PHY2208S NONLINEAR OPTICS

Midterm Test
18 February 2011
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Time: Two hours
Permitted aids: Non-programmable calculators, single-sided hand-prepared aid sheet

ANSWER ALL QUESTIONS

[25] 1. Terms and Definitions — Explain each of the following terms, and describe its significance in the context of linear or nonlinear optics. Use formulae wherever appropriate, diagrams if needed, and answers of roughly three or four sentences.

   a) nonlinear optical susceptibility
   b) parametric and nonparametric processes, in nonlinear optics
   c) phase-matching
   d) Kramers-Kronig relation
   e) optical rectification

[25] 2. Electric-Field Induced Second Harmonic — Consider a centrosymmetric and isotropic material (e.g., glass) for which \( \chi^{(3)}(\omega_4; \omega_3, \omega_2, \omega_1) \) is known. In an experimental arrangement shown below, this material is sandwiched between two parallel electrodes, and an intense laser beam is passed parallel to the electrodes.

   a) By applying a large constant (DC) voltage \( V \), some second harmonic generation \((2\omega)\) is observed. Explain how this is possible, given that you showed in your first problem-set that centrosymmetric materials make only odd-order harmonics.

   b) Assuming \( \chi^{(3)} \approx 10^{-22} \text{ m}^2/\text{V}^2 \), estimate the required voltage to produce an effective \( \chi^{(2)}_{\text{eff}} \) equivalent to \( \chi^{(2)} \) of KDP \((\approx 1 \text{ pm/V})\). The electrode spacing is \( d = 5 \text{ mm} \).

   b) In the small-signal regime \((i.e., \text{ when the incident light intensity is very low})\), show that the phase of the transmitted beam is modified by the magnitude of the applied voltage. Explain.
3. Model of optical rectification —

The system at right illustrates one simple model for the nonlinear process of optical rectification:

An electron is constrained to move along a parabolic path \( z = bx^2 \) in the x-z plane. A uniform background electric field \( \vec{E} = a\hat{z} \) acts in the \( \hat{z} \) direction, pulling the electron toward the x-axis. Therefore the electron sees a quadratic potential energy.

Now an electromagnetic field \( \vec{E} = \hat{x}E_0\left\{e^{-i\omega t} + e^{i\omega t}\right\} \) is applied to the system, driving the electron.

a) Though the electron is driven by a field along \( \hat{x} \), a current perpendicular to this, along \( \hat{z} \), will result. Write the equation of force for the electron motion along \( \hat{x} \), and solve it.

b) What is the motion of the electron along \( \hat{z} \)? What is the ratio \( \frac{|p(2)|}{|p(1)|} \)?

c) Write the nonlinear polarization response of the electron, using a tensor notation.

4. Kramers-Kronig relations —

a) In your textbook, arguments are given that the linear susceptibility \( \chi^{(1)}(\omega) \) is analytic in the upper-half complex plane. Starting from this observation, derive the Kramers-Kronig relation for the linear susceptibility:

\[
\chi^{(1)}(\omega) = \frac{-i}{\pi} \int_{-\infty}^{\infty} \frac{\chi^{(1)}(\omega') d\omega'}{\omega' - \omega}.
\]

b) In your first problem set, you considered the two-level approximation of a transition in an atom, and showed that for it the third-order susceptibility \( \chi^{(3)}(\omega) \) does not obey a Kramers-Kronig relation.

\[
\chi^{(3)} = \frac{\alpha_0(0)}{3\omega_{ba}/c} \left[ \frac{\Delta T_2 - i}{(1 + \Delta^2 T_2^2)^2} \right] \frac{1}{|E_{02}|^2}.
\]

Show this again, by explicit calculation as before.