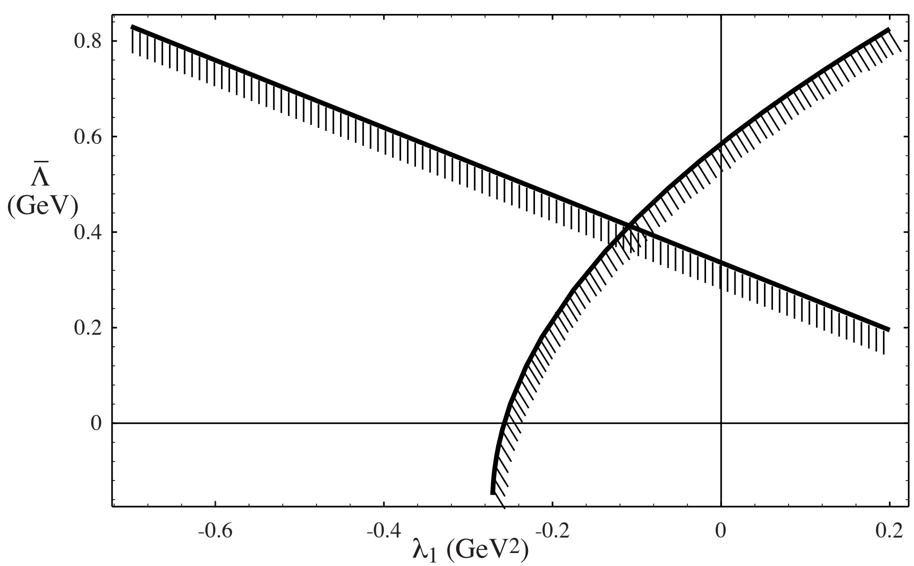
Perturbative: ~ 10%

→ This is now a PRECISION field!

Moments of B Decay Spectra:

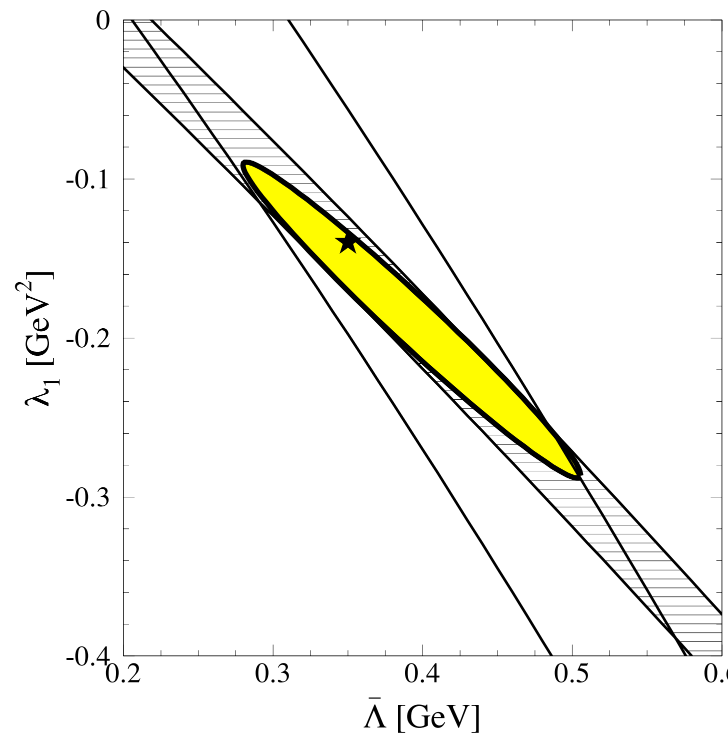
- like rate, moments of spectra can be calculated as a power series in $\alpha_s(m_b),\ \Lambda_{QCD}/m_b$, and used to determine nonperturbative parameters ... this is an old game by now.

fits c. 1995:



(Falk, ML, Savage))

hadronic invariant mass moments



(Gremm, Kapustin, Ligeti, Wise)

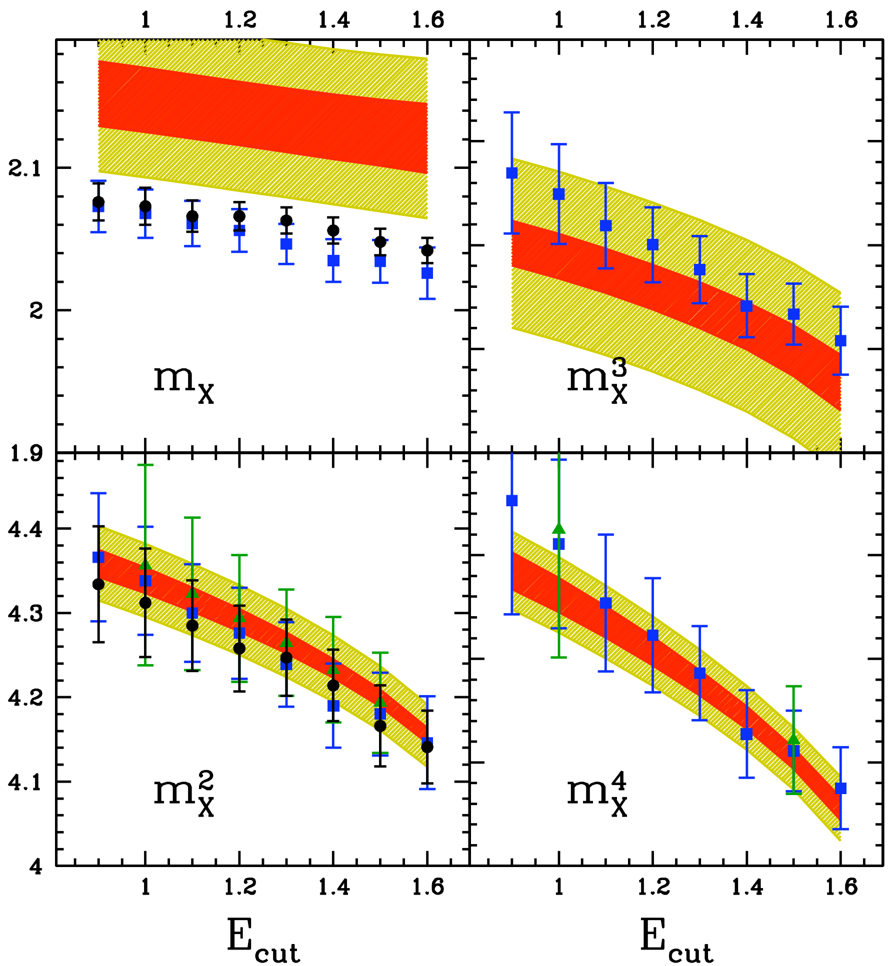
lepton energy moments

fits c. 2004:

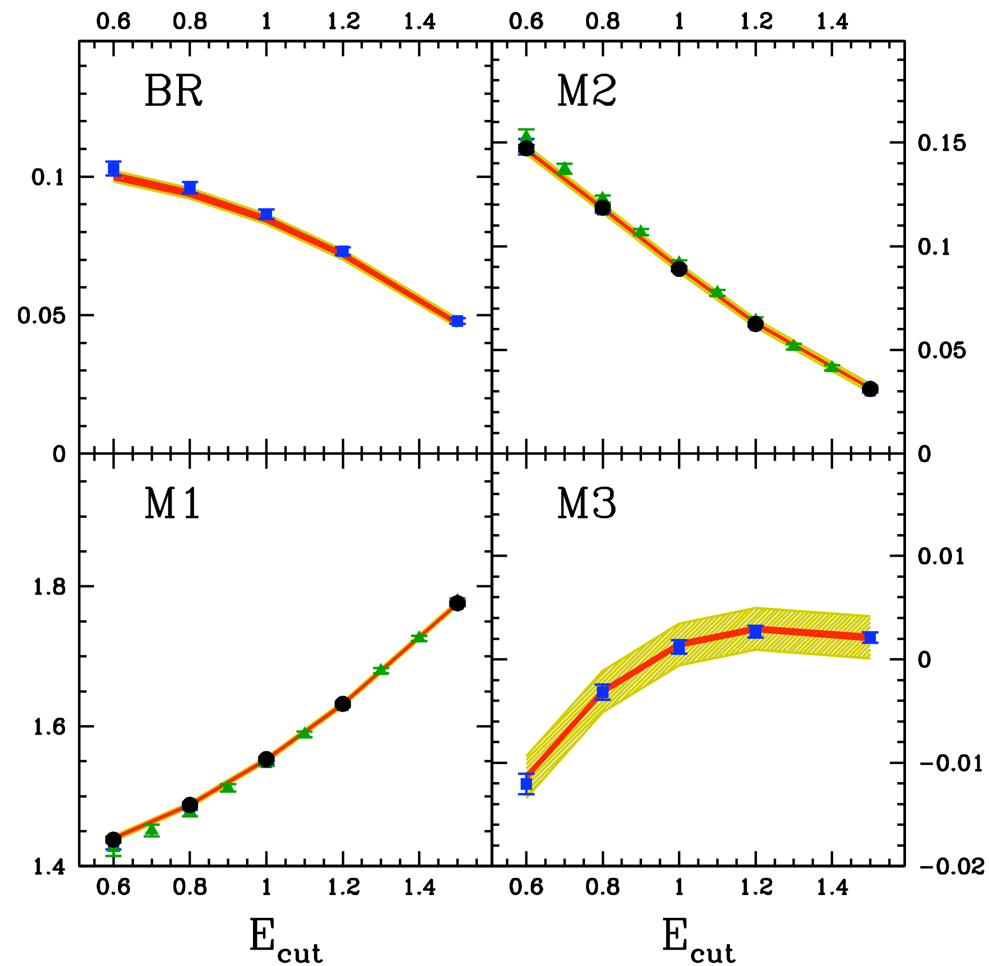
(1) Bauer, Ligeti, ML, Manohar and Trott

(up to $1/m^3$)

- fit 92 data points (spectral moments with varying lepton energy cuts - many data points strongly correlated) with 7 free parameters



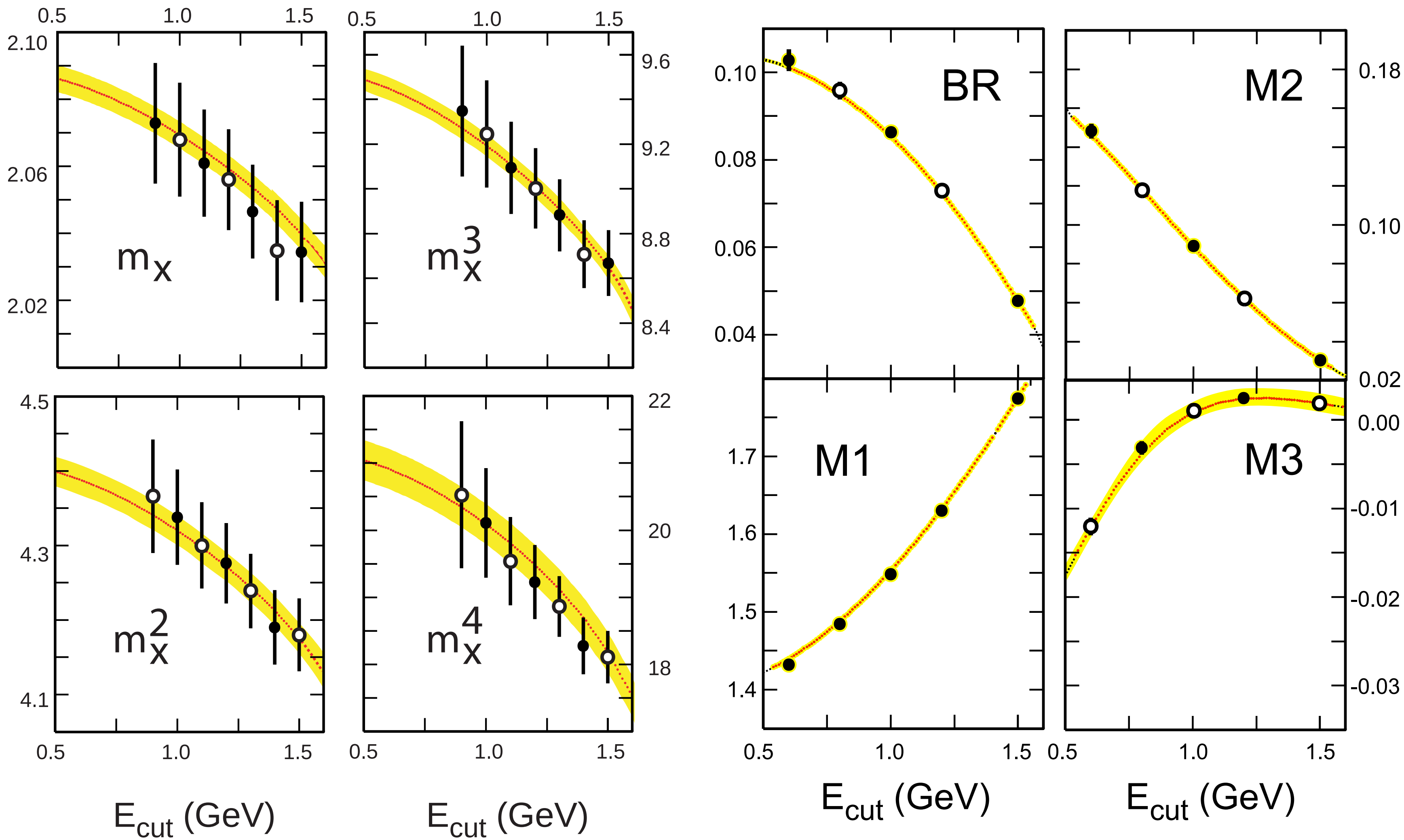
hadronic invariant mass moments



lepton energy moments

fits c. 2004:

(2) BABAR (using results of Gambino & Uraltsev)



hadronic invariant mass moments

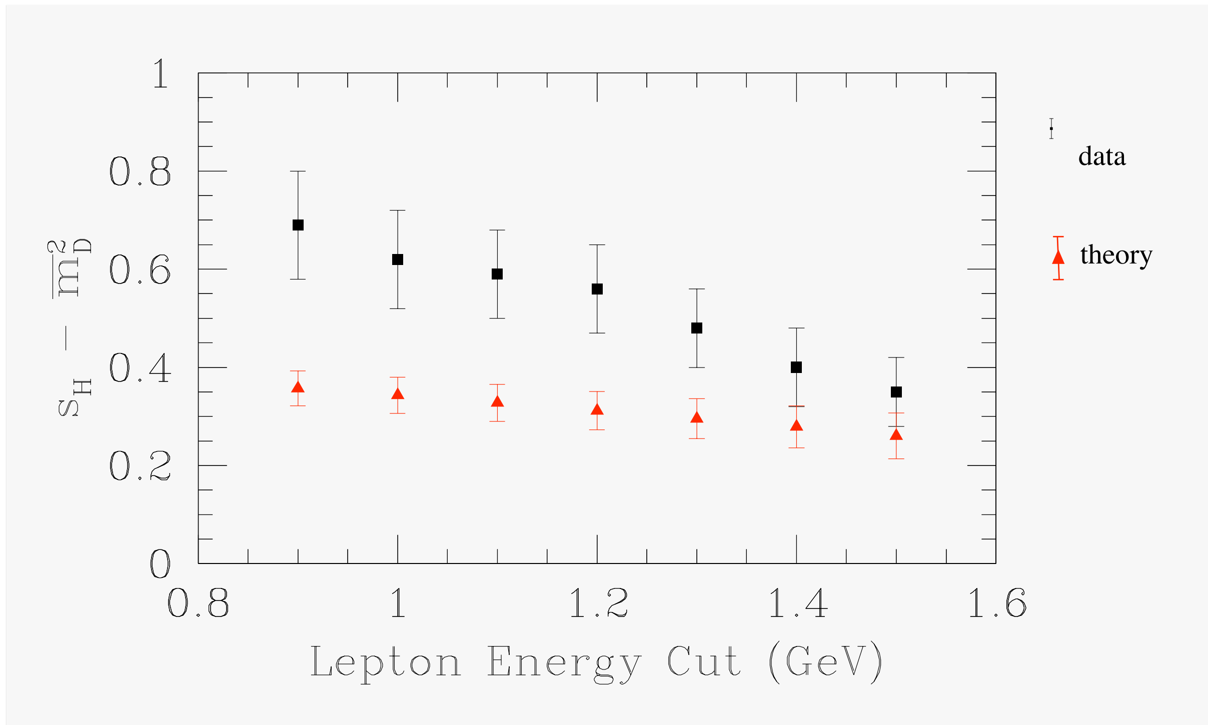
lepton energy moments

Both fits have 7 free parameters, work to $O(\Lambda_{\rm QCD}/m_b)^3$ .. differences are in details:

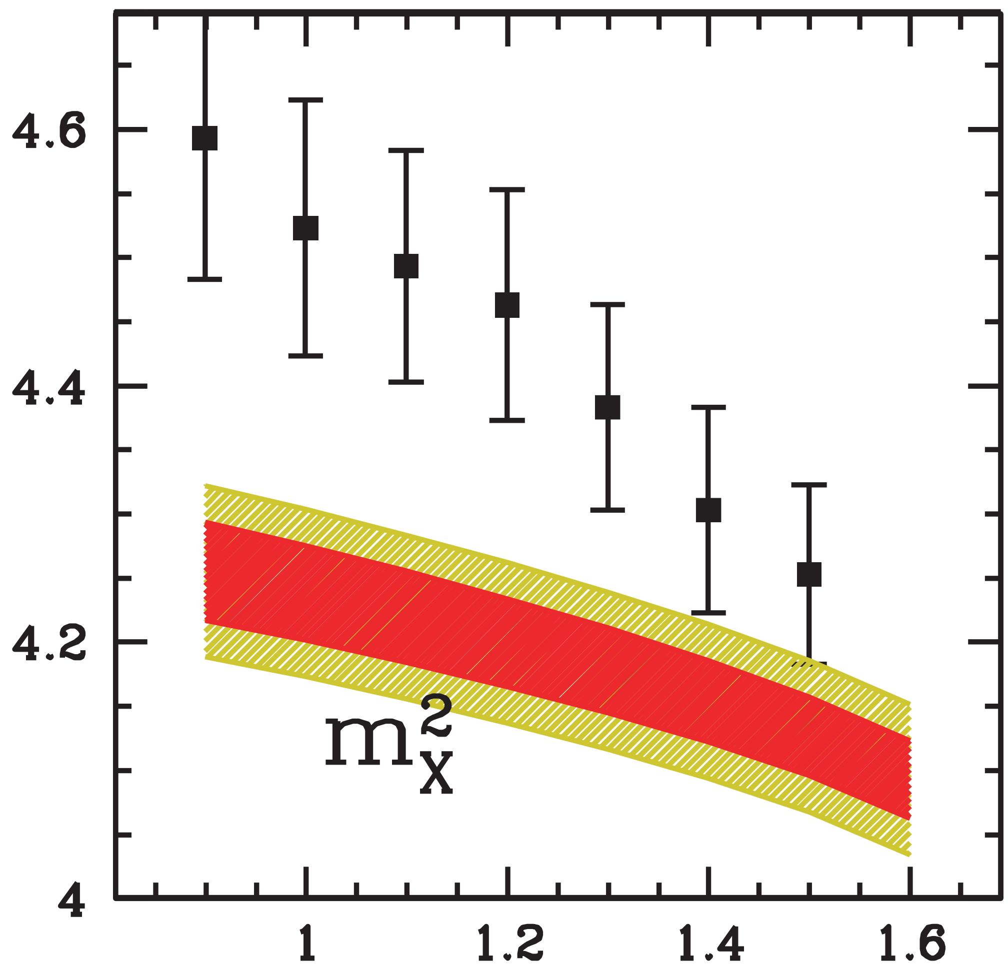
- expand in Λ_{QCD}/m_c or not in kinematics (to get m_c)
  - **+**: moves free parameter from O(1) to $O(\Lambda_{QCD}/m_b)^3$
  - **-** : introduces new expansion in Λ_{QCD}/m_c
  - Can do fit both ways; essentially no difference in fit results
- mass definitions - kinetic vs. 1S. Just scheme dependence; no significant difference in fit results
- slightly different handling of higher orders in Λ_{QCD}/m_b
- fractional hadronic invariant mass moments - results differ (BABAR fits data better; related to point above?)
  - fractional hadronic invariant mass moments intrinsically involve expansion in $\Lambda_{\rm QCD} m_b/m_c^2$ - not as clean theoretically

Hadronic invariant mass moments: From CKM '03:

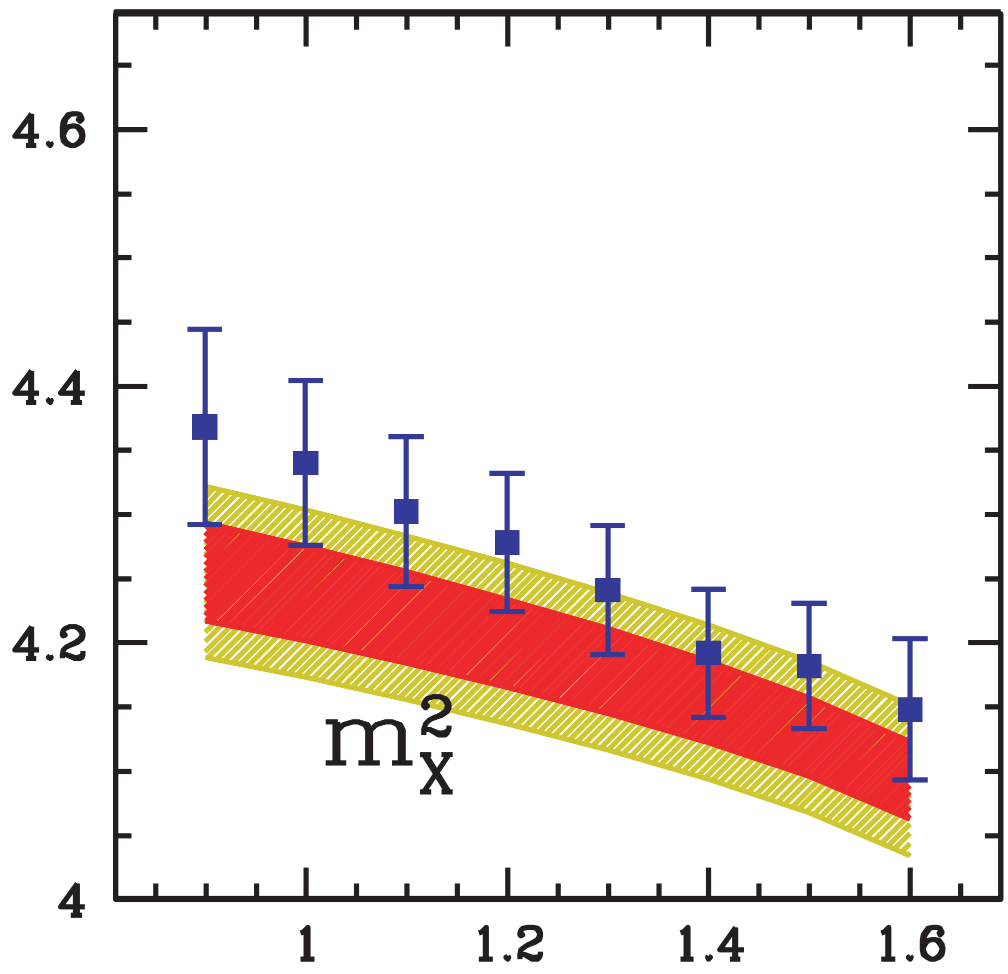
A problem? - BABAR '02:



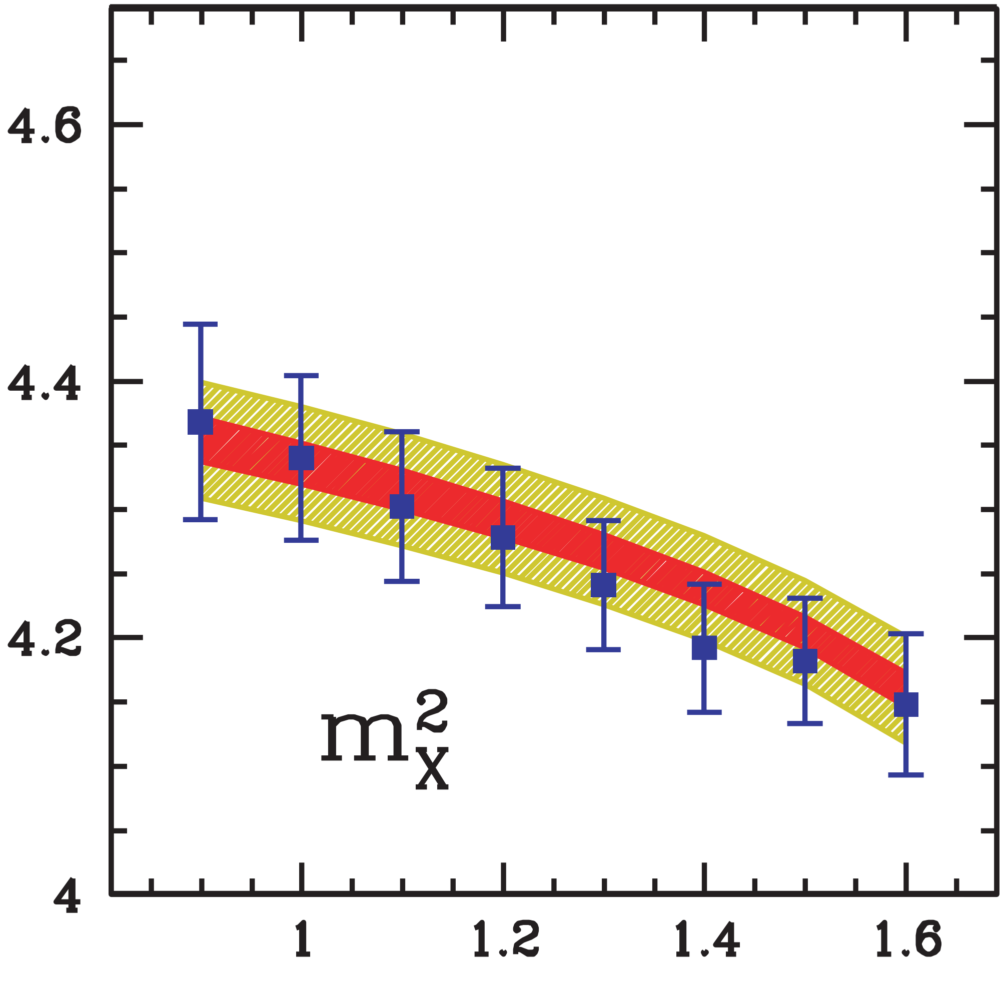
- for most values of the lepton cut, measured s_H is significantly higher than predicted



2002

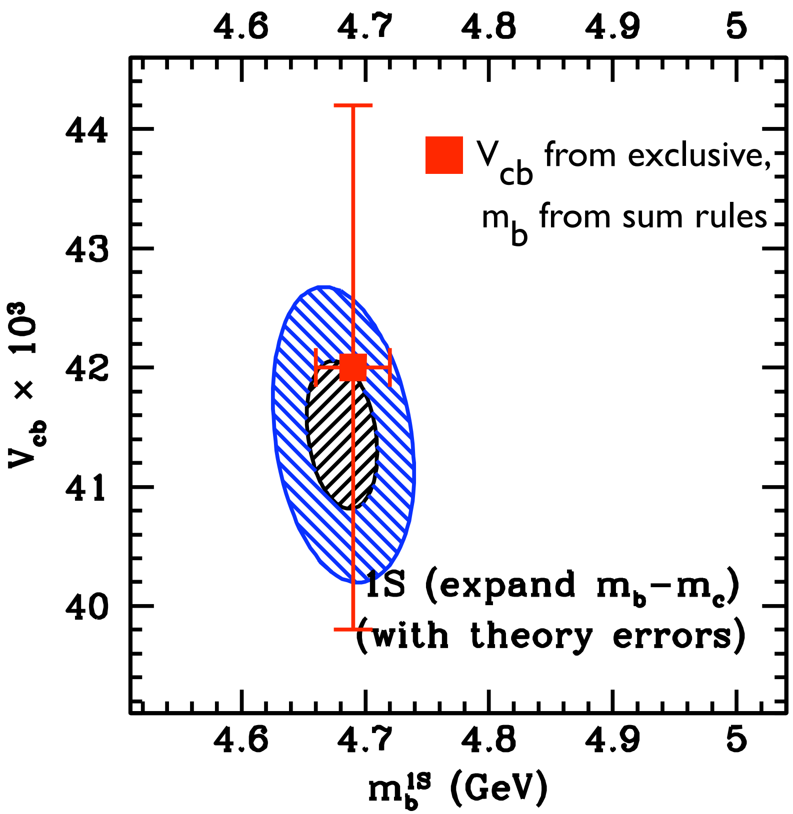


2004 - new data

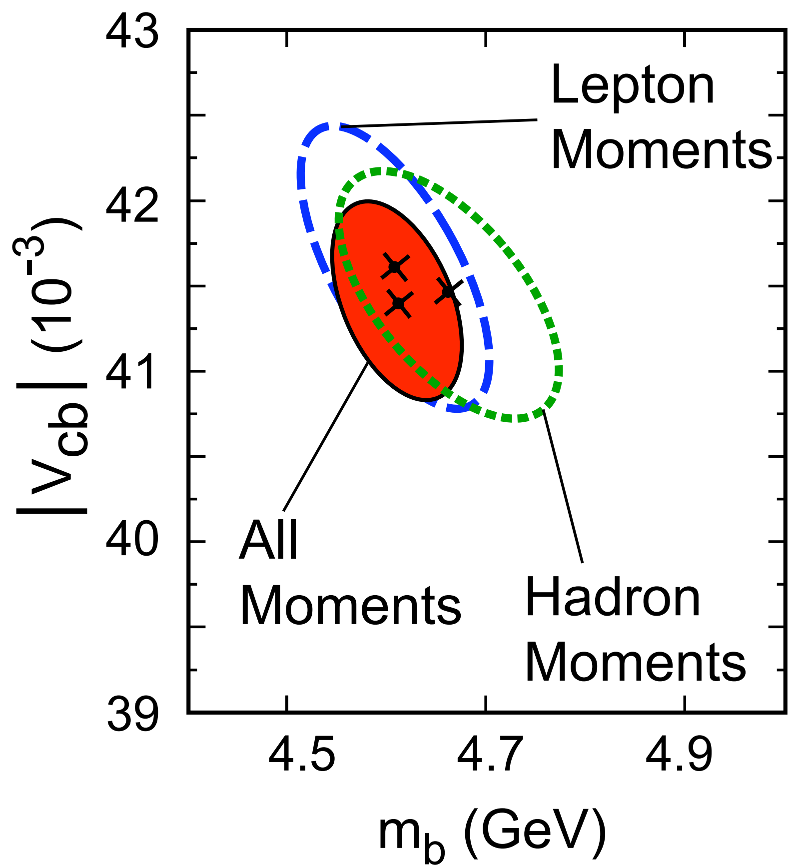


2004 - fit based on new data

Excellent agreement:



(Bauer, Ligeti, ML, Manohar and Trott)



(BABAR, using results of Gambino and Uraltsev)

$$|V_{cb}| = (41.4 \pm 0.6 \pm 0.1_{\tau_B}) \times 10^{-3}$$

$$m_b^{1S} = 4.68 \pm 0.03\,\mathrm{GeV}$$

$$\Leftrightarrow m_b(1\ \mathrm{GeV}) = (4.56 \pm 0.04)\ \mathrm{GeV}$$

$$|V_{cb}| = (41.4 \pm 0.4_{\mathrm{exp}} \pm 0.4_{\mathrm{HQE}} \pm 0.6_{\mathrm{th}}) \times 10^{-3}$$

$$m_b^{\mathrm{kin}}(1\ \mathrm{GeV}) = (4.61 \pm 0.05_{\mathrm{exp}} \pm 0.04_{\mathrm{HQE}} \pm 0.02_{\mathrm{th}})\ \mathrm{GeV}$$

Global fits also allow us to make precise predictions of other moments as a cross-check:

$$D_3 \equiv \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{0.7} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{1.5} \frac{d\Gamma}{dE_\ell}\, dE_\ell} = \begin{cases} 0.5190 \pm 0.0007 & \text{(theory)} \\ 0.5193 \pm 0.0008 & \text{(experiment)} \end{cases}$$

$$D_4 \equiv \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{2.3} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{2.9} \frac{d\Gamma}{dE_\ell}\, dE_\ell} = \begin{cases} 0.6034 \pm 0.0008 & \text{(theory)} \\ 0.6036 \pm 0.0006 & \text{(experiment)} \end{cases}$$

(BABAR)

(some fractional moments of lepton spectrum are very insensitive to $O(1/m^3)$ effects, and so can be predicted very accurately)

(Bauer and Trott)

NB: these were REAL PREdictions (not postdictions)

Hadronic physics with < 1% uncertainty!

Progress...

1995 PDG (inclusives): $|V_{cb}| = (42 \pm 2) \times 10^{-3}$

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(0.8)

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(0.8)

- looks like we're hitting a wall at 1-2% error
- but theory is passing consistency tests with flying colours - we should believe the error more now!
- complete $O(\alpha_s^2), O(\alpha_s(\Lambda_{\rm QCD}/m_b)^2)$ corrections can still usefully be done ... hard to imagine going to $(\Lambda_{\rm QCD}/m_b)^4$

V_{ub}

In principle, V_{ub} is as easy as V_{cb}:

$$\boxed{|V_{ub}| = (\underbrace{3.06 \pm 0.08}_{\text{50 MeV uncertainty on } m_b(1S)} \pm \underbrace{0.08}_{\text{perturbative uncertainty}}) \times 10^{-3} \left(\frac{B(B \to X_u \ell \bar{\nu})}{0.001} \frac{1.6\,\text{ps}}{\tau_B} \right)^{1/2}}$$

(Hoang, Ligeti, Manohar; Uraltsev)

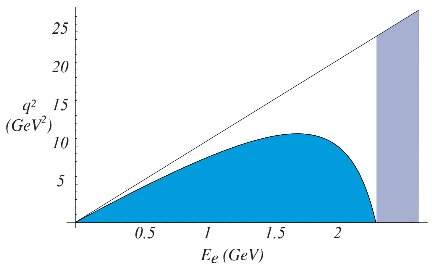
50 MeV uncertainty on $m_b(1S)$

perturbative uncertainty

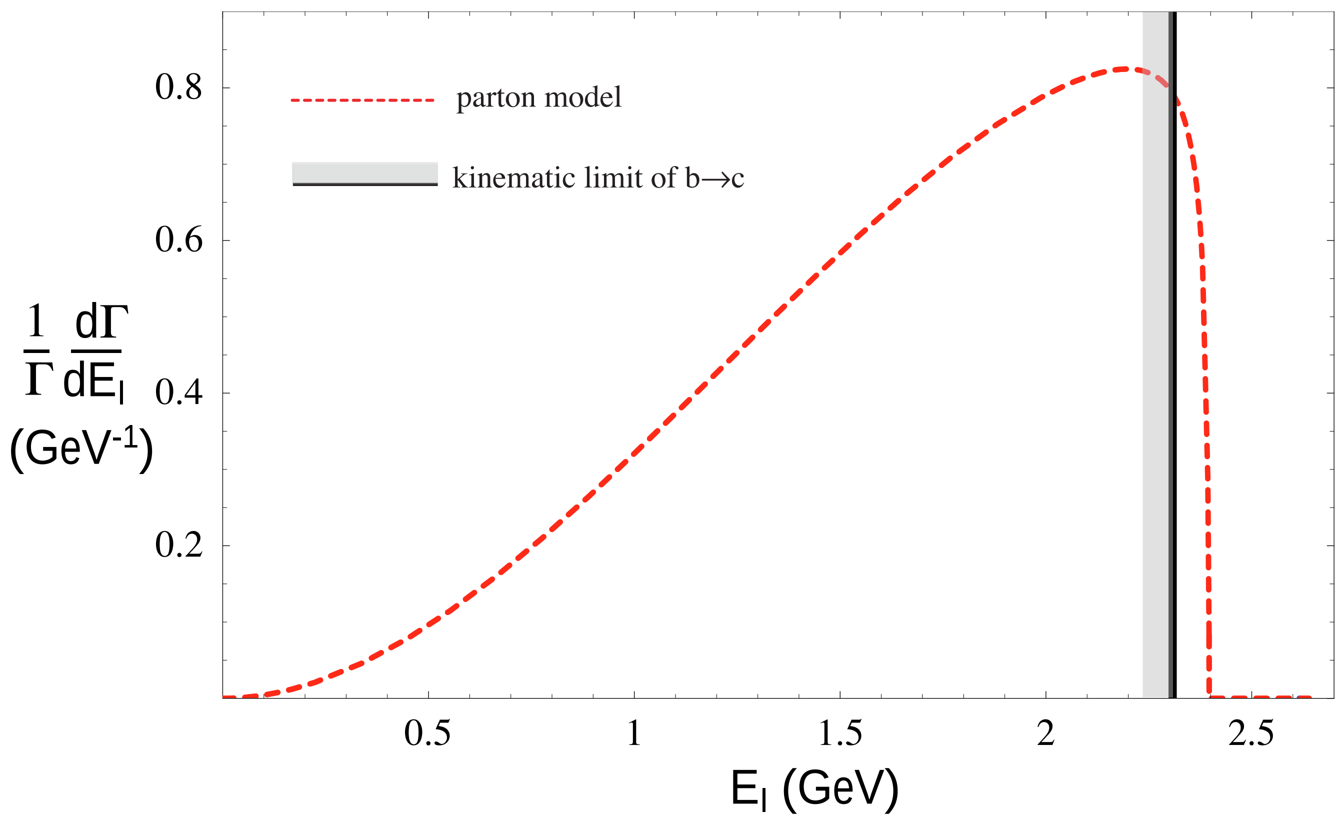
combine to a ~5% error

- very clean theoretically: greatest uncertainty is b quark mass ... nonperturbative effects are small

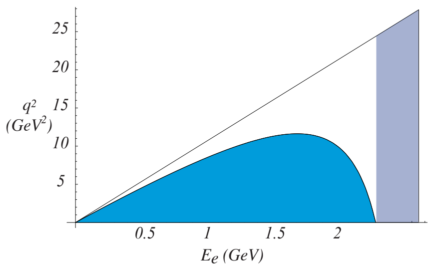
... but this requires cutting out ~100 times larger background from charm



The Classic Method: cut on the endpoint of the charged lepton spectrum



Disadvantages: • only ~10% of rate



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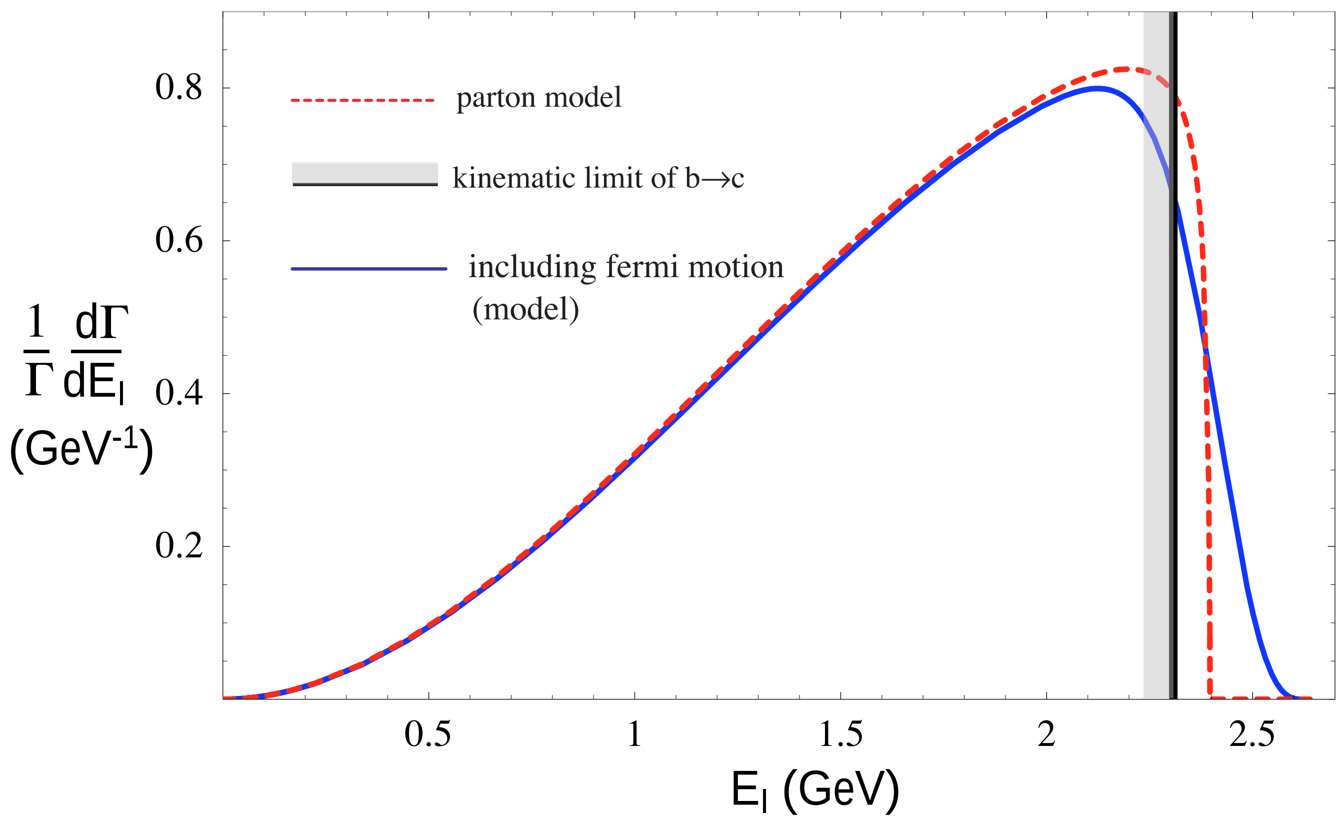
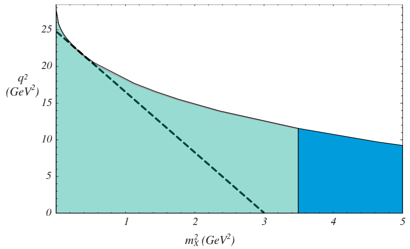


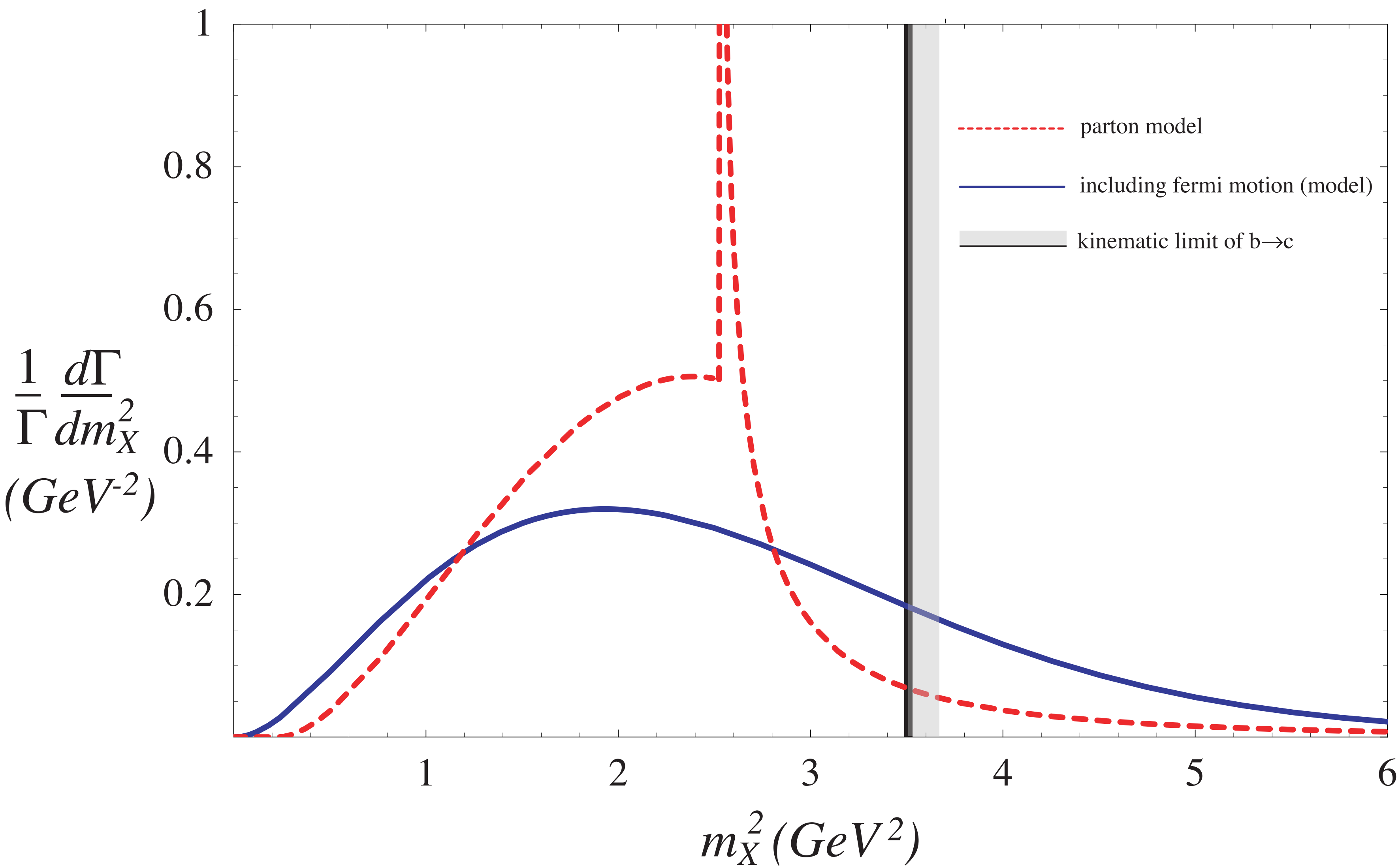
Disadvantages:

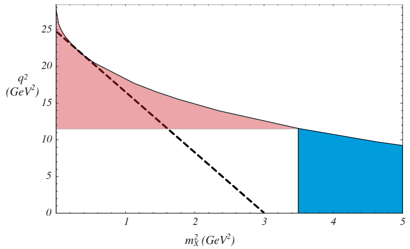
- only ~10% of rate
- sensitivity to fermi motion - local OPE breaks down



Cutting on the hadronic invariant mass spectrum gives more rate, but has the same problem with fermi motion:

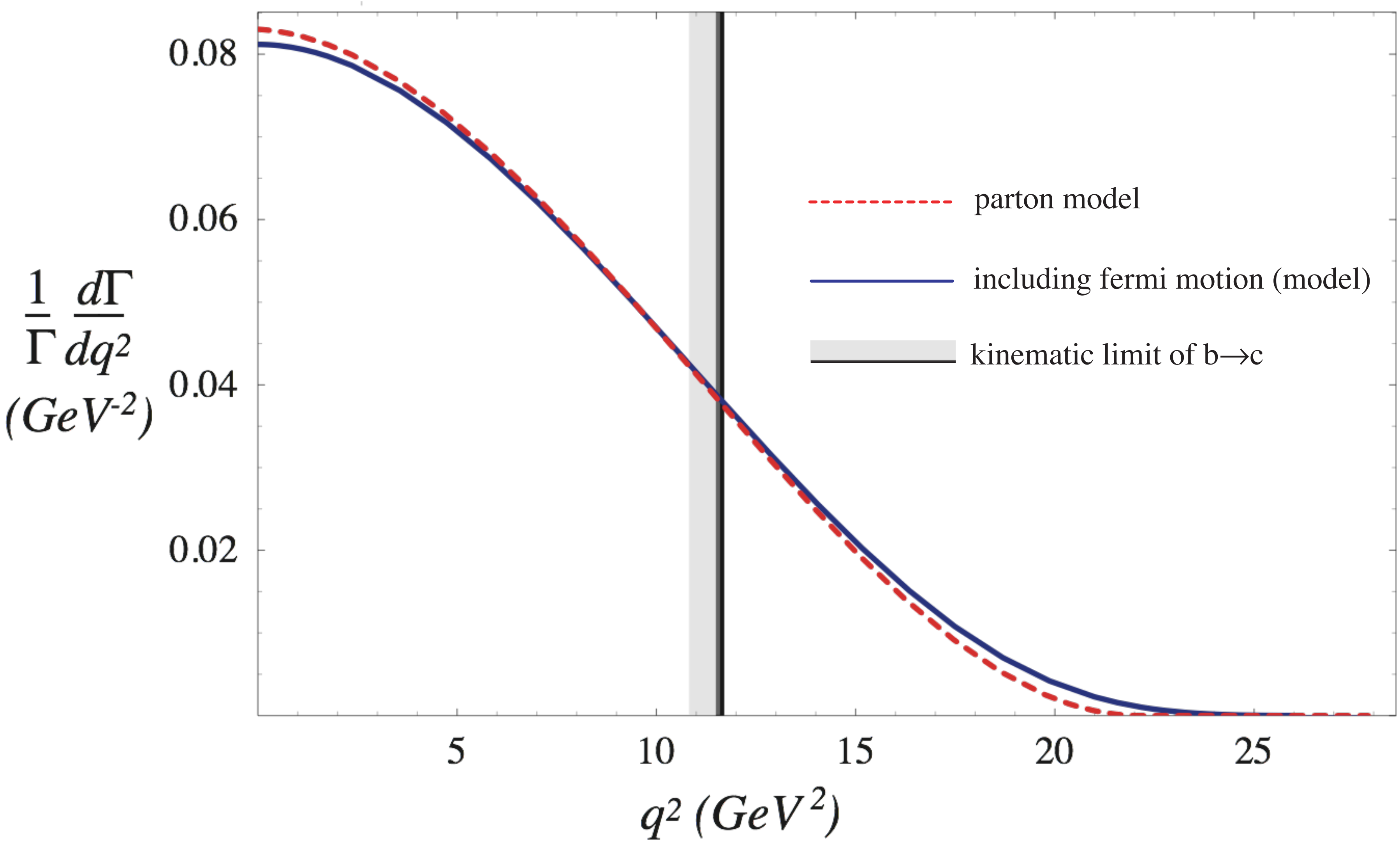
(Falk, Ligeti, Wise; Dikeman, Uraltsev)

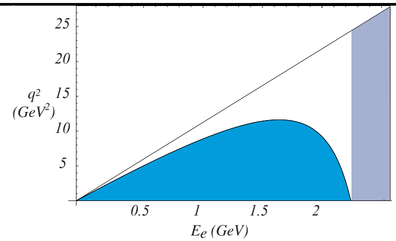
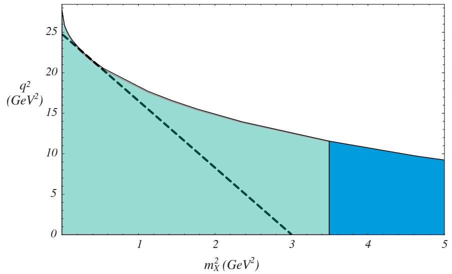
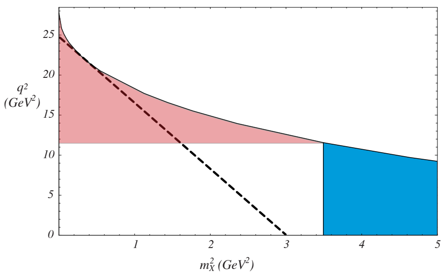
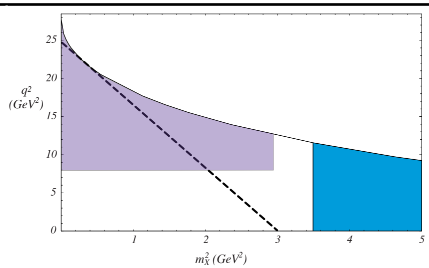
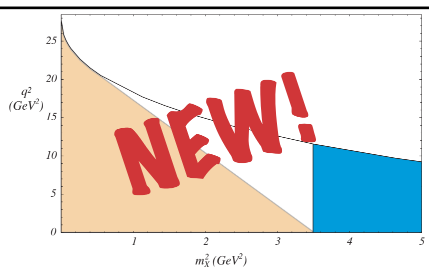




But this doesn't always happen (depends on proximity of cut to perturbative singularities) ... the local OPE holds for the leptonic q^2 spectrum:

(Bauer, Ligeti, ML)



cut	% of rate	good	bad
 $E_\ell > \frac{m_B^2 - m_D^2}{2m_B}$	~10%	don't need neutrino	
 $s_H < m_D^2$	~80%	lots of rate	
 $q^2 > (m_B - m_D)^2$	~20%	insensitive to $f(k^+)$	
 "Optimized cut"	~45%	- insensitive to $f(k^+)$ - lots of rate - can move cuts away from kinematic limits and still get small uncertainties	
 $P_+ > m_D^2/m_B$	~70%	- lots of rate - theoretically simplest relation to $b \to s\gamma$	

Theoretical Issues are much the same as in 2003:

- fermi motion - at leading and subleading order $(E_\ell,\ s_H, P_+ \text{ cuts})$
- Weak Annihilation (WA) (all)
- m_b - rate is proportional to m_b^5 - 100 MeV error is a ~5% error in V_{ub}. But restricting phase space increases this sensitivity - with q^2 cut, scale as ~ m_b^{10} $(q^2,\ \text{optimized } q^2 - s_H \text{ cuts})$
- perturbative corrections - known (in most cases) to $O(\alpha_s^2\beta_0)$ - generally under control. When fermi motion is important, leading and subleading Sudakov logarithms have been resummed. (all)

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uncertainty in m_b is now at 50 MeV level

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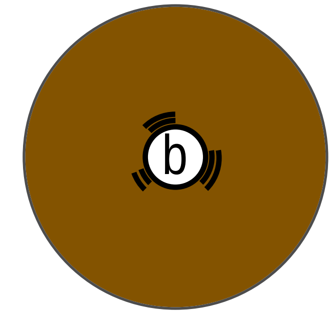
new insights into all of these

Theoretical Issues: $f(\omega) \sim \langle B|\underbrace{\bar{b}\,\delta(\omega - i\hat{D}\cdot n)b}|B\rangle$

universal distribution function
(applicable to all decays)

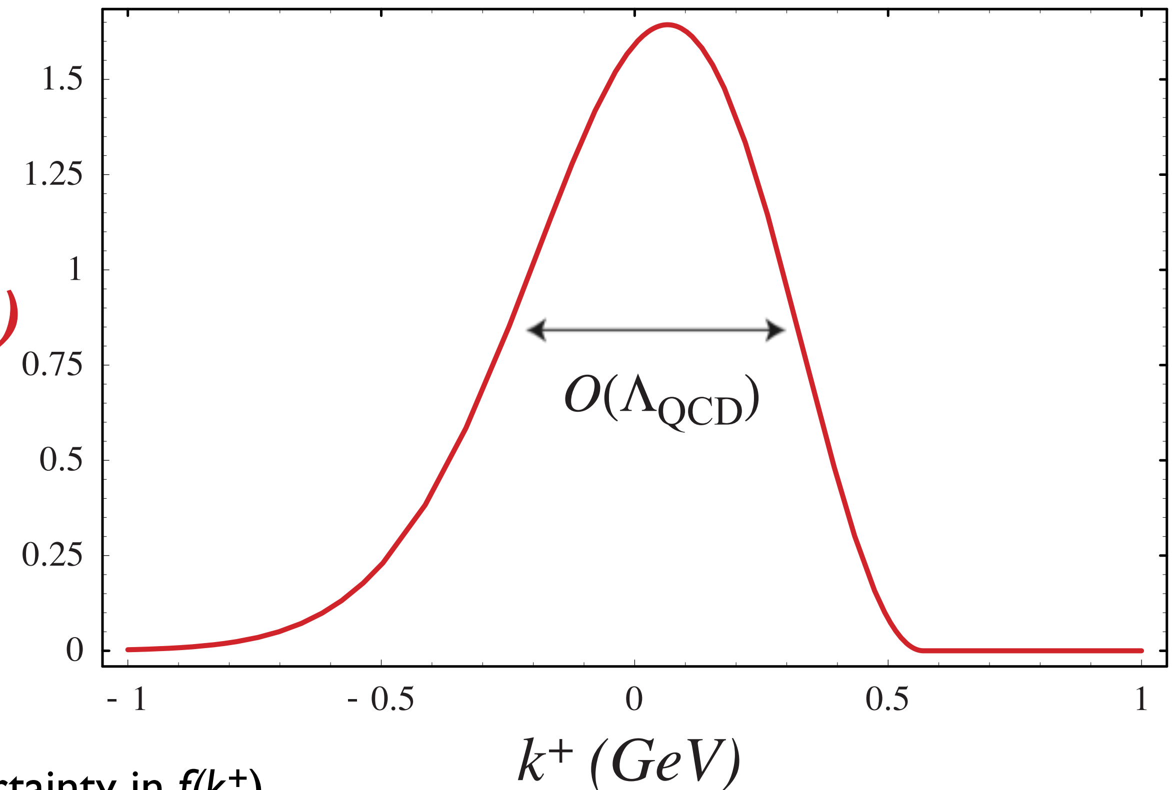


- $f(k^+)$ "shape function"

Options:

(i) model

$f(k^+)$
(model)



Ex:

$$f(k^+) = N(1-x)^a e^{(1+a)x}$$

(de Fazio and Neubert.)

a, N determined by $\bar{\Lambda},\ \lambda_1$ (gets first two moments right .. but the uncertainty in $f(k^+)$ is not simply given by the uncertainties in $\bar{\Lambda},\ \lambda_1$)

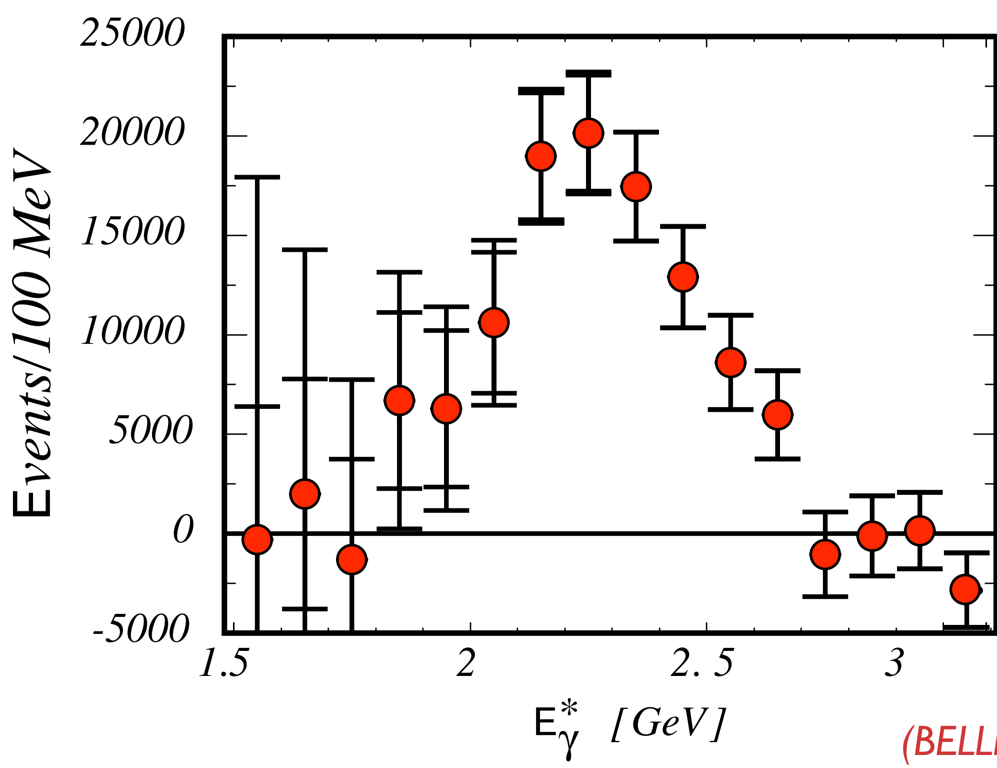
It is very difficult to determine theoretical uncertainties with this approach!

(ii) Better: determine from experiment: the SAME function determines the photon spectrum in $B \to X_s\gamma$ (at leading order in 1/m)

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{d\hat{E}_\ell}(\bar{B} \to X_u \ell \bar{\nu}_\ell) = 4\int \theta(1 - 2\hat{E}_\ell - \omega) f(\omega)\, d\omega + \dots$$
$$\frac{1}{\Gamma_0}\frac{d\Gamma}{d\hat{s}_H}(\bar{B} \to X_u \ell \bar{\nu}_\ell) = \int \frac{2\hat{s}_H^2(3\omega - 2\hat{s}_H)}{\omega^4}\theta(\omega - \hat{s}_H) f(\omega - \hat{\Delta})\, d\omega + \dots$$
$$\frac{1}{\Gamma_0^s}\frac{d\Gamma}{d\hat{E}_\gamma}(\bar{B} \to X_s\gamma) = 2f(1 - 2\hat{E}_\gamma) + \dots$$

and so can be measured from the photon spectrum in $\bar{B} \to X_s\gamma$:

(NB must subtract off contributions of operators other than O_7)



(BELLE)

NB - the “smearing” approach

$$d\Gamma = \int d\Gamma^{\text{parton}}\Big|_{m_b \to m_b + \omega} f(\omega) d\omega$$

NB - the "smearing" approach is not valid beyond tree level ...

$$d\Gamma = \int d\Gamma^{\mathrm{parton}}\Big|_{m_b \to m_b + \omega} f(\omega)\, d\omega$$

- some of the radiative corrections which are smeared should properly be included in the renormalization of the shape function

- this will cancel out in the relations between spectra, but can introduce large spurious radiative corrections in intermediate results

(iii) Best - avoid the shape function altogether, and just relate physical quantities! (leading order shape function cancels out between spectra)

ex: $$\int_0^{m_B\,\Delta_M} ds_H \frac{d\Gamma_u}{ds_H} \propto \frac{|V_{ub}|^2}{|V_{tb}V_{ts}^*|^2} \int_0^\infty dP_\gamma W_{s_H}(\Delta_M, P_\gamma) \frac{d\Gamma_s}{dP_\gamma}$$

$P_\gamma \equiv m_B - 2E_\gamma$

$$W_{s_H}(\Delta_M, P_\gamma) = \theta(\Delta_M - P_\gamma) + \theta(P_\gamma - \Delta_M) \frac{\Delta_M^3 (2P_\gamma - \Delta_M)}{P_\gamma^3}$$

$$+O(\alpha_s) + O(\Lambda_{\rm QCD}/m_B)$$

W has an expansion in powers of $\alpha_s,\ \Lambda_{\rm QCD}/m_B$, with leading term known

- (theoretical) systematic errors accumulate when you include intermediate unphysical quantities like the shape function (i.e. large perturbative corrections cancel out between spectra)
- shape function can't fit true spectra, which have resonances - only makes sense when smeared over resonance region

(iii) Best - avoid the shape function altogether, and just relate physical quantities! (leading order shape function cancels out)

$$\text{ex: } \int_0^{m_B \Delta_M} ds_H \frac{d\Gamma_u}{ds_H} \propto \frac{|V_{ub}|^2}{|V_{tb}V_{ts}^*|^2} \int_0^{\infty} dP_\gamma W_{s_H}(\Delta_M, P_\gamma) \frac{d\Gamma_s}{dP_\gamma}$$

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similarly,

$$\int_0^{\Delta_P} dP_+ \frac{d\Gamma_u}{dP_+} \propto \frac{|V_{ub}|^2}{|V_{tb}V_{ts}^*|^2} \int_0^{\Delta_P} dP_\gamma W_{P_+}(\Delta_P, P_\gamma) \frac{d\Gamma_s}{dP_\gamma}$$

(Bosch, Neubert, Lange, Paz)

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outside region of shape function validity

$$\text{ex: } \int_0^{m_B \Delta_M} ds_H \frac{d\Gamma_u}{ds_H} \propto \frac{|V_{ub}|^2}{|V_{tb}V_{ts}^*|^2} \int_0^{\infty} dP_\gamma W_{s_H}(\Delta_M, P_\gamma) \frac{d\Gamma_s}{dP_\gamma}$$

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(Bosch, Neubert, Lange, Paz)

$$P_+ \equiv m_X - |E_X|$$

- P_+ cut requires $B \to X_s\gamma$ photon spectrum over a smaller region than s_H cut
- not a big difference in practical terms (W, $f(k^+)$ both suppress large P_γ region) but theoretically cleaner

$$W_{s_H}(\Delta_M, P_\gamma) = \theta(\Delta_M - P_\gamma) + \theta(P_\gamma - \Delta_M)\frac{\Delta_M^3(2P_\gamma - \Delta_M)}{P_\gamma^3}$$

Leading logs - to sum, or not?

(Bauer, Fleming, ML; Bauer, Fleming, Pirjol, Stewart; Bauer, Manohar; Bosch, Neubert, Lange, Paz, ... also earlier work by Korchemsky and Sterman, Akhoury and Rothstein, Leibovich, Low and Rothstein)

SCET allows very elegant RGE resummation:

$$W_{P_+}^{\rm NLL}(\Delta, P_\gamma) = T(a)\left\{1 + \frac{C_F\alpha_s(m_b)}{4\pi}H(a) + \frac{C_F\alpha_s(\mu_i)}{4\pi}\left[4f_2(a)\ln\frac{m_b(\Delta - P_\gamma)}{\mu_i^2} - 3f_2(a) + 2f_3(a)\right]\right\}$$

at 2 loops:

leading log *next-to-leading log* *NNLL*

$$W_{P_+}^{(\alpha_s^2)} = \frac{C_F\alpha_s^2(m_b)}{(4\pi)^2}\left[(0.83\beta_0+3.41)\ln^2\frac{m_b}{\Delta - P_\gamma}+(4.67\beta_0-19.1)\ln\frac{m_b}{\Delta - P_\gamma}-(5.19\beta_0+c_0)\right]$$

(Hoang, Ligeti and ML)

$$\mathcal{O}(\log^2) : \mathcal{O}(\log) : \mathcal{O}(\log^0) = 1 : 0.87 : (-0.86 - 0.02c_0)$$

not a good expansion!

- large Sudakov double logs $\alpha_s^n \log^m(m_b/\mu),\ m = n+1,\ldots,2n$ cancel from *W*
- $\log m_b/\mu \sim \log 3$ is not large enough to justify leading log expansion - more justified to stick to fixed order perturbation theory (cf summing logs of m_c/m_b in exclusive $B \to D^*\ell\bar{\nu}_\ell$)

W: Nonperturbative corrections

(Bauer, ML and Mannel; Leibovich, Ligeti and Wise; Burrell, ML and Williamson; Stewart and Lee; Mannel and Tackmann; Bosch, Neubert, Lange, Paz; Beneke, Campanario and Mannel, ...)

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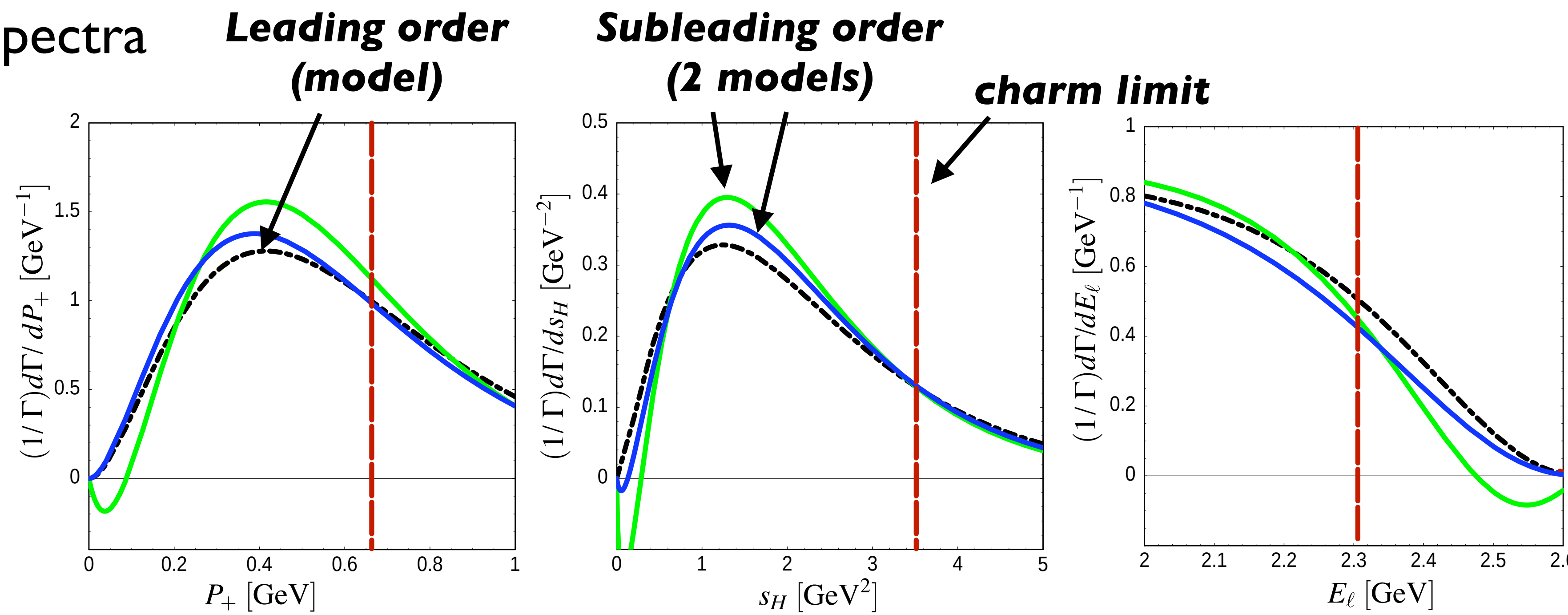
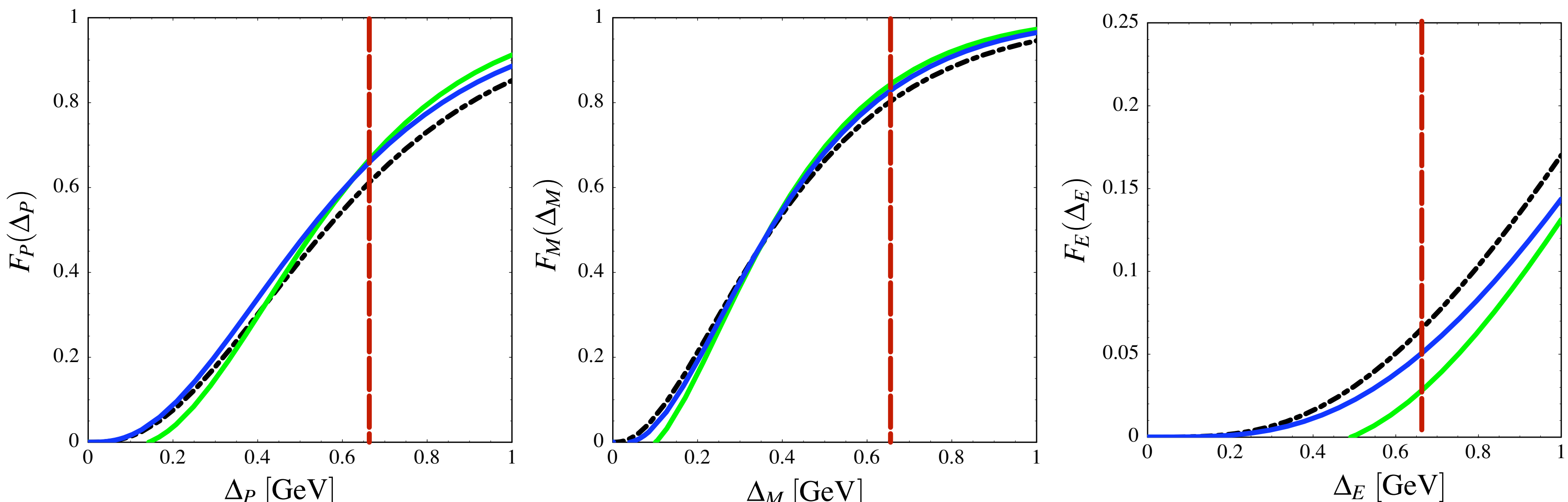
- Corrections are largest for the E_l endpoint spectrum (but improve as cuts are loosened), better for s_H and P_+
- Weak annihilation effects can be large - wide variation in estimates of size

Subleading effects (with small WA):

(a) spectra



(b) integrated spectra

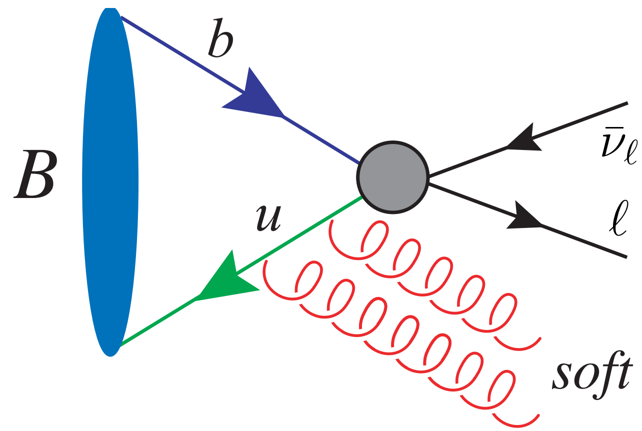


(Bosch, Neubert, Lange, Paz)

Theoretical Issues:

- Weak annihilation ... in local OPE $(q^2,\ \text{optimized}\ q^2 - s_H\ \text{cuts})$

(Bigi & Uraltsev, Voloshin, Leibovich , Ligeti, and Wise)



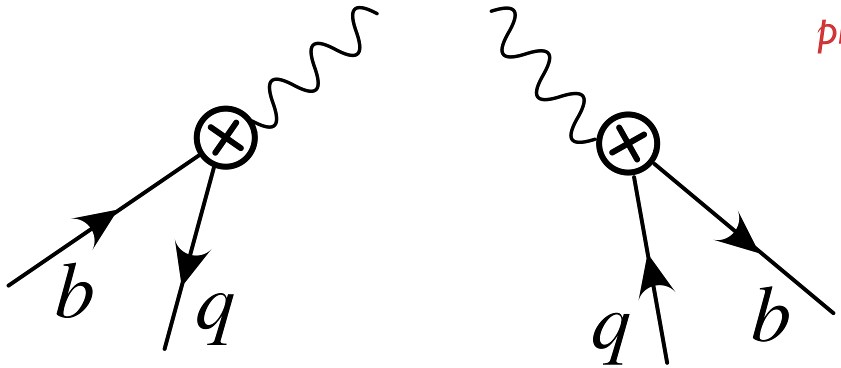
$$O\left(16\pi^2 \times \frac{\Lambda^3_{QCD}}{m_b^3} \times \text{factorization violation}\right) \sim 0.03 \left(\frac{f_B}{0.2\,\mathrm{GeV}}\right)\left(\frac{B_2 - B_1}{0.1}\right)$$

~3% (?? guess!) contribution to rate at $q^2 = m_b^2$

- an issue for all inclusive determinations
- relative size of effect gets worse the more severe the cut
- no reliable estimate of size - can test by comparing charged and neutral *B*'s, comparing D and D_s semileptonic widths

Theoretical Issues:

- Weak annihilation ... in nonlocal OPE $(E_\ell,\ s_H, P_+ \text{ cuts})$



phase space enhancement

factorization violation

$$O\left(16\pi^2 \times \frac{\Lambda_{\mathrm{QCD}}^2}{m_B^2} \times \Delta B\right)$$

sub-subleading shape function

- enhanced in shape function region to $O(\Lambda_{QCD}/m_b)^2$
- concentrated in large q^2 region
- can easily be >20% shift to integrated rate for E_l>2.3 GeV (smaller effect for other spectra since more rate included)

Theoretical Issues:

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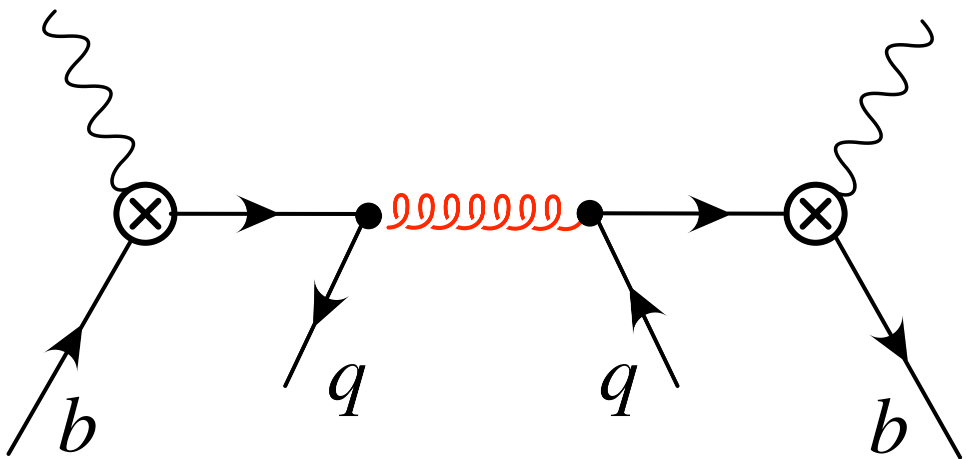


controversial

$$O\left(4\pi\alpha_s(\mu_i)\times\frac{\Lambda_{\text{QCD}}}{m_b}\times\epsilon\right)$$

only subleading!

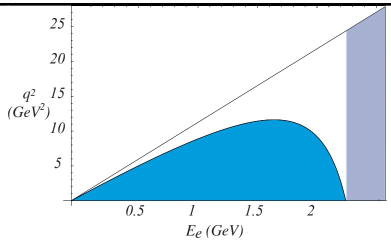
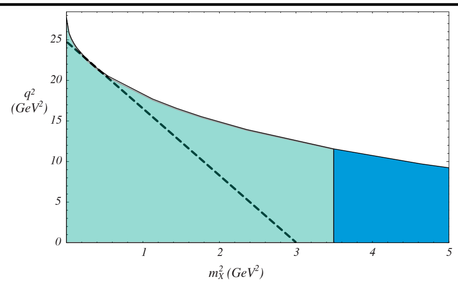
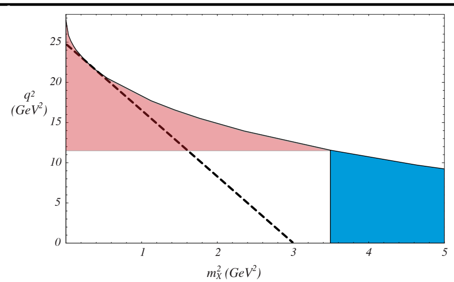
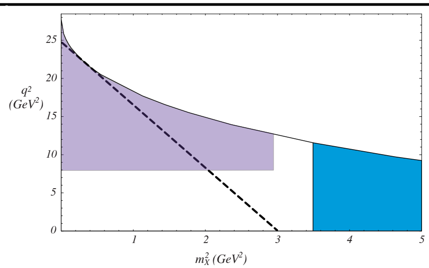
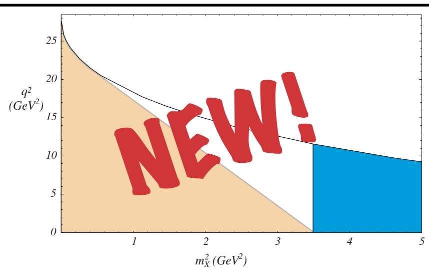
colour suppression

- hard to power count ... estimates of size vary by almost 2 orders of magnitude!

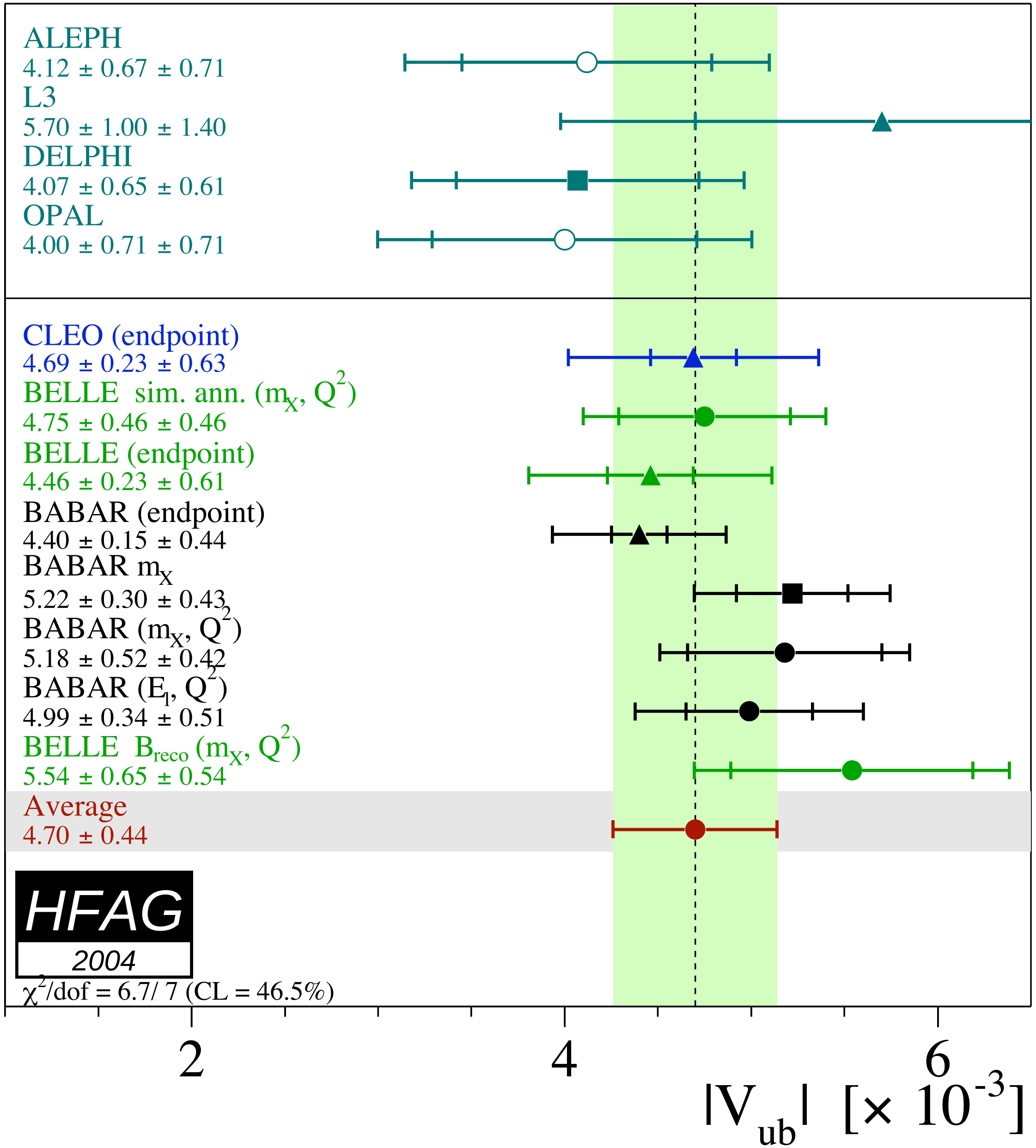
Lee and Stewart: up to 180% of LEADING term for lepton endpoint! (smaller for s_H and P_+) - would completely mess up shape function expansion

Bosch, et. al.; Neubert; Beneke et. al.: colour suppression ⇒ $\epsilon \ll 1$ + no

factor of 4 ⇒ negligible effect (smaller than other 1/m effects)

cut		% of rate	good	bad
	$E_\ell > \frac{m_B^2 - m_D^2}{2m_B}$	~10%	don't need neutrino	- depends on f(k^+) (and subleading corrections) - WA effects largest - reduced phase space - duality issues?
	$s_H < m_D^2$	~80%	lots of rate	- depends on $f(k^+)$ (and subleading corrections) - need shape function over large region
	$q^2 > (m_B - m_D)^2$	~20%	insensitive to $f(k^+)$	- very sensitive to m_b - WA corrections may be substantial - effective expansion parameter is $1/m_c$
	"Optimized cut"	~45%	- insensitive to $f(k^+)$ - lots of rate - can move cuts away from kinematic limits and still get small uncertainties	- sensitive to m_b (need +/- 60 MeV for 5% error in best case)
	$P_+ > m_D^2/m_B$	~70%	- lots of rate - theoretically simplest relation to $b \to s\gamma$	depends on $f(k^+)$ (and subleading corrections)

Experimental situation:



quoted uncertainties in any given measurement are approaching the 10% level; theoretical and experimental uncertainties are generally comparable

Bottom line(s):

- there is no "best method" - each has its own sources of uncertainty
  - local OPE: $b \to c$ experience gives us confidence in framework, but we are pushing things to lower momentum scales for V_{ub} - perturbative, nonperturbative effects are more significant
  - nonlocal OPE: reasonable model estimates suggest things are OK, but no experimental test of framework
- we only believe V_{cb} because of all the checks. Our confidence in V_{ub} will grow if different methods give compatible results.
- experiments can help beat down theoretical uncertainties
  - improved measurement of $B \to X_s \gamma$ photon spectrum - lowering cut reduces effects of subleading corrections, as well as sensitivity to details of $f(k^+)$
  - test size of WA (weak annihilation) effects - compare D^0 & D_S S.L. widths, extract $|V_{ub}|$ from $B^\pm$ and B^0 separately
- V_{ub} wall is likely to be at the ~5% level via these methods, assuming no inconsistencies

Summary:

- Theory for V_{cb} from inclusive decays is very mature - many cross-checks, corrections well understood
    - spectral moments are allowing us to test theory, fix nonperturbative corrections at the $(\Lambda_{QCD}/m_b)^3$ level
    - uncertainties are ~2% for V_{cb}, ~50 MeV for m_b - values are in excellent agreement with other methods
    - probably hitting the limits of this technique
- Model-independent determinations of $|V_{ub}|$ are possible, but require probing restricted regions of phase space - some (but not all!) regions are sensitive to nonperturbative shape function(s)
    - theory of q^2, combined q^2-m_X cut is on the same footing as for $b\to c$ decays, but at lower momentum transfer
    - much recent progress in theory of "shape function region", but not well tested experimentally
    - theoretical uncertainties of ~5% appear feasible