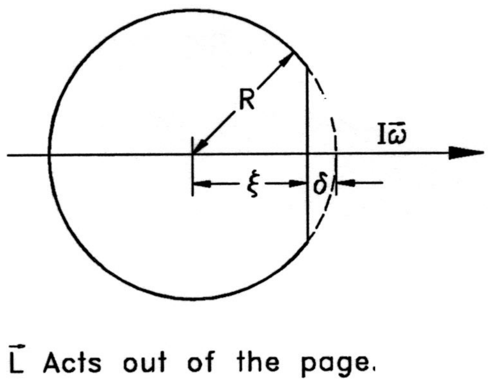
Physics
UNIVERSITY OF TORONTO

# Measuring “g” via precession of an air gyroscope




## Revisions

2023 C. Lee
2020 R. M. Serbanescu
2017 A. Liblong
1971 J. Vise

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Strobe lighting is used in this experiment. It might not be safe for those with epilepsy and other conditions sensitive to flashing light.
This experiment uses a low powered red laser. Never look into the laser beam.

# Introduction

In this experiment, the gyroscope (Figure 1) is a precisely machined sphere sitting on a thin cushion of air produced by a jet underneath it. The sphere is truncated, meaning it has been sliced to remove a spherical cap. The gyroscope is rotated with its axis of rotational symmetry along a horizontal axis and the angular velocity is maintained using an magnetic field produced by a coil connected to the A.C. mains.

Thus, the gyroscope and coil form a synchronous motor with the rotor not fixed to an axis, and the sphere is free to precess around the vertical axis. The angular momentum vector $I\vec{\omega}$ of the spinning also precesses with the sphere.

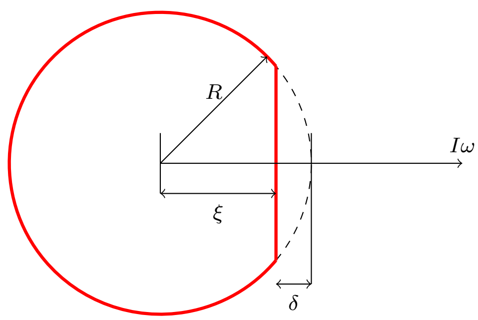


Figure 1: The gyroscope

The truncation in the sphere results in an imbalance of forces such that the Earth's gravity produces a torque meaning the Earth's gravity will produce a torque $\vec{L}$ acting perpendicular to the rotation axis and the gravitational axis. If the sphere is initially rotating around a horizontal axis, this torque causes a precession of the sphere around the vertical axis with angular velocity $\Omega = \frac{L}{I\omega}$, where $I$ is the moment of inertia of truncated sphere, $\omega$ is the angular velocity of the sphere around the (fast) rotation axis, and $\Omega$ is the angular velocity of the precession.

Assuming the sphere is symmetric around the rotation axis and has uniform density, the moment of inertia $I$ and the torque $L$ are functions of radiation $R$, density $\rho$, and the distance from the centre of the flat face to the centre of the sphere, $\xi$.

The expressions for $L$ and $I$ can be derived from appropriate double integrals. The moment of inertia $I$ can be derived assuming the sphere is composed of a series of infintesimally small disks of known density and size, shown in figure 2.

$$\begin{aligned} I &= \int r^2 dm, \\ &= \int_{x=-R}^{\xi} \int_{r=0}^{y} r^2 2\pi\rho r dr dx \end{aligned} \tag{1}$$

Check this calculation by using it to calculate the moment of inertia of a solid sphere of radius $R$ (with no truncation). Then calculate the moment of inertia of a truncated sphere. (*an alternative approach where you calculate the moment of inertia of a solid sphere and then subtract the moment of inertia of the missing cap is also possible*).

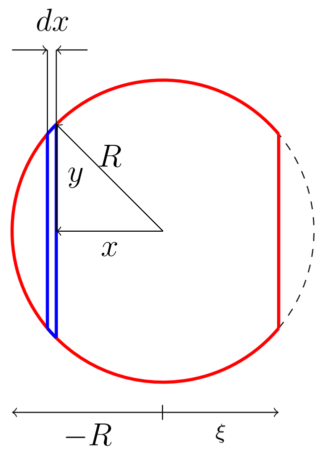


Figure 2: The integral setup that can be used for the moment of inertia calculation and the centre of mass integral needed for the torque calculation.

The torque $L$ can be derived assuming the sphere is composed of a series of infinitesimally small disks of known density and size again, but also knowing that the net torque about the centre of the sphere is equivalent to torque from a single massive particle equal to the mass of the sphere at the centre-of-mass.

$$L = \int gxdm = 2\pi g\rho \int_{x=-R}^{\xi} \int_{r=0}^{y} rxdxdr \tag{2}$$

You should complete the integral calculation as part of your lab work and show the results of your calculation. The final result for $L$ and $I$ are shown in equation 3 and 4, where the equivalence $\xi = R - \delta = R - R\epsilon$ has been used for convenience.

$$L = g\rho \frac{\pi R^4}{4} \epsilon^2 (2-\epsilon)^2, \tag{3}$$

$$I = \rho \frac{\pi R^5}{10} (2-\epsilon)^3 \left( \epsilon^2 + \epsilon + \frac{2}{3} \right) \tag{4}$$

The ratio $\frac{L}{I}$ can then be expressed in terms of $g$,$R$, and $\epsilon$:

$$\frac{L}{I} = g \frac{5}{2R} \frac{\epsilon^2}{\left(2-\epsilon\right)\left(\epsilon^2 + \epsilon + \frac{2}{3}\right)}. \tag{5}$$

Then, if the sphere is rotating with angular velocity $\omega$ and precessing with angular velocity $\Omega$, the ratio $\frac{L}{I}$ can be used to express the acceleration due to gravity $g$ in terms of $\omega$, $\Omega$, $R$, and $\epsilon$, allowing for a precise measurement of $g$.

# Procedure

The equation for $g$ requires the size parameters of the sphere ($R$ and $\xi$). Measure the truncated sphere using the calipers provided. There are multiple locations to measure the diameter of the sphere, but only one location to measure the truncated diameter. You should find that the errors for these measurements contribute significantly to the error in your final result for $g$, so take extra care to measure precisely.

> Getting the sphere to rotate in a stable manner is the most difficult part of this experiment. It is important to be patient and to take your time. You will need multiple measurements to reduce the uncertainty in your result, but once you have a found a method that is repeatable, you can take as many measurements as you need.

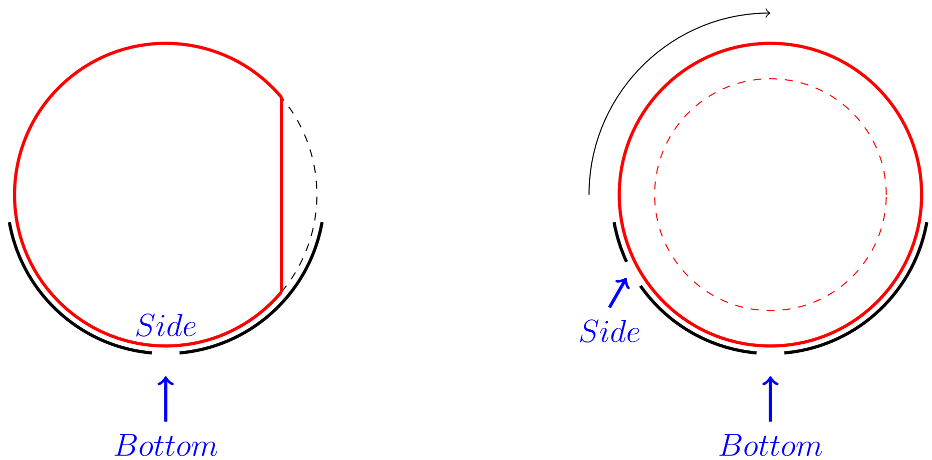


Figure 3: Schematic of the gyroscope in the experiment. The truncated sphere (red) is sitting in the base (black) with two air jet holes. The `Bottom` jet is used to separate the sphere from the base and reduce the friction. The `Side` jet is used to speed up (or slow down) the fast rotation along the horizontal axis perpendicular to the plane of the jets. (Left) front view of the setup, (right) side view of the setup showing the direction the `Side` jet will roll the sphere (thin black arrow).

The truncated sphere is placed in a base with two air jets (see figure 3). The `Bottom` jet is used to reduce the friction and the `Side` jet is used to spin the sphere along a horizontal axis. There is a coil inside the base that can be connected to the A.C. supply to produce a magnetic field at 60Hz. An example procedure to start the experiment is given below, but you can adjust the procedure as needed.

1. Place the sphere in the base and open the bottom jet a small amount (about 0.6 psi). Avoid opening the jets too far as you risk popping the air hoses.

2. Orient the sphere so that the flat face is vertical and parallel to the plane of the two jet holes.

3. While holding the sphere in this orientation, open the `Side` jet to spin the sphere in place. A pencil or other thin, blunt object can be placed against the flat face to keep the sphere aligned with the jet.

4. Once the sphere's rotation is stable, use the strobe light to track the sphere's angular speed. If the strobe frequency and the rotation frequency are the same, the sphere will look stationary under the strobe light. Make sure you don't have a rotation that is a multiple of the strobe frequency by making a small mark on the flat face. The mark should be stationary under the strobe light, but will appear in multiple places if the rotation rate is a multiple of the strobe frequency.

5. Adjust the strobe frequency to keep the image stationary while you accelerate the sphere to 60Hz.

6. Turn the magnet on to the "max" setting, then as slowly close the side jet. The sphere's rotation should lock to the field of the coil.

Once the sphere is rotating stably with no air flowing through the `Side` jet, the sphere will precess around its vertical axis with a period of about 10 minutes. The precession period can be measured by reflecting the laser off the sphere onto the surrounding walls. When the laser hits the flat surface of the sphere, it will reflect back to a fixed location on the wall and the time it takes for the laser to return to the same point on the wall is the precession period.

> It might require several attempts to get the sphere spinning in a stable manner. Once you have a stable precession, wait a few full rotations for the residual oscillations to disappear. You can calibrate the strobe against the built-in 120Hz vibrating reed if needed.

# Analysis

Calculate a value of the acceleration due to gravity using the measured rotation and precession rates, and the shape parameters of the sphere. You should measure the precession in both directions to correct for any asymmetry in the torque imparted by the air suspension due to departure from level.