Supersymmetric Standard Models in String Theory

(a) Spectrum

(b) Couplings

(c) Moduli stabilisation

Type II side [toroidal orientifolds]- brief summary- status (a),(b)&(c)

Heterotic side [Calabi-Yau compactification] – NEW

(a) globally consistent MSSM construction **Bouchard&Donagi hep-th/0512149**

(b) Coupling calculations&implications **w/Bouchard&Donagi hep-th/0602096**

Type II Side

I.Type IIA - based on toroidal orbifold constructions
w/ intersecting D6-branes;
a wealth of three-family supersymmetric Standard-like Models

non-Abelian gauge symmetry, chiral matter, family replication
& supersymmetry - geometric origin!

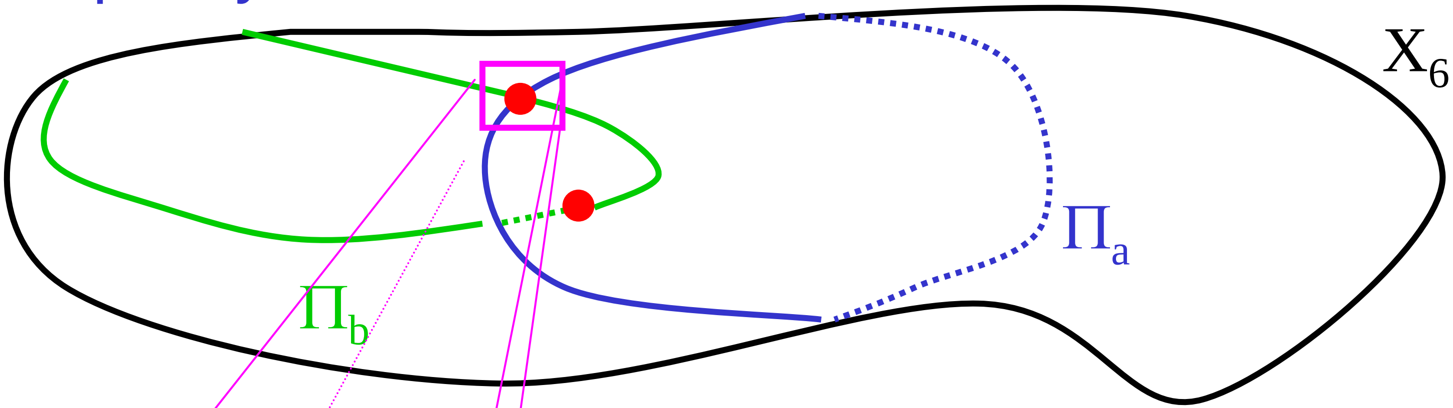
Other Type II constructions w/ SM structure:

Type II Gepner (RCFT) constructions
T.Dijkstra,L.Huiszoon&A.Schellekens,hep-th/0403196,0411126

Local Type IIB construction at obifold/orientifold singularity
H. Verlinde&M. Winjholt, hep-th/0508089 (c.f., Herman's talk)

D-branes& Massless Matter ➡ Intersecting D6-branes

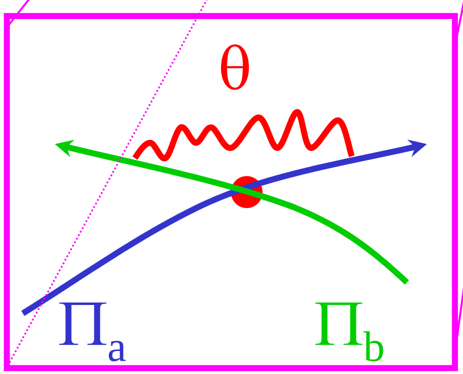
wrap 3-cycles Π



In internal space intersect at points:

Number of intersections $[\Pi_a] \circ [\Pi_b]$ - topological number

Geometric origin of family replications!

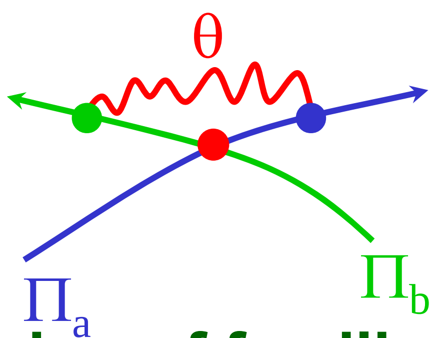


Berkooz, Douglas & Leigh ’96

At each intersection-massless 4d fermion ψ

Geometric origin of chirality!

Engineering of Standard Model



N_a- D6-branes wrapping Π_a

N_b- D6-branes wrapping Π_b

$$U(N_a) \times U(N_b)$$

$\Psi \sim (N_a, \bar{N}_b)$ - $[\Pi_a]°[\Pi_b]$ – number of families

$N_a = 3$, $N_b = 2$, $[\Pi_a]°[\Pi_b] = 3$

$$U(3)_C \times U(2)_L$$

$\Psi \sim (3, 2)$ - 3 copies of left-handed quarks

Bumenhagen et al.'00;Aldazabal et al.'00 (c.f., G. Shiu's talk)

Global consistency conditions (D6-brane charge conserv. in internal space)

& supersymmetry conditions (constraining!)

⬇

Building Blocks of Supersymmetric Standard Model

i) Supersymmetric Standard Model Constructions

primarily on Z_2xZ_2 orientifolds (CFT techniques)

a) FIRST STANDARD MODEL (1) w/G. Shiu & A. Uranga'01

branes wrap special cycles

b) MORE STANDARD MODELS (4)

branes wrap more general cycles (better models) w/I. Papadimitriou'03

c) SYSTEMATIC SEARCH FOR STANDARD MODELS (11)

based on left-right symmetric models-2 models very close to minimal SM

w/T. Li and T. Liu hep-th/0403061

d) NEW TECHNICAL DEVELOPMENTS-MORE MODELS (3)

Analysis of brane splittings/electroweak branes || w/ orientifold planes

w/P. Langacker, T. Li & T.Liu hep- th/0407178

F.Marchesano&G. Shiu hep-th/0408091

e) NEW TECHNICAL DEVELOPMENS (rigid cycles) - MORE MODELS (5)

Branes on rigid cycles w/R.Blumenhagen, F.Marchesano and G.Shiu, hep-th/0502095

w/T. Liu,work in progress

. . . .

(f) Other orientifolds: Z_4 (1) Blumenhagen, Görlich & Ott'03; Z_6 (1) Honecker & Ott '04

ii) Calculation of couplings

Yukawa couplings – fermion masses w/I. Papadimitriou'03

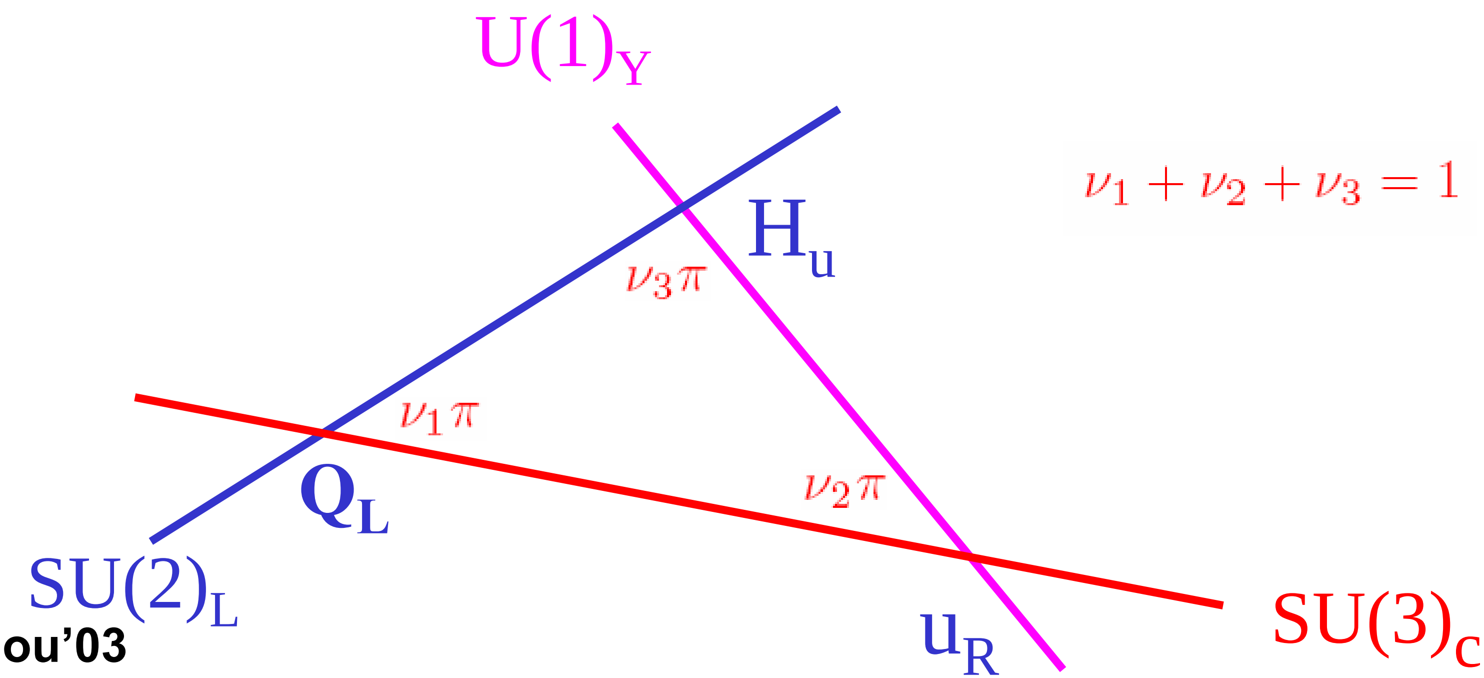
Higer order coupling Klebanov&Witten'03; Abel&Owen'03; w/R. Richter, in preparation

iii) Landscape analysis (one in 10^9) Blumenhagen,Gmeiner,Honecker&Lüst,hep-th/0510170

Z_2 x Z_2

Yukawa Couplings

Intersections in internal space (schematic on i^th^-two-torus)



$$\nu_1 + \nu_2 + \nu_3 = 1$$

w/Papadimitriou'03

(Conformal Field Theory Techniques)

$$Y = (2\pi)^{\frac{3}{2}} g_{st} \prod_{i=1}^{3} \left[\frac{\Gamma(1-\nu_1^i)\Gamma(1-\nu_2^i)\Gamma(1-\nu_3^i)}{\Gamma(\nu_1^i)\Gamma(\nu_2^i)\Gamma(\nu_3^I)} \right]^{\frac{1}{4}} \sum_I \exp\left(-\frac{A_I^1 + A_I^2 + A_I^3}{2\pi\alpha'}\right)$$

quantum part

Kähler potantial

classical part A_I^i **-triangle areas on i^th^ two-torus lattice**

Cremades Marchesano &Ibáñez '03 - detailed study

Features of explicit constructions on toroidal orbifolds

Typically:

(a) more than one Higgs doublet pairs

(b) chiral exotics (due to intersections of observable branes w/ ``hidden'' ones)

(c) couplings-realistic fermion masses ?

(d) some combination of toroidal moduli fixed by supersymmetry,

but open-sector brane-splitting and brane-recombination moduli NOT

↓

Moduli Stabilization

I. Additional (``hidden'') D-brane sectors- Strong Gauge Dynamics
- gaugino condensation/can break supersymmetry and fix closed sector (toroidal) moduli w/Langacker&Wang'03

Demonstrate for the original 3-family SM explicit construction (AdS vacuum)

↕ Related - DUAL

II. Gravity Fluxes

Type IIA

I Classification of N=1 supergravity vacua (in progress)

No example w/mild back-reaction, i.e. conformal to Calabi Yau

Nevertheless, w/SU(3) structures constrained solution of AdS_4 flux vacua w/internal spaces nearly Kähler & warp factor and dilaton fixed

Example: coset space $[SU(3)]^3/SU(2) \sim S^3 \times S^3$ allows for non-Abelian chiral intersecting D6-brane sector

w/K. Behrndt, hep-th/0308045, 0403049, 0407163
Lüst&Tsimpis, hep-th/0412250

II 4-dim Superpotential Calculation

Explicit for toroidal orbifolds w/``metric'' fluxes turned on-``twisted tori''

Derendinger,Kounnas,Petropoulos&Zwirner,hep-th/0411276,0503229

Carmara,Font&Ibáñez,hep-th/0506055

Typically leads to AdS_4 flux vacua

Type IIB:

Classification of supersymmetric flux vacua more advanced

Examples of vacua w/ mild back-reaction-conformal to Calabi Yau, (self-dual (2,1) $G_{(3)}$-form)
thus in principle allowing for a combined study of chiral D-brane gauge dynamics & moduli stabilization

Type IIB - examples of SM (magnetized D-branes) with fluxes

(Z_2 x Z_2 orientifolds) **Marchesano&Shiu hep-th/0408058,04091**

w/T.Liu/hep-th/0409032, w/T.Li&T.Liu, hep-th/0501041

Order of 20 classes of models; up to 3-units of quantized fluxes; examples of

All toroidal& open-sector moduli stabilized

Summary –Type II side

(a) Major progress (Type IIA): development of techniques for constructions on toroidal orbifolds w/intersecting D6-branes
SPECTRUM & COUPLINGS-geometric; systematic searches

(b) FLUX COMPACTIFICATON w/ SM (Type IIB)
Sizable number of semi-realistic models (on the order of 20 classes)

(c) Models not fully realistic:
typically some exotic matter; couplings not fully realistic;
only open sector& toroidal moduli stabilized
(hierarchy for SUSY breaking fluxes ?)

``Shortcomings'' possibly an artifact of toroidal orbifold constructions

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Foresee progress: Construction on Calabi-Yau threefolds

Heterotic Side

I. Calabi-Yau compactifications- algebraic geometry

holomorphic slope-stable Vector bundle constructions

Freedman,Morgan&Witten'97; Donagi'97

Large classes of supersymmetric SM-like constructions

Donagi, Ovrut, Pantev&junior collaborators `99-05

New results: globally consistent compactification w/just MSSM spectrum

Massless Spectrum — Bouchard&Donagi, hep-th/0512149

Tri-linear coupling calculation — w/Bouchard&Donagi, hep-th/0602096

II. Orbifold/free-worldsheet fermionic constructions

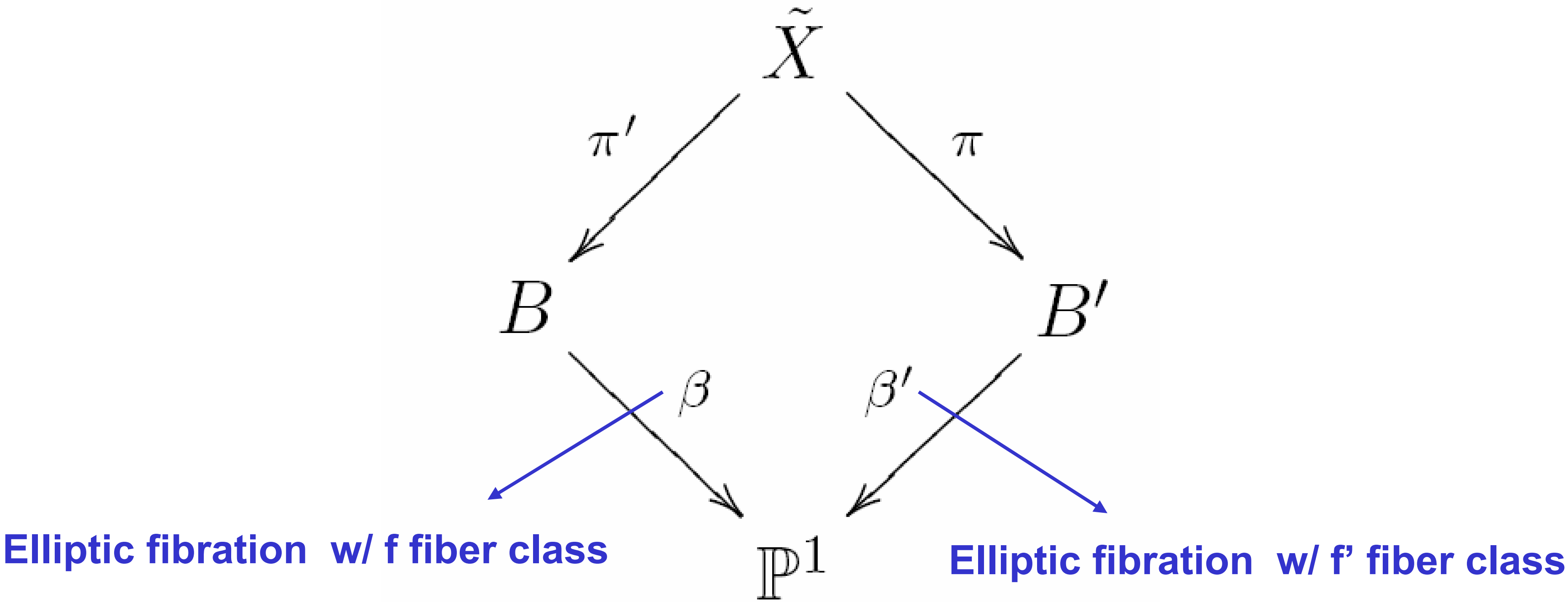
Examples of just MSSM (CFT techniques)

Faraggi et al.`01,Buchmüller et al., hep-th/0512326

No Fluxes!

Summary of the construction:

Calabi-Yau threefold $\tilde{X}$:an elliptic fibration π' over rational elliptic surface B (dP_9)



$\tilde{X}$ - fiber product $B \times_{\mathbb{P}^1} B'$

Free Z_2 action: $\mathbb{Z}_2$ involution $\tau := \tau_B \times_{\mathbb{P}^1} \tau_{B'}$

Z_2 invariant Vector Bundle $\tilde{V}$ = SU(5) vector bundle of (visible) E_8

with an action of the Z_2 involution

Gauge structure SU(5) ➡ $SU(3)_c \times SU(2)_L \times U(1)_Y$

Implementing Z_2 Wilson line

$\tilde{V}^*$ constructed as an extension:

$$0 \longrightarrow V_2 \longrightarrow \tilde{V}^* \longrightarrow V_3 \longrightarrow 0$$

V_2: rank 2 V_3: rank 3

$$V_i = \pi'^* W_i \otimes \pi^* L_i$$

W_i: rank i bundles on B L_i: line bundles on B'

(Fourier-Mukai transforms)

Donagi,Pantev,Ovrut&Waldram'00

Spectral cover construction

(Standard Model) Constraints on

Holomorphic vector bundle:

(a) slope-stable ⟺ ∃* solution (Donaldson,Uhlenbeck&Yau)

YES!

(b) SU(5) rather than U(5) bundle: first Chern class $C_1(\tilde{V})=0$

(c) 3-Chiral Families: third Chern class $C_3(\tilde{V})=-12$ (Euler Characteristic)

(d) Global consistency (Green Schwarz anomaly cancellation):

second Chern classes: $C_2(T\tilde{X})-C_2(\tilde{V})=[W]$ -effective class

$[W]= 2\, f \times pt + 6\, pt \times f'$- Yes! (M5-branes wrapping holomorphic 2-cycles

[or add hidden sector slope-stable bundle U:

$C_2(T\tilde{X})-C_2(\tilde{V})-C_2(U)=0$ (Have not done explicitly)]

Another MSSM construction: Braun,He,Ovrut&Pantev, hep-th/0512177

Based on the same Calabi-Yau threefold, but w/ $Z_3 \times Z_3$ invariant action:

Stable SU(4) vector bundle V w/MSSM spectrum [& additional U(1)]

As as it stands, **not globally consistent**:

second Chern class:

$C_2(TX)-C_2(V)=[W]$ – not an effective class

(i) Requires hidden sector slope-stable bundle U:

$C_2(TX)-C_2(V)-C_2(U)=0$

Currently, no - example of slope-stable bundle U

(previous examples – shown not slope stable Gomez,Lukic&Sols, hep-th/0512205)

(ii) Adding anti-M5branes was proposed: Braun,He&Ovrut, hep-th/0602073

[W]=holomorphic & anti-holomorphic curves ⟹ M5-anti-M5 brane annihilation;

Potential instabilities due to (anti-holomorphic) anti-M5 branes & (holomorphic) vector bundle

Back to globally consistent MSSM:

Massless spectrum (related to zero modes of Dirac operator on Calabi Yau threefolds) - **in terms of cohomology elements:**

$$Spec = \bigoplus_{q=1,3} H^q(X, \mathrm{ad}V)$$

Donagi,He,Ovrut&Reinbacher, hep-th/0411156

Long exact sequences in cohomology

Applied to specific bundle construction: ➡

MSSM w/ no exotics & n=0,1,2 massless Higgs pairs

Multiplicity	Representation	Superfield
$1 = h^3(\tilde{X}, \mathcal{O}_{\tilde{X}})_+$	$(8,1)_0 \oplus (1,3)_0 \oplus (1,1)_0$	$G, W^{\pm}, Z, \gamma$
$3 = h^1(\tilde{X}, \tilde{V}^*)_+$	$(\bar{3},1)_{-4/3} \oplus (1,1)_2$	u, e
$3 = h^1(\tilde{X}, \tilde{V}^*)_-$	$(3,2)_{1/3}$	Q
$0 = h^1(\tilde{X}, \tilde{V})_+$	$(3,1)_{4/3} \oplus (1,1)_{-2}$	exotic
$0 = h^1(\tilde{X}, \tilde{V})_-$	$(\bar{3},2)_{-1/3}$	exotic
$3 = h^1(\tilde{X}, \wedge^2\tilde{V}^*)_+$	$(\bar{3},1)_{2/3}$	d
$3 + n = h^1(\tilde{X}, \wedge^2\tilde{V}^*)_-$	$(1,\bar{2})_{-1}$	$L, \bar{H}$
$0 = h^1(\tilde{X}, \wedge^2\tilde{V})_+$	$(3,1)_{-2/3}$	exotic
$n = h^1(\tilde{X}, \wedge^2\tilde{V})_-$	$(1,2)_1$	H
$62 = h^1(\tilde{X}, \mathrm{ad}\tilde{V})_+$	$(1,1)_0$	ϕ, ν

No exotics

n Higgs pairs

R-handed ν

Table 2: The particle spectrum of the low-energy $SU(3)_C \times SU(2)_L \times U(1)_Y$ theory. Notice that all exotic particles come with 0 multiplicity, and that the spectrum include n copies of Higgs conjugate pairs, where $n = 0, 1, 2$.

+ even ; - odd representation under Z_2 action

Focus on loci in moduli space w/ n=1 and n=2 massless Higgs pairs

Tri-linear superpotential couplings:

$$\lambda_{ijk} \sim \int_X \Omega \wedge \Phi_i \wedge \Phi_j \wedge \Phi_k \,.$$

CY (3,0)-form (0,1)-forms

Classical calculation (triple pairings of co-homology groups):

$$(\mathbf{d}) \quad H^1(\wedge^2 \tilde{V}^*)^{(3,3+n)} \times H^1(\wedge^2 \tilde{V}^*)^{(3,3+n)} \times H^1(\tilde{V}^*)^{(3,3)} \longrightarrow H^3(\wedge^5 \tilde{V}^*)^{(1,0)} \simeq \mathbb{C},$$

$$(\mathbf{u}) \quad H^1(\tilde{V}^*)_+^{(3,0)} \times H^1(\tilde{V}^*)_-^{(0,3)} \times H^1(\wedge^2 \tilde{V})_-^{(0,n)} \longrightarrow H^3(\mathcal{O})^{(1,0)} \simeq \mathbb{C},$$

$$(\mu) \quad H^1(\mathrm{ad}\tilde{V})_+^{(62,0)} \times H^1(\wedge^2 \tilde{V}^*)_-^{(0,3+n)} \times H^1(\wedge^2 \tilde{V})_-^{(0,n)} \longrightarrow H^3(\mathcal{O})^{(1,0)} \simeq \mathbb{C},$$

(d) down-quark, charged lepton couplings, R-parity(Lepton, Baryon)-violating

(u) up-quark couplings

(μ) coupling w/ vector bundle moduli: μ-parameter & neutrino masses

Calculation Involved:

(i) exact spectral sequences, filtration & explicit basis for cohomology elements

(ii) Detailed study of vector bundle moduli space; specifically at n=1,2 loci

(d) Triple Pairing: Down-Sector and R-parity Violating Yukawa Couplings

ZERO! – ranks of cohomology groups- incompatible

$$\lambda_l^{ij} e_i L_j \bar{H} + \lambda_d^{ij} Q_i d_j \bar{H}$$

Charged leptons & down quarks massless

$$\alpha_1^{ijk} L_i L_j e_k + \alpha_2^{ijk} L_i Q_j d_k + \alpha_3^{ijk} u_i d_j d_k$$

R-parity (Lepton & Baryon no.) violating

Terms ABSENT!

(u) Triple Pairing: Up-Sector Yukawa Couplings

Locus w/ n=1 massless Higgs pair:

$$\lambda_u^{ij} Q_i u_j H \qquad \lambda_u = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & 0 \end{pmatrix}$$

Symmetric rank 3 matrix

(function of vector bundle moduli on n=1 locus)

Can obtain realistic mass hierarchy

(not quantitative- physical Yukawa couplings depend on Kähler pot.)

Locus w/ n= 2 massless Higgs pairs: two copies of the matrix above

(μ) Triple Pairing: μ-terms and Neutrino Yukawa Couplings

Locus w/ n=1 massless Higgs pair:

Moduli space transverse to n=1 locus 2-dimensional: Φ_1 and Φ_2

Non-zero triple pairing: $\lambda_1 \phi_1 H \bar{H} + \lambda_2 \phi_2 L \bar{H}$.

(a) Small deformation transv to n=1 locus: e.g. < Φ_1> << 1
μ-parameter for the Higgs pair at EW scale (``fine tuning'')

Φ_2- right-handed neutrino & L-lepton doublet ➡ 1 massive neutrino

(b) On n=1 locus, both terms generate masses for 2 neutrinos and

no μ-parameter

Locus w/ n=2 massless Higgs pair: Moduli space transverse to n=1 locus

6-dimensional: Φ_{ij} (i=1,2.3: i=1.2)

Non-zero triple pairing:

$$W = \sum_{i=1}^{3} \sum_{j=1}^{2} \lambda_{ij}\, \phi_{ij}\, L_i\, \bar{H}_j$$

Can generate μ-parameters and/or up to 3 neutrino masses !

Conclusions:

An Heterotic MSSM passed crucial tests at the classical level of couplings:

(a) Up-quark sector: rank 3 matrix –possible realistic mass hierarchy

(b) Down-quark&charged lepton sector -massless

(c) R-parity (L&B) violating couplings –absent (proton stable)

(d) Vector bundle moduli (transverse to n=1,2 massless Higgs pair locus):

Can generate μ-parameters (non-zero VEV's) and/or play a role of right-handed neutrinos with up to 3 Dirac neutrino massive

Further test at quantum (worldsheet instanton) level:

Masses for down-quark&charged lepton sector

Absence of R-parity violating couplings may impose constraints on

Vector bundle moduli space

Is this THE model?

One of a large number with (semi)-realistic features - tip of the iceberg

Expect many more constructions on Calabi Yau threefolds both on Heterotic & Type II side

(employing algebraic geometry & CFT techniques)