Lobe-Cleft Patterns in the Leading Edge of a Gravity Current

by

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Abstract

Gravity currents appear in various geophysical fluid flows such as oceanic turbidity currents, avalanches, pyroclastic flows, etc. A laboratory analogue of these flows was constructed using a lock-exchange tank (150×60×30 cm), where the density difference was produced by the addition of titanium dioxide powder to water. This setup was then applied to study gravity currents driven by a variety of density differences, measuring the mean front speed and the shape of the leading edge. Planform views of the head of the gravity current were taken and the evolution of a spanwise lobe-cleft pattern was measured as a function of time. Growth and decay of the lobes, as well as lobe splitting, were observed in time-evolution diagrams. The observed structure of the front was compared to a theoretical linear stability treatment of the lobe-cleft instability by HärTEL et al. In this theory an unstable density inversion occurs when the leading edge of the denser gravity current overrides the less dense ambient fluid. The predictions of this theory were tested against the observations of the dominant wavenumber versus Grashof number, the ratio of buoyancy to viscous forces. The lobe-cleft pattern was found to first develop in a regime of elongated fingers with wavenumbers greater than the predictions of the linear stability analysis and subsequently transform to the shifting lobe-cleft regime with initial wavenumbers less than those predicted by HärTEL et al.
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Chapter 1

Gravity Currents in the Natural Environment

1.1 Introduction

Gravity currents are buoyancy driven fluid flows which occur in the environment and industry. These flows are driven by gravity acting upon small density differences between a lighter fluid and a denser fluid. The direction of propagation is typically perpendicular to the direction of gravitational attraction. While the driving force behind these flows is rather simple, the qualitative behavior of a gravity current can be quite rich. The study of gravity currents has, until recently, focused primarily on the two-dimensional, cross-sectional behavior of gravity currents and has been reviewed in an excellent book by Simpson [1]. This report will investigate an aspect of the three-dimensional structure of a gravity current, namely the dynamics of a pattern of shifting lobes and clefts found at the front of an intruding gravity current.

1.2 Gravity Currents in Nature

Gravity currents occur in the atmosphere, in suspension currents on land, in the ocean and in industrial situations. Gravity currents have been studied both as they occur within natural contexts and as laboratory models of these flows. While many of these gravity currents share similar properties, the specific dynamics of any given gravity current depend heavily on the particulars of their environment. With this in mind, many laboratory experiments have sought to develop a deeper understanding of simplified versions of the flows found in nature.
Gravity currents often develop within the atmosphere as in the case of thunderstorm outflows. Warm moist air flowing up the top of the thunder cloud can impinge upon the tropopause causing a gravity current flow along the boundary leading to the well known anvil shaped cloud formations. In addition a gravity current of cold air carrying hail or rain can form at the base of the thunder cloud. Atmospheric gravity currents are also found in sea breeze fronts. Here, cooler sea air forms a gravity current as it passes into air heated by the warmer land. These flows are often highlighted through the suspension of dust and other aerosols as well as insects and birds. Measurements of these flows have been conducted using satellite imagery, balloon flights, and observations as these flows pass by an instrumented tower.

One particularly devastating form of gravity currents is the avalanche. Avalanches have effected populated mountainous areas all over the world. These gravity currents can be caused by a suspension of snow or rock, or through soil suspension in water as in the case of mud flows or lahar. One particularly explosive source of avalanches is related to volcanic activity. Basaltic lava streams and pyroclastic flows of hot ash are all examples of highly destructive volcanic gravity currents.

Gravity currents can also be found in oceanic and river systems. Here, density differences between fresh water river efflux and saline oceanic water can lead to a river plume and a salt wedge. In addition, large suspensions of sediment and organic material, often triggered by an earthquake, can lead to underwater turbidity currents. These flows, which are the dominant mode of deep submarine sediment transport, can travel hundreds of kilometers transforming the topography of the ocean floor over which they pass.

Finally, gravity currents can be found in industrial settings with applications in many fields. In the mining industry, dangerous gasses can travel rapidly through mine shafts as gravity currents. Knowledge of these flows is essential to proper ventilation and safety. Spreading oil slicks also behave like a gravity current as do dense gases such as liquid natural gas. The study of these fluid flows can help regulate transport and cleanup of these dangerous materials.

1.3 The Anatomy of a Gravity Current

The anatomy of gravity currents has been studied extensively both through theoretical and numerical methods. Typical gravity currents form a frontal zone where there is a distinct dividing line between the intruding and ambient fluids. This leading zone is
followed by a region of intense mixing called the tail. The front of the gravity current tends to be deeper than the following flow and has a raised 'nose' at its leading edge as the current progresses over a no-slip boundary. This profile is unstable to two instabilities as shown in Fig 1.1: a Kelvin-Helmholtz roll instability which contributes to mixing behind the head and a lobe-cleft instability at the leading edge of the current.

Figure 1.1: The profile of the front of a gravity current is unstable to a Kelvin-Helmholtz roll instability (top) and a lobe-cleft instability (bottom) (from Simpson [1]).
1.4 Two-Dimensional Theory of Gravity Currents

Inviscid-fluid theory, in which viscous forces are absent, was the first attempt to quantitatively predict the dynamics of gravity currents. In this theory, the frictional effect of the ground is lost while a Kelvin-Helmholtz instability remains. This flow was studied by Benjamin in 1968 [2] in the case of a cavity flow (an intrusive flow into ambient fluid bounded on both top and bottom, see Fig 1.2). Benjamin used the equation of conservation of mass and Bernoulli’s equation applied along the interface to relate the depth of the flowing layer \( h_2 \) to the total depth \( H \), resulting in the solution

\[
    h_2 = H/2. \tag{1.1}
\]

Solutions with \( h > H/2 \) are not possible, and solutions with \( h < H/2 \) involve wave breaking since the loss of energy at the front exceeds that from wave breaking.

![Figure 1.2: Front of a frictionless gravity current studied by Benjamin [2] in the case of cavity flow. Here an gravity flow of height \( h \) intrudes along the top surface of a cavity of height \( H \) with a contact angle of 60°.](image)

The effects of mixing on the dynamics of a gravity current were then investigated both theoretically and experimentally by Britter and Simpson [6]. In their theory a region of mixing downstream of the gravity current head was investigated for the same inviscid, incompressible, steady flow with no-slip boundary conditions. These results were experimentally tested with an apparatus which used a flexible conveyor belt to...
ensure a uniform flow velocity throughout the channel at the gravity current head. Their results show a set of two-dimensional Kelvin-Helmholtz billows developing and an absence of the lobe-cleft instability.

1.5 Current Investigations into the Dynamics of Gravity Currents

The first major experimentation of the lobe-cleft instability was undertaken by Simpson [5] in which a shifting pattern of lobes and clefts was observed. Simpson further investigated this phenomenon by initially suppressing the instability by moving the bottom surface over which the gravity current flowed at speeds in excess of the gravity current. This showed that the instability was due to an unstable stratification of the intruding dense fluid and the less dense ambient fluid. Related research into fingering instabilities in viscous gravity currents was done by Snyder and Tait [7, 8]. In these experiments a viscous gravity current intruded into a less dense, more viscous ambient fluid. Using scaling arguments and dimensional analysis Snyder and Tait found the fingering in these currents were caused by both an unstable density inversion and through a viscous fingering mechanism analogous to a Saffman-Taylor instability in a Hele-Shaw cell. Further experiments by Didden and Maxworthy [9] report seeing the lobe-cleft pattern at the front of a less dense viscous current intruding at the upper boundary of a denser ambient fluid. The lobe-cleft pattern is also evident in the particle image velocimetry measurements of Alahyari and Longmire [10]. Further unpublished experiments into the lobe-cleft instability were conducted by Marcel Fehr and Thomas Osinga under the direction of Dr. HärTEL and Prof. Rösen at ETH Zurich [11].

The aim of this report is to investigate the dynamics at the front of a gravity current, specifically to experimentally test the linear stability analysis of HärTEL et al. [3] which attempts to understand the evolution of the lobe-cleft pattern at the leading edge of a gravity current. Previous to their analysis there has been little theoretical investigation of this instability, although the phenomenon has been observed experimentally.
Chapter 2

Theory

2.1 Introduction

Recent theoretical work on the dynamics at the front of an intruding gravity current by Härtel et al. [3, 4] provide a first approach to understanding the lobe-cleft instability at the leading edge of gravity currents. This theoretical work investigated the dynamics at the leading edge of a gravity current using a three-dimensional direct numerical simulation (DNS) [3] and subsequently performed a linear stability analysis on this system [4] using the results from the DNS as the base flow being perturbed.

2.2 Theoretical Tank Geometry

The simplest geometry from which to investigate the dynamics of a gravity current is the lock-exchange tank pictured in Fig 2.1. Here a heavy fluid intrudes into a lighter fluid from right to left in a closed tank with length $L_1$ and height $2h$. The fluids are initially separate and evolve with time after release of the lock gate as depicted by the dotted line. The coordinate system used describes the flow direction as $x_1$, the cross-flow direction as $x_2$, and the vertical direction (direction of gravitational acceleration) as $x_3$. The case of small density differences was investigated by Härtel et al. enabling the use of the Boussinesq approximations, and all boundary conditions were no-slip.
2.3 Base Equations

The base equations which Härtel et al. used to model the intruding gravity current are the standard fluid dynamics equations for small density differences where the Boussinesq approximations have been adopted. The non-dimensionalized equations express conservation of mass, conservation of momentum and conservation of energy as

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_k}{\partial x_k} = -\frac{\partial p}{\partial x_i} + \frac{1}{\sqrt{Gr}} \frac{\partial^2 u_i}{\partial x_k \partial x_k} + \rho e_i^g, \quad (2.2)$$

$$\frac{\partial p}{\partial t} + \frac{\partial (\rho u_k)}{\partial x_k} = \frac{1}{\sqrt{Gr Sc^2}} \frac{\partial^2 \rho}{\partial x_k \partial x_k}, \quad (2.3)$$

respectively. Here $u_i$ is a velocity vector, $x_k$ is the position, $p$ is the pressure, $\rho$ is the density and $e_i^g$ is a unit vector in the vertical direction ($e_i^g = (0, 0, -1)^T$). The quantities in equations 2.1–2.3 are made dimensionless by the channel half-height $\hat{h}$, the average density $\hat{\rho}_a = (\hat{\rho}_{\text{max}} + \hat{\rho}_{\text{min}})/2$, and the buoyancy velocity $\hat{u}_b$ defined as

$$\hat{u}_b = \sqrt{\hat{g}' \hat{h}}, \quad (2.4)$$

where $\hat{g}'$ is the reduced gravity, defined with respect to the gravitational acceleration $\hat{g}$ as

$$\hat{g}' = \hat{g} \frac{\Delta \hat{\rho}}{\hat{\rho}_a}, \quad (2.5)$$

The non-dimensionalized pressure and density are given by

$$p = \frac{\hat{p}}{\hat{\rho}_a \hat{u}_b^2}, \quad (2.6)$$

$$\rho = \frac{\hat{\rho} - \hat{\rho}_{\text{min}}}{\Delta \hat{\rho}}, \quad (2.7)$$
where $\Delta \tilde{\rho} = \tilde{\rho}_{\text{max}} - \tilde{\rho}_{\text{min}}$.

The non-dimensionalized Boussinesq equations are thus governed by two dimensionless parameters. The Grashof number $Gr$ is the ratio of buoyancy forces to viscous forces defined as

$$Gr = \left( \frac{\tilde{u}_b\tilde{h}}{\tilde{\nu}} \right)^2 = \frac{\tilde{g}\Delta \tilde{\rho} \tilde{h}^3}{\tilde{\rho}_a \tilde{\nu}^2},$$

(2.8)

and the Schmidt number $Sc$ is the ratio of kinematic viscosity $\tilde{\nu}$ to molecular diffusivity $\tilde{K}$ defined as

$$Sc = \frac{\tilde{\nu}}{\tilde{K}},$$

(2.9)

although in most of the numerical calculations of Härtel et al. this parameter was set to unity. In addition, the Reynolds number is defined with respect to the front velocity $\tilde{u}_f$ and the height of the foremost part of the front $\tilde{h}_f$ such that

$$Re = \frac{\tilde{u}_f \tilde{h}_f}{\tilde{\nu}}.$$

(2.10)

Finally, the Froude number measures the ratio of the front velocity to the buoyancy and is defined as

$$Fr = \frac{\tilde{u}_f}{\tilde{u}_b} = \tilde{u}_f \sqrt{\frac{\tilde{\rho}_a}{\tilde{g} \Delta \tilde{\rho} \tilde{h}}}.$$  

(2.11)

### 2.4 Three-Dimensional Direct Numerical Simulation of Gravity Current Head

The fluid flow at the head of a gravity current has been computationally studied in the first of a pair of papers by Härtel et al. [3]. Here, a three-dimensional direct numerical simulation (DNS) of the flow of a gravity current in the lock-exchange geometry shown in Fig 2.1 is performed using equations 2.1–2.3. Calculations were performed at a number of Grashof numbers and it is the fluid property profiles from these calculations which provide the basis for the subsequent linear stability analysis. In addition, the full three-dimensional DNS clearly show the presence of the lobe-cleft instability as shown in Fig 2.2.
2.5 Linear Stability Analysis

In a companion paper to the three-dimensional DNS Härte et al. [4] performed a linear-stability analysis of the flow at the head of two dimensional gravity current fronts. Basing their investigation on the full DNS simulation of the gravity current they found that the front traveled at an essentially constant velocity after a short initial transient. For this reason a translating coordinate system was defined at the velocity of the front for the purposes of the linear stability analysis. The two-dimensional base flows were obtained from the DNS simulation for five different Grashof numbers (Gr=3 × 10^4, 9 × 10^4, 9 × 10^6, 9 × 10^7, and 4 × 10^8).

Applying standard linear-stability theory, Härte et al. then decomposed the flow into a two-dimensional, steady-state mean flow (overbar) and an infinitesimal disturbance (prime) as follows:

\begin{align*}
    u_i(x_i, t) &= \overline{u_i}(x_1, x_3) + u'_i(x_1, x_2, x_3, t), \quad (2.12) \\
    p_i(x_i, t) &= \overline{p_i}(x_1, x_3) + p'_i(x_1, x_2, x_3, t), \quad (2.13) \\
    \rho_i(x_i, t) &= \overline{\rho_i}(x_1), x_3 + \rho'_i(x_1, x_2, x_3, t). \quad (2.14)
\end{align*}

Assuming that the instability is wavelike in the span wise (x_2) direction with wavenumber \( \beta \) and that the dominant mode grows exponentially at rate \( \sigma \) the disturbances then take the general form.
Figure 2.3: Stability diagram of HärTEL et al. [4] showing the growth rate $\sigma$ vs. wavenumber $\beta$ for the five different Grashof numbers $Gr$ listed in the legend.

\begin{align}
  f' &= \hat{f}(x_1, x_3) \cos(\beta x_2) e^{\sigma t} \text{ for } f' = u'_1, u'_3, p', \rho', \\
  f' &= \hat{f}(x_1, x_3) \sin(\beta x_2) e^{\sigma t} \text{ for } f' = u'_2.
\end{align}

where $\hat{f}$ is an amplitude function that does not depend on time. Inserting these definitions into the general equations (2.1–2.2), subtracting the mean flow equations and neglecting all terms which are quadratic in disturbances, leads to five linear partial differential equations describing the evolution of the disturbances. In addition, Sc is assumed to be unity since HärTEL et al. found only a weak dependence of $\sigma$ on Sc in a precursor study.

HärTEL et al. then use a computational approach which discretizes the stability equations in a rectangular sub domain encompassing the foremost part of the front thus transforming them into a general algebraic eigenvalue relation. Solving this eigenvalue relation gave the stability diagram shown in Fig 2.3.

The stability diagram shown in Fig 2.3 has been numerically calculated for the five Grashof numbers indicated. Only the results of the first mode of instability are discussed although HärTEL et al. report that at times more than one unstable mode was obtained from the analysis. From the diagram it can be clearly seen that there is a broadening of the unstable band of wavenumbers as the Grashof number is increased. It should also be noted that HärTEL et al. found that the three-dimensional
The instability investigated here occurs even at Grashof numbers where the front is stable to the Kelvin-Helmholtz instability leading to rolls following the head of the current. Assuming that the maximum wavenumber $\beta_{\text{max}}$ with the highest growth rate $\sigma_{\text{max}}$ corresponds to the observed wavelength at the onset of the instability, Härtel et al. plot $\beta_{\text{max}}$ and $\sigma_{\text{max}}$ versus $Gr$ and compare it with the dominant wavenumber from the lobe-cleft experiments of Simpson [5] as shown in Fig 2.4.

The results of the linear stability analysis showing the variation of $\beta_{\text{max}}$ with $Gr$ are subsequently tested experimentally in chapter 5. The underlying assumption of this test is that the wavenumber with the maximum growth rate, $\sigma_{\text{max}}$, will dominate and thus be the characteristic wavelength observed in the experimental apparatus. While it would also be advantageous to explore the variation of $\sigma_{\text{max}}$ with Grashof number experimentally, the process of assessing
Chapter 3

Experiment

3.1 Introduction

The apparatus used to experimentally replicate gravity currents consisted of a lock-exchange tank and a digital camera output to a computer. Gravity currents were produced by increasing the density of the fluid behind the lock through the addition of titanium dioxide powder. The currents were released by raising the gate. Their subsequent advance down the length of the tank as well as the evolution and dynamics of the front of the gravity currents were captured using the digital camera. Images where subsequently processed using Matlab’s image processing toolbox in concert with Matlab software developed in-house.

3.2 Lock-Exchange Tank

The lock-exchange tank of outside dimensions $150 \times 60 \times 30$ cm was composed of three main components: the lock, the gate, and the tank as shown in Fig 3.1 and Fig 3.2. At the beginning of every data run both the lock and the tank sections were filled with water, and the gate was closed. Titanium dioxide, a white powder, was then added to the lock section, thus making the fluid behind the gate opaque and heavier than that in the tank section. The settling time of the titanium dioxide powder is much slower than the time during which the fluid flows are investigated so that particle dynamics within the gravity current can be neglected.

The gate itself was composed of three pieces as shown in Fig 3.3. Two outside members were attached to the tank walls and formed a track through which the middle piece could slide. The track pieces were designed with two open sections at the top
and the bottom divided by a solid middle section, while in the gate piece the middle
section was open and the top and bottom thirds were solid. This meant that as the
gate was raised two channels connecting the lock and the tank were simultaneously
opened at the top and bottom of the tank with an ultimate cross-section which was
two-thirds of the cross-section of the lock-exchange tank. This geometry enabled
the gravity current to enter at the bottom of the tank while a return current from
the tank flowed into the top of the lock, thus conserving flow and minimizing initial
perturbations from the raising of the gate. While this design minimized the effect
of opening the gate on the flow it also needed to be able to separate the two fluids
while closed long enough for the residual currents within the tank to die out. For this
reason flexible tubing was stretched around the end of the gate to provide a better
seal to the bottom and sides of the lock-exchange tank. In addition to the tubing
motor grease was applied to all gate seams before the lock-exchange tank was filled
to minimize lock to tank leakage. Finally, the track pieces were reinforced for greater
rigidity thus providing a tighter seal to the gate piece by preventing outwards bowing.

The lock-exchange tank, of dimensions $150 \times 60 \times 30$ cm, was made entirely
from lexan (a polycarbonate plastic). The bottom plate was made from 1” thick
double-laminate lexan and was screwed to the side walls which were all 1/2” thick.
These joints were bonded together with bonding glue and were further sealed with
silicone sealant. All gate pieces where also made from 1/2” lexan. The lid was
made of two 1/2” lexan pieces which were bolted to an aluminum L-bracket frame
surrounding the lock-exchange tank. A neoprene gasket at the top of the side walls
of the lock-exchange tank helped to prevent leaks at the joint with the lid. A dexeon
superstructure attached to the aluminum frame held the digital camera and lighting
was provided from the sides of the tank by four 250W halogen lights also attached to
the aluminum frame.
Figure 3.1: Schematic of the lock-exchange tank.
Figure 3.2: Planwise view of the lock-exchange tank.
Figure 3.3: Schematic of the gate showing the three pieces in expanded view. A central, moving gate piece is held in place by two reinforced track pieces. When open the holes line up and a channel is opened between the lock and the tank.

3.3 Moving Bottom Plate

To compare the results of our experimental investigation of the lobe-cleft pattern with the theoretical results of Härtel et al. [4] the leading edge of the gravity current needed to initially be in an unperturbed state. This smooth initial condition was not easily attainable as there was an appreciable amount of stirring caused by raising the gate. For this reason the front was smoothed after its release using the following scheme.

Following the example of Simpson [5] a moveable bottom plate was constructed. The main idea behind the moveable bottom plate was to produce an artificial free-slip bottom boundary condition by moving the bottom plate at, or slightly above, the velocity of the intruding current. This process of moving the bottom boundary effectively damped out the lobe-cleft instability by clamping the leading edge of the gravity current to the bottom plate thus eliminating the unstable density inversion in which the lighter ambient fluid was over-ridden by the denser, intrusive gravity current.

The moving bottom plate, as shown in Fig 3.4, was realized with a set of lexan tracks with a machined groove running down either side of the length of the tank. A cart, with two wheels riding in these groves, was then pulled down the length of the tank using a fishing line fed through a pulley to a DC motor mounted outside of the tank. By varying the voltage applied to the DC motor the speed at which the
Figure 3.4: Schematic of the moving bottom plate. The bottom plate slides on a pair of guide wheels fitted into a set of machined tracks. A DC motor is used to wind a fishing line through a pulley to pull the moving bottom plate across the floor of the tank.

The bottom plate was pulled along the bottom of the tank could be crudely controlled over the course of the experiment. Because the moveable bottom plate was only used to initialize the gravity current by damping out the lobe-cleft instability the velocity of the bottom plate was neither recorded nor feedback controlled.

### 3.4 Visualization

Visualization of the intruding gravity currents was performed by taking images of the opaque fronts from above using a digital camera as shown in Fig 3.5. Lighting was provided by four 250W halogen lights positioned at the four corners of the tank to provide uniform lighting throughout the lock-exchange tank. These lights were angled obliquely into the tank from the sides to ensure that the base image was entirely black. A piece of black cloth was placed beneath the tank to provide a dark background against which the pictures where taken. This ensured a high contrast between the intruding gravity current and the background. Enhancing the contrast of the intruding gravity current with the background image through physical means proved to be one of the simplest ways of increasing the effectiveness of the image processing routine.
A Dalsa camera took digital pictures of the intruding gravity current. It was a non-interlaced, 512 × 512 pixel, black and white digital camera. A wide angle lens was used to image a 55 × 55 cm section of the floor of the lock-exchange tank. Image warping due to the wide angle lens was not significant. This camera was controlled using a QNX machine (Unix-like PC) running in-house software which enabled the user to take multiple pictures at pre-defined intervals. These images were then stored on the hard disk and stamped with both their sequence number and the time at which they were taken. This time was used in data analysis to evaluate the front speed of the current.

3.5 Experimental Procedure

The experimental procedure was developed to ensure the reproducibility of results and to obtain the smoothest possible initial conditions. The experimental procedure was comprised of two steps: the filling and preparation of the tank and the release and recording of a gravity current event.

A typical data run was started by closing and sealing the gate and subsequently filling the tank end of the lock-exchange tank. Filling the tank section first through
the hose connection in the tank lid ensured that any leaks through the gate would be from the tank section to the lock section. To produce a known density contrast needed to initiate a gravity current, titanium dioxide powder was weighed and added to a small quantity of tap water and placed in the lock section. The lid of the lock section was then screwed into place and the lock section was filled with tap water. Cold tap water was used to fill both the tank and the lock to minimize density differences due to temperature variation within the lock-exchange tank. In addition, the filled lock-exchange tank was allowed to sit for several minutes to minimize the presence of large scale flows initiated during the filling procedure. While this wait time was not long enough for all residual currents to die out, any residual currents at the time of the experiments where small enough that they did not have an appreciable effect on the intruding gravity currents. Longer wait times were not feasible since titanium dioxide sediments out on a time scale of around one hour. This effect was neglected in the gravity current flows as they occur on a much shorter time scale than the sedimentation.
Chapter 4

Analysis

4.1 Introduction

The results of the experimental gravity currents were analyzed as follows. The leading edge of the intruding gravity current was extracted from the raw images and compiled into a time-evolution diagram. This diagram was then used to determine the average front velocity, the height of the gravity current $h$ and its Grashof number $Gr$. Determination of the dominant wavelength of the flow was done both manually and automatically and the results were made non-dimensional by the height of the gravity current. The dominant wavenumber and associated Grashof number were then compared to the linear stability analysis of Härtel et al. as discussed in chapter 5.

4.2 Edge Detection

Data processing of the raw gravity current images (seen at the bottom of Fig 4.1) was done with Matlab’s image processing toolbox. For each data run a series of tiff format images were produced showing the gravity current as a lightly colored intrusion flowing across an otherwise black background. A Matlab program was written to read the tiff images produced in a data run from their folder and produce a Matlab cell structure containing the time of the image, and a vector of points at the leading edge of the current. Figure 4.1 shows the results of the edge detection program produced from a series of images taken during a data run. The edges produced from these images, shown below, are highlighted in red in the time-evolution diagram.

The edge detection program first reads the tiff images from the input directory. The first of these images is defined as the reference image and ideally contains an
image of the black background just prior to the intrusion of the gravity current. Each subsequent image of the intruding gravity current is processed by first subtracting the reference image using Matlab’s imsubtract function. This process eliminated any systematic image structures leaving only those changing in time. Matlab’s medfilt2 function was then used to assign the median value of a surrounding square of 5x5 input pixels to each output pixel in order to smooth out some of the high frequency noise inherent in the images. The intensity spectrum of the resulting image was shifted using the imadjust function thus clipping the dark end of the intensity spectrum and skewing the rest towards the bright end. This process of re-adjusting the intensity spectrum greatly enhances contrast and greatly refines the output of the Matlab’s edge function. The edge function used the Sobel approximation of the derivative of intensity to determine the position of the leading edge. This function outputs a binary matrix of equal size to the input image depicting the location of edges within the original image. This edge information was compiled into a cell structure containing the time at which the picture was taken and a vector containing the position of the intruding gravity current. As a final image processing step only the foremost detected edge was used in any time step to avoid multiple definitions of the leading edge as well as noise contamination from the turbulent billows following the gravity current head.
4.3 Current Velocity Measurements

Processing of the experimental gravity current data began with the determination of the average front velocity of the gravity current. The time-evolution diagrams produced by the edge detection algorithm shown in Fig 4.1 plot the outline of the front taken at regular intervals. By averaging the position along this outline for each time slice a graph of average position versus time can be constructed as shown in Fig 4.2. Fitting a straight line to this graph thus gives a measure of the average front velocity.
velocity of the gravity current. Measurements of average front velocity versus the density difference are shown in Fig 4.3. Note that there is a strong trend in the front velocity versus density data.
4.4 Determination of Dimensionless Parameters

The determination of dimensionless parameters is a process which is inherently ambiguous. While some fluid dynamics problems possess certain obvious characteristic length scales, others do not. However, analysis of phenomenon with similar, but different, physics requires a method of comparing various experiments and theories. To this end most theoretical and experimental investigations non-dimensionalize the parameters governing these phenomenon in an effort to map all results onto one another. In this way, various theoretical and experimental results can be quantitatively compared.

Intruding gravity currents are parameterized by the dimensionless Grashof and Froude numbers defined in equation 2.8 and equation 2.11 respectively. To estimate values of these quantities the density difference, $\Delta \rho$, and average front velocity, $\bar{u}_f$, were measured while the gravitational acceleration, $\bar{g}$, and viscosity, $\bar{\nu}$, were known.

Figure 4.3: Plot of the measured, non-dimensionalized density difference vs. the average front velocity for all runs.
quantities. Hartel et al. [3] conducted a series of numerical experiments on gravity currents in which they determined the variation of the Froude number with the Grashof number. Using these theoretical results the characteristic height $h$ was evaluated by fitting the experimental results to a first order fit of the computational Fr versus Gr curve as shown in Fig 4.4. The scaled heights produced with these results provide reasonable estimates of the actual gravity current height. As shown in Fig 4.5 the median current height is 81mm with a standard deviation of 5mm.

In the scaling of the gravity currents the characteristic length scale was the mean height of the current formulated by fitting the known data to the theoretical model of Hartel et al. While this provides a means of comparison, differences in tank geometries may effect scaling of this parameter. Most importantly, the fact that Hartel et al. use a theoretical tank geometry where each fluid fills half the tank may effect the precise scaling of $h$. However, this ambiguity is somewhat accounted for by the fact that for a large range of Grashof numbers the theoretical predictions of Hartel et al. follow lines of constant $h$ as seen in Fig. 4.6.
Figure 4.5: Gravity current height vs. index using Härtel et al. [3] theoretical result to solve for the gravity current height.
Figure 4.6: Plot of wavenumber vs. Grashof number. Theoretical plot from Härtel et al. as well as lines of constant $\bar{h}$ are shown.
4.5 Manual Cleft Detection

The moving bottom smoothing technique discussed in section 3.3 produced a two dimensional gravity current which was drastically stretched. Initially, as the bottom plate came to rest, the nose of the intruding current was pinched to the moving bottom plate and formed an elongated wedge behind. This profile was unstable to a series of lobes with long, protruding clefts seen faintly in Fig 4.7. Because this initial pattern was remarkably faint it was not detected using the automated cleft detection software discussed in section 4.6. For this reason, this initial pattern was characterized by manually locating the clefts and taking the average distance between them as the dominant wavelength. Results of this analysis are discussed in chapter 5.
Figure 4.7: Initial pattern of lobe and clefts after moving floor is stopped and the instability is allowed to grow.
Figure 4.8: Clefts are picked by looking at time series of five points (here denoted $x_1$, $x_2$, $x_3$, $x_4$, $x_5$). Two averages are constructed from the outliners: $a_{12} = (x_1 + x_2)/2$ and $a_{45} = (x_4 + x_5)/2$. A cleft at location $x_3$ must then satisfy the condition that $x_3 < a_{12}$ AND $x_3 < a_{45}$.

4.6 Automated Cleft Detection

4.6.1 The Cleft Detection Routine

To further analyze of the time-evolution diagrams produced by the edge detection algorithm a cleft picking algorithm was developed. Because of the large noise component in the signal, as well as the limited size of the sequence (512 pixels across) a real-space cleft detection algorithm was implemented. A real-space based algorithm was used instead of more traditional Fourier analysis techniques because the gravity current front was noisy and clearly not sinusoidal. The real-space algorithm used as input the fronts picked using the edge detection program. These fronts are first smoothed by convolution with a boxcar of width 10 (see section 4.6.2). The subroutine then picks out clefts from this smoothed time series using the following cleft definition depicted in Fig 4.8. Clefts are defined by the condition where, in a five point series, the middle point is smaller than both the average of the two points to the left and the average of the two points to the right. A sample output of the cleft detection routine is plotted along with the time-evolution diagram used as input in Fig 4.9.
Figure 4.9: Sample output of the cleft picking algorithm. Cleft picks are shown as red stars against the output of the edge detection algorithm for the may26b data set.

4.6.2 Determination of the Convolution Box Size

The automated cleft detection routine first smoothed the front profile produced by the edge detection routine before applying the cleft detection algorithm. This smoothing was achieved by convolving a variable width boxcar, of height unity, with the front profile. To determine the best box car size the mean and median cleft sizes, as well as number of cleft picks versus convolution box size, were plotted as shown in Fig 4.10.

Ideally, for some range of convolution box size, the median and mean cleft sizes, as well as the number of clefts picked, should be relatively stable. This technique presumes that for some range of smoothing box sizes the cleft picking algorithm will only detect actual clefts. At convolution box sizes which are too small the cleft picking algorithm detected noise in the signal, and for overly smoothed fronts the algorithm will miss clefts which have been smoothed out. However, as can be seen from Fig 4.11 this criterion was only marginally effective. Variability between data runs makes the choice of $b$ less than obvious. Thus, a value of $b = 10$ was somewhat arbitrarily
chosen to minimize noise and maximize the signal. From Fig 4.11 it can be seen that this value corresponds to a region of relative, but not absolute, stability. Using these techniques for cleft detection a statistical approach was then used to evaluate the dominant wavelength of the lobe-cleft pattern as discussed in section 5.1.2.
Figure 4.10: The two graphs are guides for determining the best convolution box size in the process of smoothing the front profile. The graph at top shows mean and median cleft size vs. convolution box size, and the graph at bottom shows the number of clefts picked vs. the convolution box size. Data analyzed in both graphs is from the may26b data set.
Figure 4.11: Each graph shows the result of the cleft picking algorithm for varying convolution box sizes. From top to bottom these convolution box sizes are $b = 4, 10, 30$. 
Chapter 5

Results and Discussion

5.1 Lobe-Cleft Pattern

5.1.1 Qualitative Aspects of the Lobe-Cleft Pattern

Gravity currents exhibit a characteristic pattern of lobes and clefts at their leading edge as can be clearly seen in Fig 5.1. The lobes are the protruding regions of the flow and are separated by sharp cusps called clefts as shown in Fig 5.2. While the fluid flows at the head of the gravity current can be fairly turbulent (see Fig 5.3) the pattern of lobes and clefts is reasonably robust as can be seen in a sampling of the time-evolution diagrams (see Fig 5.4). The stability of the lobe-cleft pattern is readily apparent in the time-evolution diagrams produced by the front detection algorithm. Over time lobes can be seen to grow and to die away. In fact, lobes are observed to grow to a typical maximum size after which they split into a series of smaller lobes through the lobe-cleft instability. These new lobes then similarly grow or decay as the current moves down the tank. Similarly, some lobes can be seen to die away as they are pinched off by the larger lobes surrounding them.

The onset of the lobe-cleft pattern from a uniform initial condition reveals two regimes of the lobe-cleft pattern. At first, a short wavelength pattern develops in the shallow, stretched intruding gravity current as seen in Fig 4.7. Here lobes are seen to be separated by relatively long clefts which run deep into the intruding current. Subsequently this pattern evolves into a longer wavelength pattern of lobes and clefts.
Figure 5.1: Photograph showing the lobe-cleft pattern at the front of the intruding gravity current. The lobe-cleft pattern is shown at a later stage of development far from onset.

Figure 5.2: Photograph showing the cleft structure at the front of the intruding gravity current.
Figure 5.3: Side view of the intruding gravity current showing the turbulent nature of the flow. Notice the presence of the lobe-cleft structure at the front.
Figure 5.4: Time-evolution diagrams for three data runs (from top) mar15b, apr12c, apr14a. The shifting pattern of lobes and clefts can be seen to clearly evolve with time as the gravity current intrudes from left to right.
5.1.2 Statistical Approach to Dominant Wavelength Determination

Comparison of the lobe-cleft pattern and linear stability analysis required a method of determining the wavelength of the pattern of lobes and clefts at the leading edge of the gravity current. However, because of the highly turbulent nature of the flow and the difficulty in producing exactingly precise initial conditions this pattern was rarely perfect. Even when a well ordered pattern did develop it was not sinusoidal because of highly developed non-linearities. In addition, size constrictions limited the length of the data series used to evaluate the dominant wavelength. For these reasons the method of Fourier analysis proved ineffectual in analyzing the pattern at the front of the advancing gravity current. Instead real-space methods were used to identify the location of the clefts in the advancing current as described in section 4.6.1.

A further complication in the picking of clefts arose from the difference in image quality and spatial distribution of clefts between the initial pattern of lobes and clefts to the fully developed phenomenon. For this reason the lobes and clefts shown in Fig 4.7 were analyzed manually, and the subsequent evolution of lobes and clefts was then studied with the algorithms described in section 4.6.

The output of the cleft detection routine is a series of space-time points indicating the location of the clefts as the gravity current traverses the length of the lock-exchange tank. A measure of the lobe sizes at each time slice can be achieved by finding the distance between adjacent spatial points at each time slice. While this method produces a direct measure of the lobe sizes present it is highly vulnerable to noise as some lobes may be missed by the routine while others may be falsely picked. However, by compiling a histogram of the lobe sizes over several time slices a histogram with a clear peak can be obtained for sections of the time-evolution diagram as shown in Fig 5.5.

A further complication in the analysis of the dominant wavelength through the use of statistical methods arose from the fact that the histograms produced using the method described above had a non-Gaussian distribution. Because of this fact, an iterative smoothing method was developed to determine the dominant wavenumber in the histograms. In this method a series of histograms were constructed using a large number of histogram box widths. For each histogram the lobe size corresponding to the maximum value of the histogram was recorded. These lobe widths were then averaged over the series of histograms. A typical plot of dominant lobe size
Figure 5.5: Histogram of lobe sizes for the entire data set may26b. The time-evolution
diagram of data set may26b with the clefts marked is shown in Fig 4.9.

(corresponding to maximum histogram value) versus the number of histogram boxes
(inverse to the histogram box width) is shown in Fig 5.6.
Figure 5.6: Plot of the maximum histogram value vs. number of boxes per histogram is shown for data set may26b. Box size varies inversely with numbers of boxes per histogram with the minimum box size set at the sampling interval.
5.2 Comparison With Linear Stability

Analysis of the data using the methods described in chapter 5 have produced a preliminary comparison with the numerical results of HärTEL et al. as shown in Fig 5.7. All data lies within a range of Grashof numbers from $1 \times 10^6$ and $1 \times 10^7$. The results show the theoretical linear stability analysis of HärTEL et al. [4], the experimental results from Simpson [5] and the experimental results of the current study. The initial instability as reported both in this study and by Simpson lies consistently above the theoretical results of HärTEL et al. However, onset of the classical shifting lobe-cleft structure occurs at wavenumbers significantly lower than those predicted by the linear stability analysis. One possible explanation for this discrepancy is that HärTEL et al. perform a linear stability analysis on a fully developed gravity current profile. Thus the characteristic height in their analysis is the non-dimensionalized height of the fully developed front. In contrast, the initial instability in all experimental techniques surveyed occurs in a front which has been stretched into a thin wedge of intrusive material implying that the scaling height is much smaller than the height in the fully developed case. As the pattern of shifting lobes and clefts then develops the average cross-sectional flow profile becomes similar to the base state of the HärTEL et al. calculation. However, at this point the three-dimensional profile already has developed a high degree of spanwise structure and therefore cannot be directly compared to the onset wavelengths HärTEL et al. predict.
Figure 5.7: Variation of wavenumber vs. Grashof number for the numerical results of Härtil et al. [4] with experimental results of Simpson [5] and those reported in this study.
Chapter 6

Conclusion

An experimental analysis of the lobe-cleft instability at the leading edge of a gravity current was performed. Experimental gravity currents were observed in a $150 \times 60 \times 30$ cm lock-exchange tank and were driven by density differences produced by adding titanium dioxide powder. The leading edge of these currents were observed by lighting the opaque currents from the side with four halogen lights and taking digital images from above. The leading edge was extracted from these images and analyzed using a real space cleft picking algorithm. These programs, in addition to some manual processing of images, allowed a dominant wavelength to be found at an early stage in the development of the gravity current front. For comparison with the stability analysis of Härtel et al. [4] the leading edge of the gravity current was first smoothed with a moveable bottom plate. This bottom plate was pulled in the direction of the intruding current at a velocity greater than, or equal to, the front velocity of the current to first smooth the front. The bottom plate was then stopped and the three-dimensional structure at the leading edge of the gravity current was allowed to develop.

Comparison of the dominant wavelength present in the early stages in the development of a smoothed, experimental gravity current head was made with the linear stability analysis of Härtel et al. [4]. It was found that initially a pattern of small lobes with elongated clefts appeared which subsequently transformed into a longer wavelength, shifting pattern of lobes and clefts. The initial pattern of lobes and clefts had wavenumbers consistently higher than those predicted by Härtel et al. while at the onset of the shifting pattern of lobes and clefts the wavelength was found to be consistently lower. All measurements were made within a range of Grashof numbers from $1 \times 10^6$ and $1 \times 10^7$. 
The transition at onset from wavenumbers above the theoretical predictions of Härtel et al. to ones below may be explained by analyzing the smoothing techniques applied to produce the initial two-dimensional gravity current. As the moving bottom plate smoothes the intruding gravity current it also stretches the profile into a narrow wedge of dense material underlying the lighter ambient fluid. The characteristic height of this fluid is much smaller than that of a fully developed gravity current used as the base state in Härtel’s linear stability analysis thus producing higher onset wavenumbers. As the flow profile develops the wavenumbers decrease and broaden into the shifting lobe-cleft regime. A comprehensive comparison with the results of Härtel et al. would thus require either more realistic initial conditions in the linear stability analysis or an experimental method of smoothing the front which maintained a closer approximation of the fully developed flow profile.
Bibliography


