I won’t try to replace the solution assigned to the TA, but some quick notes may be helpful:

Q1: must be proper definitions, not a collection of attributes, properties, i.e., must be the "defining" attributes and properties, which distinguish each thing asked from any other. For each, 3 points for definition and 2 points for significance in context of NLO. Undergrads sometimes say “This is a very important phenomenon in optics”, and that naturally gets no points. I’m looking to see that you understand — precisely — what something is, and how it fits in, why it matters.

_some_ key items:

a) OR: looking to see that “rectified” is made clear — the oscillating nature is removed in OR and a DC component results…

b) DFG: must refer to w1, w2 and w3 = w2 - w1; normally should show a level diagram to illustrate

c) phase-matching: must describe phase relation of propagating wavefronts, and refer to different indices of refraction, must supply an example of how to address obstacle that normal dispersion almost always presents

d) param/non-param: must show understanding of energy conservation or non-conservation in light field field; level diagrams normally are expected, showing virtual and real levels

e) two-photon absorption: usually must reference virtual process, need for two photons to arrive essentially at the same time therefore binary (nonlinear) process; significance could be detector like Si which does not see 1550nm photons, but does see TWO 1550nm photons arriving within H.U.P. time of each other.

Q2: see text §1.4.1, through to equation 1.4.21

Q3:

a) like the GRENOUILLE problem on PS#2, the lens will make a cone of rays which will hit the crystal over a range of different angles, both tangential and sagittal (horizontal and vertical planes) and typically a rough alignment will bring you close enough that one bundle of rays within this cone will meet the phase-matching conditions. If you have it perfect, SHG will exit exactly along the central rays through the lens, which are not deviated. Otherwise, the location on the screen of the SHG will indicate for you what bundle does have the right angle, and thus precisely how to change tilt of the crystal to be perfect for the next shot.

Note that you must be in the geometric limit (ray limit) — far from being focussed — so that different k-vector directions may be well-defined, and identifiable.
b) Type II, so the laser should be polarized at 45 degrees to fast & slow axes. Factors/
issues like frequency selectivity for GRENouille…
Q4 Electron constrained to parabolic motion  
For problems of constrained motion like this, the method of Lagrange's is often very helpful
\[ L = T - V ; \quad \frac{\partial L}{\partial q_i} = \frac{\partial \dot{q}_i}{\partial t} \]  
generalized coordinates \( q_i \)
\[ T = \frac{1}{2} m (\dot{x}^2 + \dot{z}^2) ; \quad \dot{z} = bx^2 \Rightarrow \dot{z} = 2bx \dot{x} \]
\[ = \frac{1}{2} m \dot{x}^2 (1 + 4b^2 \dot{x}^2) \]
\[ V_{\text{field}} = -\int_{x_0}^{x} e(y) \, dy = eax = eax^2 \]
\[ V_{\text{drive}} = -\int_{x_0}^{x} e\dot{x} \, dx = -E_0 2\cos \omega t \cdot \dot{x} = E_0 2\cos \omega t \cdot x \]
\[ L = T - V = \frac{1}{2} m \dot{x}^2 (1 + 4b^2 \dot{x}^2) - eabx^2 - E_0 2\cos \omega t \cdot x \]
\[ \frac{\partial L}{\partial \dot{x}} = m\dot{x} (1 + 4b^2 \dot{x}^2) ; \quad \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = m\dot{x} (1 + 4b^2 \dot{x}^2) + m\ddot{x}(8b^2 \dot{x} \dot{\dot{x}}) \]
\[ \frac{d^2 x}{dx^2} = \frac{1}{2} m\ddot{x} (8b^2 \dot{x}) - 2eab \dot{x} - 2E_0 \cos \omega t \]
\[ = 4m\dot{x}^2 c^2 b^2 - 2eab \dot{x} - 2E_0 \cos \omega t \]

Thus the equation of motion
\[ m\ddot{x} (1 + 4b^2 \dot{x}^2) + 4m\dot{x}^2 c^2 b^2 = -2eab \dot{x} - 2E_0 \cos \omega t \]
\[ m\ddot{x} + 4b^2 \dot{x} \ddot{x} + 4b^2 \dot{x}^2 = -2eab \dot{x} - 2E_0 \cos \omega t \]

* By inspection, the 2nd & 3rd terms are 3rd order (you can use \( x = \lambda x^{(1)} \) + \( \lambda^2 x^{(2)} \) + \( \lambda^3 x^{(3)} \) ... and find explicitly)

To second-order:
\[ x^{(1)}: \quad m\ddot{x}^{(1)} = -2eab \dot{x}^{(1)} - 2E_0 \cos \omega t \]
\[ x^{(2)}: \quad m\ddot{x}^{(2)} = -2eab \dot{x}^{(2)} \]

The first is a driven undamped SDO, which has as solution the non-driven (homogeneous) oscillator plus the particular solution. The second is just the non-driven solution again and adds nothing - not surprising, because this oscillator is symmetric!

\[ x^{(1)}: \quad \ddot{x}^{(1)} + \frac{2eab}{m} \dot{x}^{(1)} = 0 \quad x^{(1)} = A \cos (\omega_0 t + \phi) , \quad \omega_0^2 = \frac{2eab}{m} \]

* Particularly: try \( B \cos (\omega_0 t + \phi) \), then
\[ -\omega^2 B \cos (\omega_0 t + \phi) + \omega_0^2 B \cos (\omega_0 t + \phi) = -2eE_0 \cos \omega t \]
\[ \Rightarrow x = 0 \quad \therefore B = \frac{-2eE_0}{\omega_0^2 - \omega^2} \cos \omega t \]

Thus to second order
\[ x(t) = A \cos (\omega_0 t + \phi) - \frac{2eE_0}{m(\omega_0^2 - \omega^2)} \cos \omega t \]

only the 2nd term is driven by \( E_0 \)
b) along \( z \): \( z = bx^2 \) (constrained)

Take for now \( A = 0 \), then

\[
\begin{align*}
\bar{\rho} = b \left( -\frac{2eE_0}{m(w_0^2-w^2)} \cos \omega t \right)^2 = \frac{4e^2 E_0^2 b}{m^2(w_0^2-w^2)^2} \cos^2 \omega t \\
= \frac{2e^2 E_0^2 b}{m^2(w_0^2-w^2)^2} + \frac{2e^2 E_0^2 b}{m^2(w_0^2-w^2)^2} \cos 2\omega t
\end{align*}
\]

\[
\left| \frac{\rho_y^{(2)}}{\rho_x^{(1)}} \right| = \frac{eE_0 b}{m(w_0^2-w^2)}
\]

There's no \( \rho^{(2)} \) in the \( x \)-direction, only in the \( z \)-direction; there's no \( \rho^{(1)} \) in the \( z \)-direction.

c) \( \rho_x(2w) = \epsilon_0 \chi^{(2)}(2w; w, w) E^2_y(w) \) \( \gamma' \).

\[
\begin{align*}
\rho_y(2w) &= \frac{2e^2 b}{m^2(w_0^2-w^2)} E_0^2 \\
\rho_x(2w) &= 0
\end{align*}
\]

Only applying \( E_x \) here, could write

\[
\bar{\rho}(2w) = \epsilon_0 \left[ \begin{array}{c} 0 \\
\frac{2e^2 b}{m^2 \epsilon_0(w_0^2-w^2)} \end{array} \right] E_x E_x
\]

Or formally like 1.5.26

\[
\left[ \begin{array}{c} \rho_x(2w) \\
\rho_y(2w) \end{array} \right] = 2\epsilon_0 \left[ \begin{array}{ccc} d_{11} & d_{12} & d_{13} \\
d_{21} & d_{22} & d_{23} \end{array} \right] \left[ \begin{array}{c} E_x(w)^2 \\
E_y(w)^2 \\
E_z(w) E_y(w) \end{array} \right] \\
\]

\( d_{11} = \frac{e^2 b}{m^2 \epsilon_0(w_0^2-w^2)} \)

\( d_{12} = \frac{e^2 b}{m^2 \epsilon_0(w_0^2-w^2)} \)

\( d_{21} = \frac{e^2 b}{m^2 \epsilon_0(w_0^2-w^2)} \)

\( d_{22} = \frac{e^2 b}{m^2 \epsilon_0(w_0^2-w^2)} \)

Others not determined

II - ALTERNATIVE solution:

Note: The problem can be solved by forces also; only component along wire accelerates electron.

\[
\bar{F} = -ze \theta
\]

1) Field resolved along wire

\[
\text{Field sin} \theta \quad \text{Field sin} \theta \cos \theta
\]

\[
\text{Field} = -2eE_0 \cos \omega t
\]

1) Field resolved along wire

\[
\text{Field} \cos \theta
\]

2) \( x \)-component of this

\[
\text{Field} \cos \theta \cos \theta
\]

\[
\tan \theta = \frac{dy}{dx} = 2bx \\
\cos \theta = \frac{1}{\sqrt{1+4b^2x^2}} \\
\sin \theta = \frac{2b}{\sqrt{1+4b^2x^2}}
\]
The electron is free to move along the wire but not along \( x \), so the same component analyzers must be used. Note that even constant speed along wire means acceleration in \( x \) & \( z \):

\[
\dot{s}^2 = \dot{s}^2 x^2 + \dot{s}^2 z^2 \quad \Rightarrow \quad \dot{s} = s \sqrt{1-4b^2x^2}
\]

\[
\dot{s} = \frac{s}{\dot{s}} x \sqrt{1+4b^2x^2}
\]

\[
\ddot{s} = \ddot{s} x \sqrt{1+4b^2x^2} + \dot{s} x \frac{1}{2} (1+4b^2x^2)^{-\frac{1}{2}} (8b^2x \dot{s})
\]

\[
(\dot{s})_x = \dot{s} \cos \theta = \dot{s} \frac{1}{\sqrt{1+4b^2x^2}} = \ddot{s} + \frac{4b^2x \dot{s}}{1+4b^2x^2}
\]

Then the force equation

\[
m(\ddot{s})_x = F_{\text{field}} \sin \theta \cos \theta + F_{\text{drive}} \omega \dot{s}
\]

\[
m \ddot{s} + \frac{4mb^2x \dot{s}^2}{(1+4b^2x^2)} = -\frac{ea}{(1+4b^2x^2)} 2b \dot{x} - \frac{2eE_0 \cos \omega t}{(1+4b^2x^2)}
\]

\[
m \ddot{s} (1+4b^2x^2) + 4mb^2x \ddot{s} = -2ea bx - 2eE_0 \cos \omega t
\]

as the Langrange method, but much more likely to make a mistake figuring it all out.