Toward the End of Time

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Goal of this Talk

- To describe the physics of cosmological singularities
- To come closer to an understanding of the origin and fate of time

Based on the following papers:

hep-th/0602???, Daniel Robbins, Emil Martinec + S. S.

hep-th/0601062, Ben Craps, Arvind Rajaraman + S. S.

which further develop the models appearing in:

hep-th/0509204, Daniel Robbins & S. S.

hep-th/0506180, Ben Craps, Erik Verlinde & S. S.

There are two models that we will consider today. Both are generalizations of the original BFSS Matrix model. Both are null cosmologies.

- Matrix Big Bang – description of a light-like linear dilaton in type IIA string theory

I want to describe the leading quantum mechanical effects in both models.

- Null-Brane Matrix model – description of the null-brane “funnel” space-time in M-theory

I Matrix Big Bang

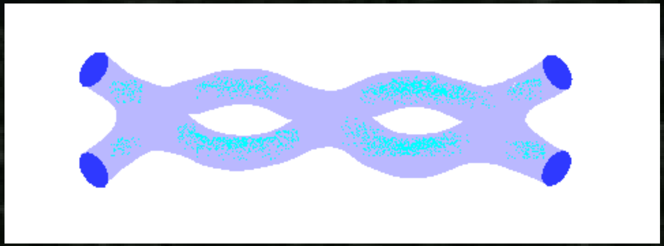
The space-time theory is the light-like linear dilaton.

$$g_s = e^{\phi}; \qquad \phi = -QX^+$$

The string metric is flat while the Einstein metric sees a “crunch”

$$ds_E^2 = e^{QX^+/2} ds_{10}^2.$$

Big Bang



Strong string coupling

Weak coupling

$X^+ = -\infty$

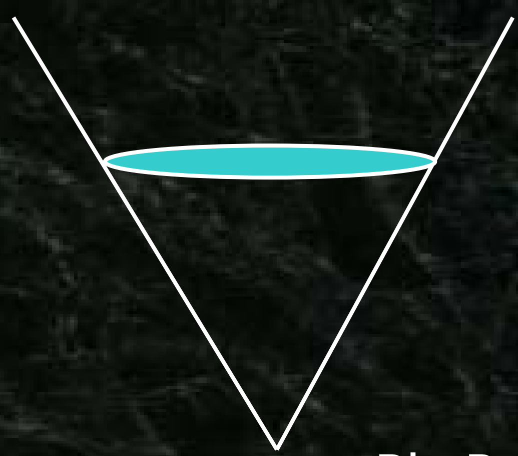
$X^+ \to \infty$

No a priori definition of string theory on this background.

We will use Matrix theory to provide a definition.

On decoupling, we find a non-perturbative definition of this background in terms of Matrix strings on the Milne orbifold,

$$ds^2 = e^{2Q\tau}(-d\tau^2 + d\sigma^2); \qquad \sigma \sim \sigma + 2\pi\ell_s.$$

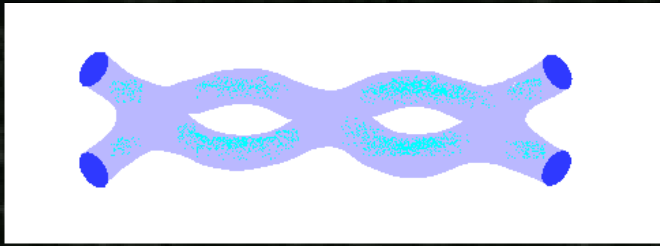


$$S = \frac{\ell_s^2}{2\pi} \int^3 \frac{1}{2}(D_\mu X^i)^2 + \bar{\Psi} D\!\!\!/\, \Psi + e^{-2Q\tau}\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4\pi^2} e^{2Q\tau}[X^i, X^j]^2$$

$$+ \frac{1}{2\pi} e^{Q\tau}\bar{\Psi}\gamma_i [X^i, \Psi] .$$

We identify $\hbar$ with $\frac{1}{\ell_s^2}$.

The value of Q has no invariant physical meaning. It is either zero or non-zero.



Matrix Big Bang

Weakly coupled Yang-Mills

Strongly coupled
Matrix strings

X^{+} = ¡ 1

X^{+} ! 1

RG flow corresponds to time evolution.

The singularity is replaced by a non-abelian gluon phase.

$$g_{Y M} = \frac{1}{g_s \text{`}\ s}$$

Let us examine the leading quantum mechanical effects.

Remarkably little is known about interacting field theories on time-dependent spaces.

We would like to define “effective physics” at low-energies. However, the usual Wilsonian procedure always breaks down at sufficiently early times.

Expand around a vacuum: $\mathrm{Tr}(X)^2$ » b^2.

M_W^2 » $e^{2Q\text{¿}}b^2$

Always below the cut-off for sufficiently early times.

Characteristic break down time: $\tau_{non-abelian} \sim \frac{1}{Q} \ln(\Lambda/b)$

At this time, the theory is non-abelian. We will see that the 1PI effective action does appear to make sense.

On the other hand, perturbation theory breaks down at late times,

$$g_{YM}/b \sim 1; \qquad \tau_{string} \sim \frac{1}{Q} \ln(\ell_s b)$$

At this point, perturbative string theory should take over.

Supersymmetry is broken: does Matrix theory make sense?
Do we have flat directions?

The Milne orbifold is non-Hausdorff. Does this lead to problems in defining field theory? etc.

We will see that there appear to be no problems in defining and studying the quantum mechanics of this theory.

Further, the theory appears to be more effective computationally than we perhaps had a right to expect.

First one can check that integrating out particles with a time-varying mass does not introduce non-localities in the 1PI effective action.

Let’s take that as given.

How do we compute quantum effects?

We will use the description of the Milne space as an orbifold of flat space.

$»^{§} = p\frac{1}{2Q} e^{Q(¿ § ¾)} \longrightarrow ds^{2} = ¡\ 2d»^{+} d»^{i}$

Boost identification: $»^{§} » »^{§} e^{§ 2¼Q`_{s}} :$

In this frame, particles of non-zero spin are not periodic under the identification.

A boost invariant wavefunction satisfies:

$Á_{s}(e^{2¼Q`_{s}} »^{+} ; e^{i\ 2¼Q`_{s}} »^{i}) = e^{2¼Q`_{s} s} Á_{s}(»^{+} ; »^{i}):$

The leading quantum mechanical effect should be a potential for the impact parameter, b.

So we want to compute a time-dependent version of the Coleman-Weinberg potential.

Integrate out quadratic fluctuations. This generates a determinant. For example, for a massive boson we want to evaluate:

$$\det^{(-1/2)}(H) = \det^{(-1/2)}\left(2\frac{\partial^2}{\partial\xi^+\,\partial\xi^-} + b^2\right)$$

$$\det{}^{(-1/2)}(H) = \exp\left(\frac{1}{2}\int d^2\xi \int dt\, \frac{e^{-it(H-i\epsilon)}}{t}(\xi;\xi)\right)$$

Expressed in terms of the heat kernel.

Usually SUSY guarantees that all contributions to the potential vanish but not here!

$$e^{-itH_s}(\xi;\xi) =$$

$$\sim \sum_n \frac{1}{(2\pi)2t} \exp\left(-itb^2 + i\frac{\xi^- \xi^+}{2t}(1 - e^{2\pi Q\beta_s n})(1 - e^{-2\pi Q\beta_s n}) + 2\pi Q\beta_s ns\right)$$

The UV divergences from small t are regularized.

Taking ghosts, fermions, gauge-fields and bosons into account gives a spin-dependent factor:

$$6f(0) - 4f(1/2) - 4f(-1/2) + f(1) + f(-1) = \sum_{n=-\infty}^{\infty} \tanh^2\left(\frac{2\pi Q \ell_s n}{4}\right).$$

where

$$f(s) = -\sum_{n \neq 0} \frac{e^{2\pi Q \ell_s n s}}{(1 - e^{2\pi Q \ell_s n})(1 - e^{-2\pi Q \ell_s n})}.$$

On regulation, we see that this prefactor is positive. The potential tends to lift the flat directions.

Back in the (σ, τ) frame:

$$-\int V_{eff} \approx -\int d\sigma d\tau\, (2Qb)\, e^{Q\tau} \int dt\, \frac{1}{t^2} \exp\left(-\frac{be^{Q\tau}}{2Q}\left(t + \frac{1}{t}\right)\right).$$

The t-integral defines a Bessel function of the second kind,

$K_1(be^{Q\tau}/Q)$.

$$-\int \lim_{\tau \to -\infty} V_{eff} \approx -\int d\sigma d\tau \left(b^2 e^{2Q\tau} \{Q\tau + \ln(\tfrac{b}{2Q})\} + \ldots \right).$$

This makes sense since the theory is becoming free.

$$-\int \lim_{\tau\to+\infty} V_{eff} \sim -\int d\sigma d\tau\, (2Qb) e^{Q\tau} \left(\sqrt{\frac{Q}{b}}\, e^{-\frac{Q\tau}{2}} e^{-\frac{b}{Q} e^{Q\tau}} + \dots \right)$$

Decays superfast at late times!

Supersymmetry and the flat directions are restored at late times. Our usual picture of gravity emerges from Matrix theory.

$$-\int \lim_{\tau\to+\infty} V_{eff} \sim -\int d\sigma d\tau\, (2Q) \left(\sqrt{\frac{Qb}{g_s}}\, e^{-\frac{b}{g_s Q}} + \dots \right)$$

D-brane induced late time potential?

A final comment about general covariance.

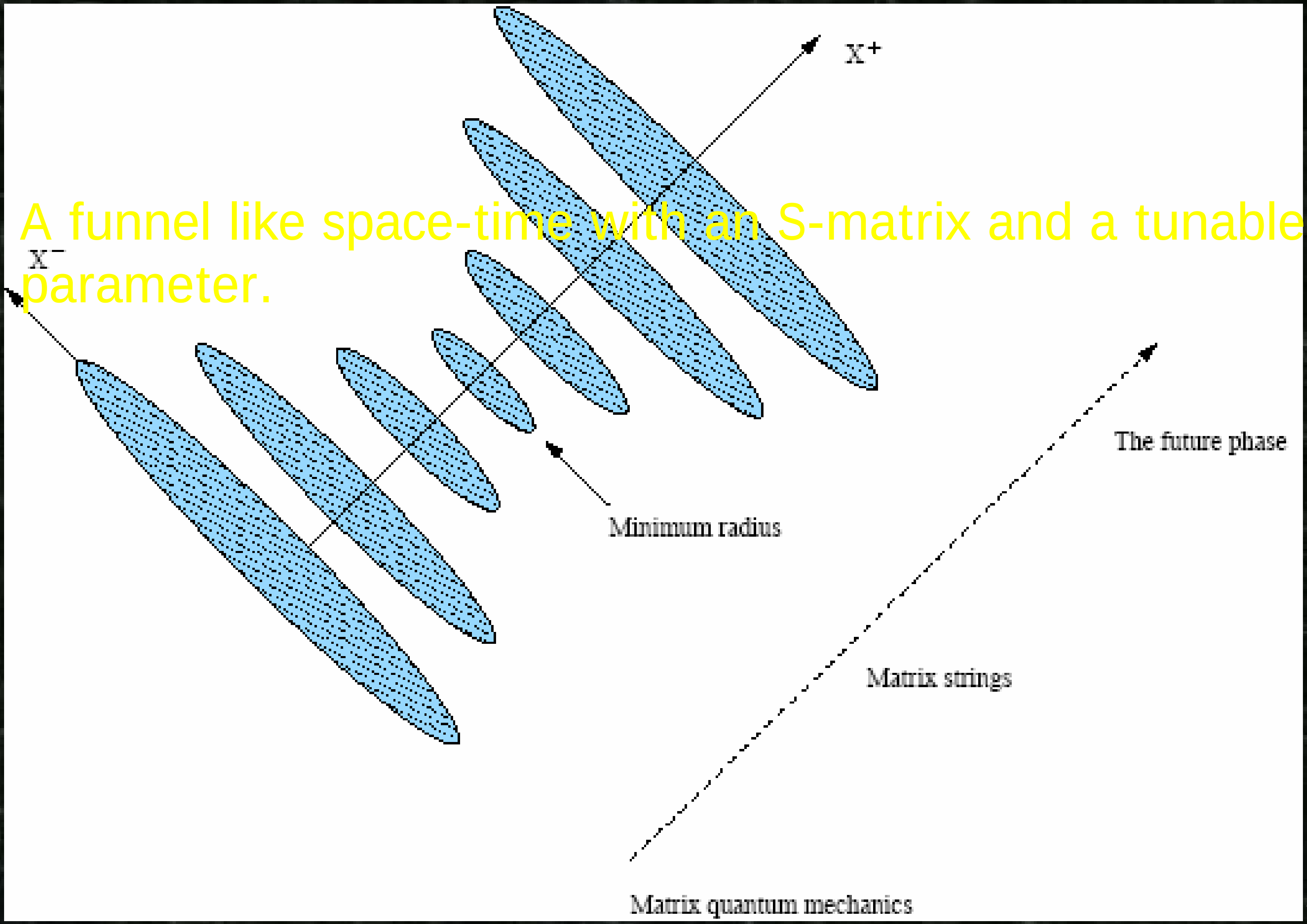
We should be able to couple Matrix strings to gravity without any local anomalies. So the breaking of Poincare invariance is spontaneous.

$$S_{eff} = \int \sqrt{-g}\,\left(\partial\phi\, g^{-1}\,\partial\phi + V^{(0)}(\phi) + R(g)\,V^{(1)}(\phi) + \dots\right);$$

This is not the form of the potential we found!

II Null-brane Matrix Model

A funnel like space-time with an S-matrix and a tunable parameter.



Orbifold description:

$$X = \begin{pmatrix} x^+ \\ x^i \\ x^- \\ z \end{pmatrix} \to g \cdot X = \begin{pmatrix} x^+ \\ x^i + 2\pi\lambda x^- + 2\pi^2\lambda^2 x^+ \\ x^- + 2\pi\lambda x^+ \\ z + 2\pi L \end{pmatrix}.$$

Depends on (λ, L).

Can again compactify $X^i \sim X^i + 2\pi R$,

and consider the Matrix description.

This is the theory of an array of boosted D0-branes.

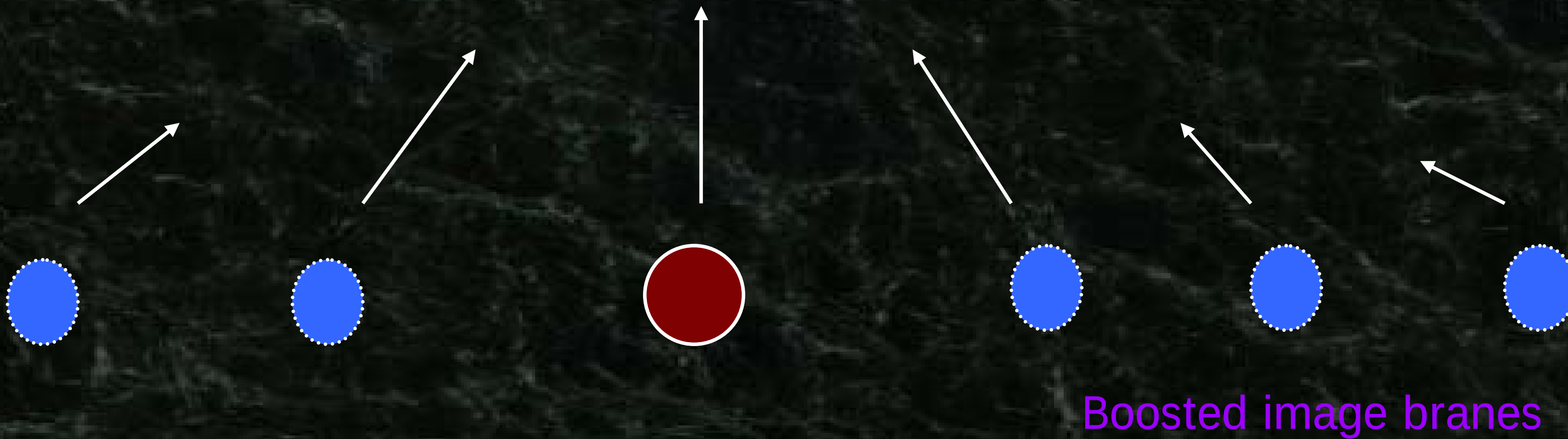
$$S \sim \int d\tau\, d\sigma\, \mathrm{Tr} \Big((D_0 x^i)^2 + (D_0 x)^2 - L^2 (D_1 x^i)^2 - L^2 (D_1 x)^2$$

$$- \sqrt{2}(x - \tau D_0 x) F + (L^2 + \tau^2) F^2 + \tfrac{1}{2}\left[x^i, x^j\right]^2 + \left(\left[x, x^i\right] + \tfrac{-\tau}{\sqrt{2}} D_1 x^i\right)^2 \Big)$$

$\tau = L = 0$ is the crunch.

A new flat direction opens up where sigma excitations become light.

$$(L^2 + \tfrac{1}{2}\tau^2)(D_1 x^i)^2 \to 0.$$



D-branes will interact via their velocity-dependent interaction which will generate a static potential.

Classically, the brane and its images will collapse into a black hole whose size grows with L. (Horowitz & Polchinski)

The potential is computed in much the same way as before except that the operators are nicer.

$$\exp\left(\int d\tau d\sigma V_{\text{eff}}\right) = \det{}^{-1/2}(H) = \det{}^{-1/2}\left(-\partial_\tau^2 - n^2(L^2+\tau^2/2)\right).$$

$$\det{}^{-1/2}(H) = \exp\left(\frac{1}{2}\int d\tau d\sigma \int \frac{dt}{t} e^{-tH}(\tau;\tau)\right)$$

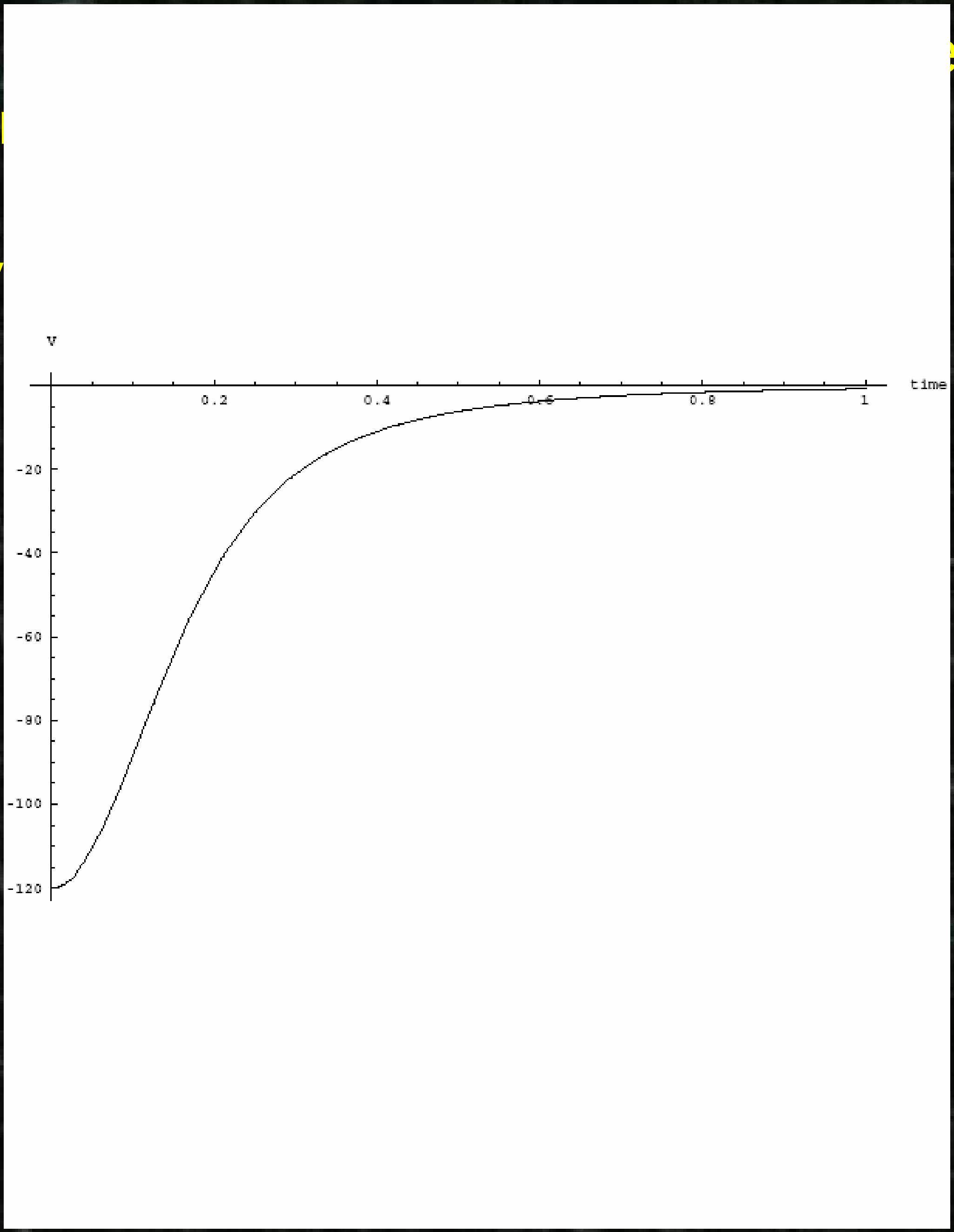
The heat kernel is the SHO kernel given by Mehler's formula:

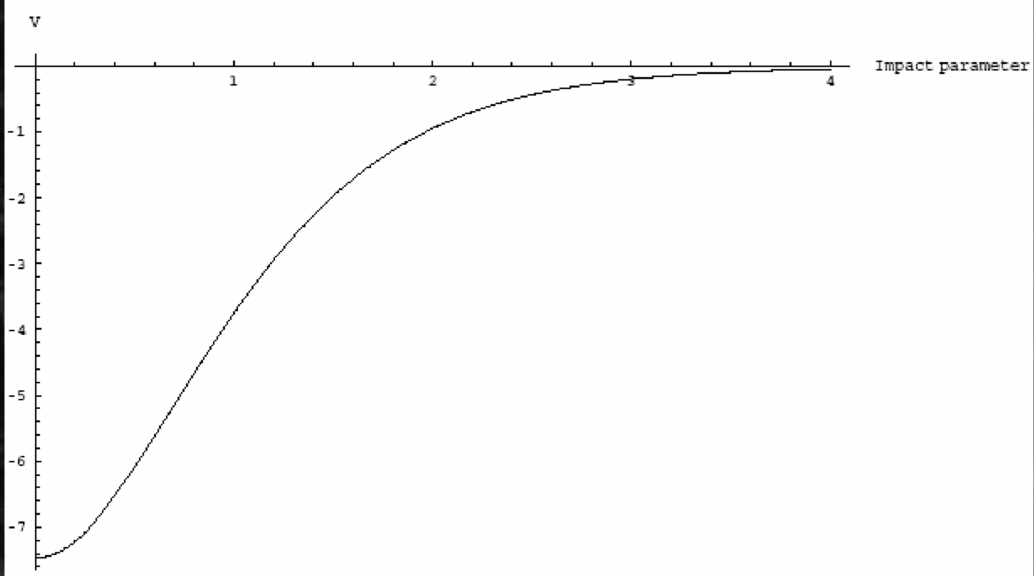
$$e^{-tH}(\tau;\tau) = e^{-n^2L^2t}\left(\frac{-n}{2\pi\sqrt{2}\sinh(\sqrt{2}nt)}\right)^{1/2}\times$$
$$\exp\left[\frac{-n}{\sqrt{2}\sinh(\sqrt{2}nt)}\left(\tau^2(\cosh(\sqrt{2}nt)-1)\right)\right]$$

The potential [partially obscured] ading

velocity-depe[partially obscured]

Schematically




V
Impact parameter
1
2
3
4
-1
-2
-3
-4
-5
-6
-7

The well becomes infinitely deep as L goes to zero and we approach the crunch.

It tentatively appears we never escape from the non-abelian gluon phase and time ends.

I haven't discussed particle production but it is also encoded in the imaginary part of the phase shift.

Holography ⟷ **Field theory on cosmological backgrounds**

Steps toward an understanding of:

- The origin and fate of time
- The definition of string theory in cosmological backgrounds
- A theory of quantum cosmology and initial conditions (recoupling gravity to Matrix theory?)