1. Boyd Chapter 2, Question #1
   (I suggest you read Jaanimagi et al., Opt. Lett. 4, pp. 45–47 (1979))
2. Boyd, Chapter 2, Question #4
3. Nonlinear susceptibilities
   Two optical beams $E_1$ and $E_2$ with wavelengths of 1.0 µm and 0.6 µm, respectively, propagate together through a nonlinear material.
   a) Assuming a $\chi^{(2)}$ nonlinearity, what new wavelengths can possibly be generated in this material?

   The above nonlinear material is now replaced with a centrosymmetric material for the remaining part of this problem.
   b) What is the dominant nonlinear susceptibility?
   c) Assuming $\chi^{(3)}$ nonlinearity, what new wavelengths $\lambda_j$ can possibly be generated in this material that simultaneously involve the interaction of both $E_1$ and $E_2$ beams? Write down the corresponding nonlinear polarization $P(\lambda)$ including their $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ terms (ignore Cartesian indices).
   d) If $|E_1| >> |E_2|$, identify the most dominant terms in part (c).
   e) Write down the nonlinear polarization terms associated with self- and cross phase modulation of each beam (identify $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ terms)
   f) Under what condition the simultaneous presence of both beams leads to a nonlinear attenuation (absorption) of both beam? Describe this process, the required energy resonance (use diagrams), and the nature of the complex susceptibility $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ (with respect to part (e)).
   g) Under what condition the simultaneous presence of both beams leads to a nonlinear attenuation (absorption) of one beam (which?) and gain in the other (which?)? Describe this process, the required energy resonance (use diagrams), and the nature of the complex susceptibility $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ (with respect to parts (e) and (f)).

4. Phase sensitivity of parametric amplification (beware possible formalism changes)
   In this problem we consider the effect of the phase $\phi$ in parametric amplification. Assume a strong pump laser $E_2 = E_2 \sin(2\omega t - k_2 z)$ is incident on a $\chi^{(2)}$ crystal, together with a signal field $E_1 = E_1 \sin(\omega t + \phi - k_1 z)$. Furthermore, phase-matching is satisfied, i.e., $k_2 = 2k_1$. Using the complex notation (i.e., $E_i = \text{Re}(E_i e^{i\omega t + \phi_i})$) solve the equations of motion in the presence of $\phi$. 
Show that for \( \phi = 0 \) (or \( \pi \)) the signal experiences exponential amplification, while for \( \phi = \pi/2 \) the signal is attenuated. This result illustrates the nature of phase-sensitive amplification, i.e., only the “in-phase” component of the field (i.e., \( E_1 \propto \cos(\omega t) \)) is amplified, whereas the “out of phase” or “quadrature” component of the signal field is exponentially damped. This effect gives rise to the generation of non-classical states of light (“squeezing”, cf. Breitenbach, Schiller, Mlynek, Nature, Vol. 387, p.471, 1997.)

5. Frequency-resolved optical gating — The technique of autocorrelation has clear limitations: the result is always symmetric, even if the pulse being measured is not, so clearly information is lost. In fact, a family of different pulses can all give the same autocorrelation result – the operation is projective and therefore not invertible. Significant errors can result, even in finding the duration of the pulse.

Interestingly, this can very largely be solved by a simple step: keep the spectrum of the autocorrelation, also. With this extra information, it is possible to invert the data nearly to recover the original complex-valued field \( \tilde{E}(\omega) = |E(\omega)| \exp\{-i\phi(\omega)\} \), including amplitude and phase. The result is unique, up to a set of measure zero. The family of techniques that uses this principle is called frequency-resolved optical gating, or FROG.

One approach is to start with a background-free autocorrelator, illustrated at right. After making the autocorrelation, a grating resolves the second-harmonic light spectrally, in the direction perpendicular to the autocorrelation-time axis.

Another approach uses phase-matching to create spectral resolution: a horizontal cylindrical lens focusses the beam onto a horizontal line, to make the autocorrelation, but this implies a range \( \Delta\theta \) of angles of incidence. By choosing a thick frequency doubling crystal, a given frequency is phase-matched only for a very narrow fan \( \delta \theta \) within this range of angles. But a slightly different frequency will have similar behaviour for another very narrow fan at a slightly different angle. Thus, different angles out of the crystal double different frequencies, and the spectrum is dispersed automatically. This technique is called GRENOUILLE. Read the details here:

http://www.swampoptics.com/PDFs/tutorials_GRENOUILLE.pdf

The SHG crystal is lithium iodate (LiIO\(_3\)), for Type I angle-tuned frequency doubling.
a) For 35 fs pulses from a Ti:sapphire laser (\(\lambda=800\) nm), what is bandwidth of wavelengths that must be resolved?

b) Approximately what Fresnel prism apex-angle should be used to prepare the single-shot background-free autocorrelation?

c) What arrangement of polarization, crystal orientation and average angle of incidence are needed to produce frequency doubling in this case?

d) If a resolution of \(\Delta\lambda = 2\) nm is needed, what f/number of cylindrical focus and what thickness of crystal are required?