Linear Confinement from AdS/QCD

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Excited Rho Mesons

(from PARTICLE DATA BOOK)
Excited Rho Mesons

\[ m_n^2 \gg n \]

As expected for a vibrating open string.

So how does a string theory of QCD deal with this piece of data?

\[ m_n^2 \gg n \] is a general feature of \textit{almost} any confining large N gauge theory.
Linear Confinement

\[ E \sim \sigma L \]

via semiclassical quantization:

\[ \int pdq \sim EL \sim \frac{E^2}{\sigma} \sim n \]

implies:

\[ m^2 \sim \sigma n \]
In real QCD we do not expect rho mesons to exist with asymptotically high $n$.

Due to the presence of fundamental quarks the string can snap.

However at large $N$ snapping is suppressed by $1/N$ and therefore all mesons are stable. Rho mesons exist as stable particles for all $n$. 

In the real world strings snap!
High Spin and Regge trajectories.

A similar argument is very familiar for high spin mesons.

$$m_S^2 \gg S$$

\[ M^2, \text{GeV}^2 \]

rho mesons

Regge Trajectories
Asymptotic mass formula for highly spinning and/or highly excited mesons

\[ m_{n,S}^2 \sim \sigma(n + S) \]

Thickness of highly excited fluxtube is negligible. String is described by Nambu-Goto action. Vibration and rotation frequency identical.

Universal property of almost any large N confining gauge theory!
What kind of confining large $N$ theory can show different behavior?

Confining gauge theory governed by scale of mass gap $\Lambda$; meson masses are of order $\Lambda$

Typically string tension $\sigma \gg \Lambda^2$

Introduce a large parameter $\lambda \gg 1$ such that

$$\sigma \sim \sqrt{\lambda} \Lambda^2$$
What kind of confining large N theory can show different behavior?

\[ \sigma \sim \sqrt{\lambda} \Lambda^2 \quad \lambda \gg 1 \]

Meson masses still of order \( \Lambda \).
Stringy behavior (Linear Confinement and Regge Physics) only set in for

\[ m^2 \sim \sigma \gg \Lambda \]

Theory not stringy at all! Typically:

\[ m_n^2 \sim n^2 \quad \text{instead of} \quad m_n^2 \sim n \]
Linear Confinement and AdS/CFT

- Unfortunately this includes ALL field theories which have a gravity dual.
- These theories are just not string theories, neither on the field theory side nor on the “string theory” side. This is what made the gravity dual tractable.
- The universal mass formula provides a window into a property of a large class of field theories with a string theory dual.

\[ m_{n,S}^2 \sim \sigma (n + S) \]
Very heavy mesons are stringy!

- Using flavor brane heavy mesons with $m \sim \sigma$ can be represented by semiclassical strings.
- Regge behavior from spinning strings.
- No vibrating string solutions known.
- In any case, huge hierarchy between string and gravity modes !!!!
Bulk dual of a QCD like theory?

New approach is necessary!

a) construct string theory on highly curved AdS spacetime with RR-flux.

b) Assume such a theory exists. Assume it has an effective local description even though $R \sim \ell_s$
A local 5d dual of QCD?

The local bulk field theory should be thought of as the corresponding string field theory, fields of arbitrary spin are included as elementary fields.

Such a description has been postulated as a theorem before!

Such a description gives excellent (~ 10%) agreement with QCD data!
BY MAKING A KALUZA-KLEIN EXPANSION ON $S^5$, WE CAN CONVERT THIS TO A LOCAL FIELD THEORY ON $A{DSS}_5$ (WITH INFINITELY MANY FIELDS OF INCREASING MASS)

HOWEVER, THE LARGE N LIMIT AT FIXED $g^2 N$ ALWAYS FACTORIZES SO ACCORDING TO THE "THEOREM" AT ANY $g^2 N$ THERE IS A LOCAL FIELD THEORY ON $A{DSS}_5$

L = $\int d^5 x \sqrt{g} (f g^2 N^2 R + \ldots)$

THE NONTRIVIAL STATEMENT HERE IS LOCALITY: EVEN THOUGH WE ARE NOT EXPANDING NEAR $\phi = 0$, THERE SHOULD BE A LOCAL DESCRIPTION FOR $N \to \infty$.

(E. Witten, talk at JHS60)
A local description of zero curvature AdS.

Witten’s goal was to argue that free N=4 SYM is dual to a higher spin gauge theory.

In the free limit the N=4 has many new conserved higher spin currents, the corresponding bulk gauge fields are massless.

The bulk action is uniquely determined, assuming we only allow two derivative terms!
A local 5d model for QCD vector mesons.

Erlich, Katz, Son and Stephanov, hep-ph/0501128

\[ S = \int d^5 x \sqrt{-g} \left( -|D X|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right) \]

- SU(N_f) x SU(N_f) gauge fields + mass=-3 scalar field (D-anti D tachyon) dual to $<\bar{\psi} \psi>$ mass and condensate.
- Background is AdS with hard IR wall.
- Decouples into vector (L+R) modes (rho mesons) and a coupled axial (L-R) + scalar sector (pions + axial mesons)
A local 5d model of QCD vector mesons. (AdS/QCD)

- Quadratic order in action determines masses and decay constants.
- Unlike in other QCD models, all consequences of chiral symmetry are hard wired into the model.
- Current-current 2-point function fixes

\[ g_5^2 = \frac{12\pi^2}{N_c} \]
Meson masses and interactions.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Measured</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\pi$ (in MeV)</td>
<td>139.6 ± 0.0004</td>
<td>141</td>
</tr>
<tr>
<td>$m_\rho$</td>
<td>775.8 ± 0.5</td>
<td>832</td>
</tr>
<tr>
<td>$f_\pi$</td>
<td>92.4 ± 0.35</td>
<td>84</td>
</tr>
</tbody>
</table>

Fixes the three parameters in model:
- **QCD-scale** (position of IR wall)
- 2 Parameters of **flavor brane**
  (mass and **chiral condensate**)
Meson masses and interactions.

But then the model predicts the following:

<table>
<thead>
<tr>
<th>Observable</th>
<th>Measured</th>
<th>Model (In MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{a_1}$</td>
<td>1230 ± 40</td>
<td>1220</td>
</tr>
<tr>
<td>$F_{a_1}^{1/2}$</td>
<td>433 ± 13</td>
<td>440</td>
</tr>
<tr>
<td>$F_{\rho}^{1/2}$</td>
<td>345 ± 8</td>
<td>353</td>
</tr>
<tr>
<td>$g_{\rho \pi \pi}$</td>
<td>6.03 ± 0.7</td>
<td>5.29</td>
</tr>
</tbody>
</table>

Eigenvalues of waveguide modes, normalization and overlap of eigenfunctions.
Meson masses and interactions.

Including higher spin mesons (= massless higher spin gauge fields) one has zero additional inputs and 6 more predictions:

\[ (f_2, f_2 \rightarrow \gamma \gamma, \omega_2, f_4, \rho_5, f_6) \]

(Katz, Lewandowski and Schwartz)

Including strange quark: one more input (strange mass) 5 more predictions:

\[ (f_k, m_k, m_k^*, m_\phi, m_{k1}) \]

(Erlich, Katz, Son and Stephanov)
Simple change of variables brings mode wave equation into Schrödinger form.

- Hard IR Wall
- $1/z^2$ potential
- $m_n^2 \sim n^2$
- $m_S^2 \sim S^2$
Excited rhos and high spin mesons

Hard wall gives the same spectrum as stringy models; **no linear confinement; no Regge**.

In many papers it was argued that this is a **general problem** of AdS/QCD.

However from the picture it is obvious that it is rather a **feature of the IR wall**!
Building a better IR wall.

We know precisely what we need to get linear confinement: Harmonic Oscillator!

\[ V(z) = \frac{1}{z^2} + z^2 \]

The large \( z \) asymptotic form has to be \( V \sim z^2 \)

\[ m_n^2 = 4(n + 1) \]
Building a better IR wall.

In terms of the action we have two options:

\[ S = \int d^5x \, e^{-\Phi} \sqrt{-g} \left( -|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right) \]

Deformed geometry:

\[ e^A = \frac{e^{-z^2}}{z^2} \]

Non-trivial dilaton background on pure AdS:

\[ \Phi = z^2 \]
A better IR wall for spinning mesons?

With the quadratic dilaton on AdS the modes for massless higher spin gauge fields come for free with the desired Regge behavior

$$m_{n,S} = 4(n + S)$$

For large $z$ one can see that mass terms for the spin fields do not change this behavior.

**Quadratic Dilaton is universal IR wall.**
String Theory interpretation?

It is generally believed that the bulk interpretation of confinement is tachyon condensation.

Conjecture:
Closed string tachyon condensation on AdS gives rise to a spatial tachyon profile supporting a quadratic dilaton with the background metric unchanged.

To leading order in $\alpha'$, a linear T with $V = -T^2$ potential naively could support a quadratic dilaton …
Local 5d models (AdS/QCD) successfully reproduce many features of QCD.

Linear Confinement and Regge physics can be built in by proper model of IR wall; choice is unique.

Given that currently 4d Nambu Goto action is used in the Monte Carlo simulations analyzing LHC data (PYTHIA), 5d models should be developed further!
Summary – part II

As long as it is true that large N physics leads to local lagrangians in the bulk we can use generic properties of confining gauge theories as well as QCD DATA to learn about closed string tachyon condensation and maybe other stringy processes on highly curved spaces.