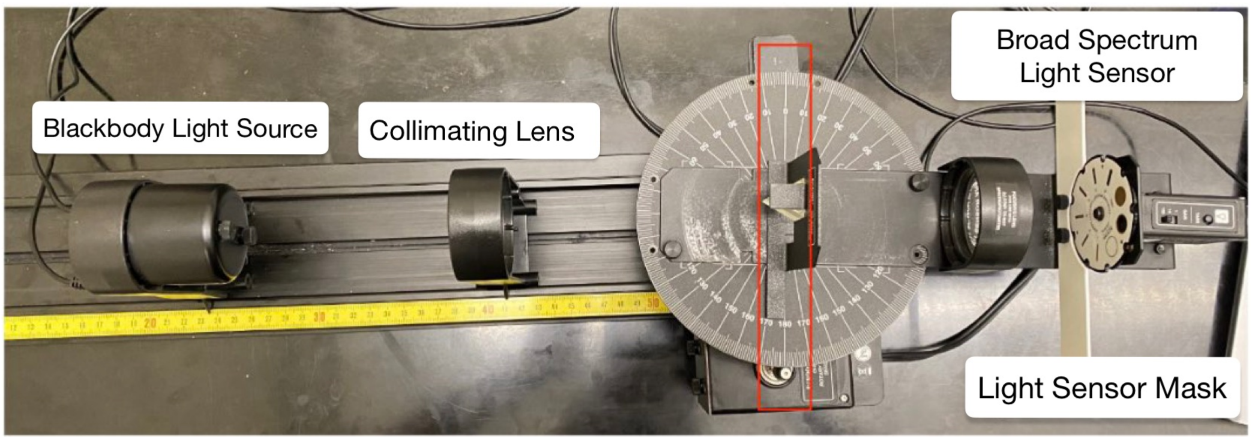
Physics
UNIVERSITY OF TORONTO

Blackbody Radiation



Revisions

2023 T. Vahabi, C. Lee
2022 M. Nardin, E. Horsley
2013 R. Serbanescu

current revision: 3fcb776
date: March 17, 2025

Introduction

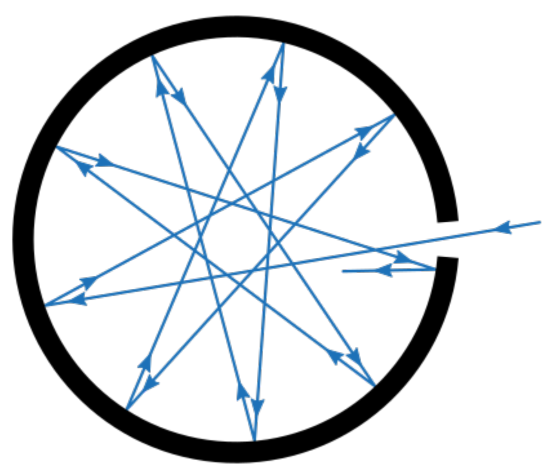
All material objects with a temperature above absolute zero emit electromagnetic radiation. The radiation represents a conversion of a body's thermal energy into electromagnetic energy, and is therefore called *thermal radiation*. Conversely all matter absorbs electromagnetic radiation to some degree.

An object that absorbs all radiation falling on it at all wavelengths is called a *blackbody* **?**.

It is well known that when an object, such as a lump of metal, is heated, it glows; first a dull red, then as it becomes hotter, a brighter red, then bright orange, then a brilliant white. Although the brightness varies from one material to another, the color (strictly spectral distribution) of the glow is essentially universal for all materials, and depends only on the temperature. In the idealized case, this is known as *blackbody*, or *cavity*, *radiation*. At low temperatures, the wavelengths of thermal radiation are mainly in infrared. As temperature increases, objects begin to glow red.

The concept of the blackbody is an idealization, as perfect blackbodies do not exist in nature. Experimentally, blackbody radiation may be measured as the steady state equilibrium radiation from a cavity in a rigid body. A good approximation of blackbody absorption is a small hole leading to the inside of a hollow object.

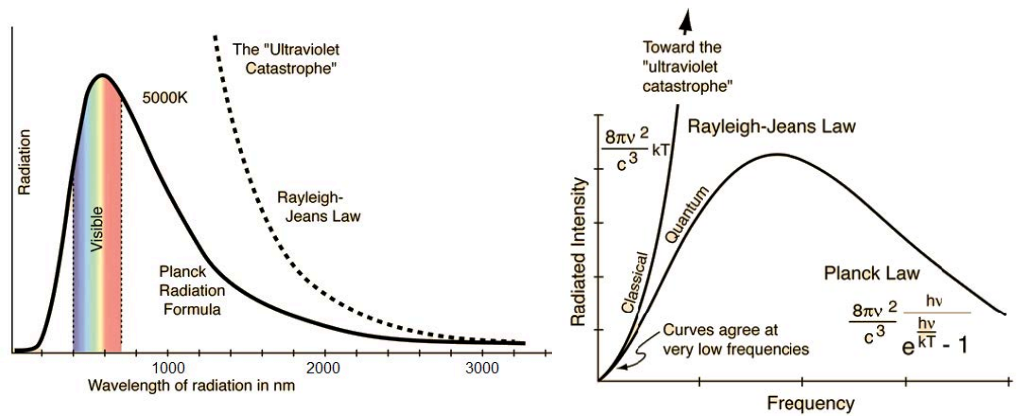
A closed oven of graphite walls at a constant temperature with a small hole on one side produces a good approximation to ideal blackbody radiation emanating from the opening. Blackbody radiation becomes a visible glow of light if the temperature of the object is high enough. At 1000 K, the opening in the graphite oven looks red; at 6000 K, it looks white. No matter how the oven is constructed, or of what material, as long as it is built such that almost all light entering is absorbed, it will be a good approximation to a blackbody, so the spectrum, and therefore color, of the light that comes out will be a function of the cavity temperature alone. Classical physics suggested that all modes had an equal chance of being produced, and that the number of modes went up proportional to the square of the frequency. The predicted continual increase in radiated energy with frequency was named "the ultraviolet catastrophe" (Fig.1).



Approximation of a blackbody in a closed oven of graphite walls with a small hole. (Image from https://en.wikipedia.org/wiki/Black_body

Theory

The energy of blackbody radiation varies with temperature and wavelength. As the temperature increases, the peak of the intensity of blackbody radiation shifts to higher energies



(a) (b)

Figure 1: : The amount of radiation as a function of wavelength (a) and frequency (b) of thermal radiation: classical (Rayleigh-Jeans Law) and quantum (Planck Law) theories. (Images from http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html)

(lower wavelengths). This explains why heated objects change appearance by glowing red at first and bluer/whiter at higher temperatures.

The peak wavelength of light emitted by a blackbody obeys **Wien's displacement law:**

$$\lambda_{max} T = 2.898 \cdot 10^{-3} m \cdot K \tag{1}$$

where λ_{max} is the wavelength of the peak and T is the absolute temperature of the emitting object.

Another experimental fact is that the total amount of energy emitted by a body increases with temperature as following the **Stefan-Boltzmann Law:**

$$P/A = I = \epsilon \cdot \sigma \cdot T^4 \tag{2}$$

where P is power, A is the surface area of the body; I is the intensity of radiation at the object surface, ϵ is the emissivity of the surface of the body, and σ is the Stefan-Boltzmann constant ($5.67 \times 10^{-8} W \cdot m^{-2} \cdot K^{-4}$). For the majority of objects, $\epsilon < 1$, and they are called "grey bodies", while $\epsilon = 1$ for the blackbody.

Blackbody radiation is emitted in the form of a spectrum, where intensity is a function of frequency or wavelength. Intensity emitted in a unit of solid angle in the wavelength interval $d\lambda$ can be defined as: $I(\lambda, T) d\lambda$. In the classical model of blackbody radiation, the **Rayleigh-Jeans Law** takes into account that cavity atoms are modeled as oscillators emitting electromagnetic waves of all wavelengths:

$$\frac{dI}{d\lambda d\Omega} = I(\lambda, T) = \frac{2\pi ckT}{\lambda^4} \tag{3}$$

where k is Boltzmann's constant and c is the speed of light in free space. However the Rayleigh-Jean Law is an approximation that is valid only at long wavelengths **(Fig.1a)**. According to this equation the intensity approaches infinity as λ approaches zero, causing the "ultraviolet catastrophe".

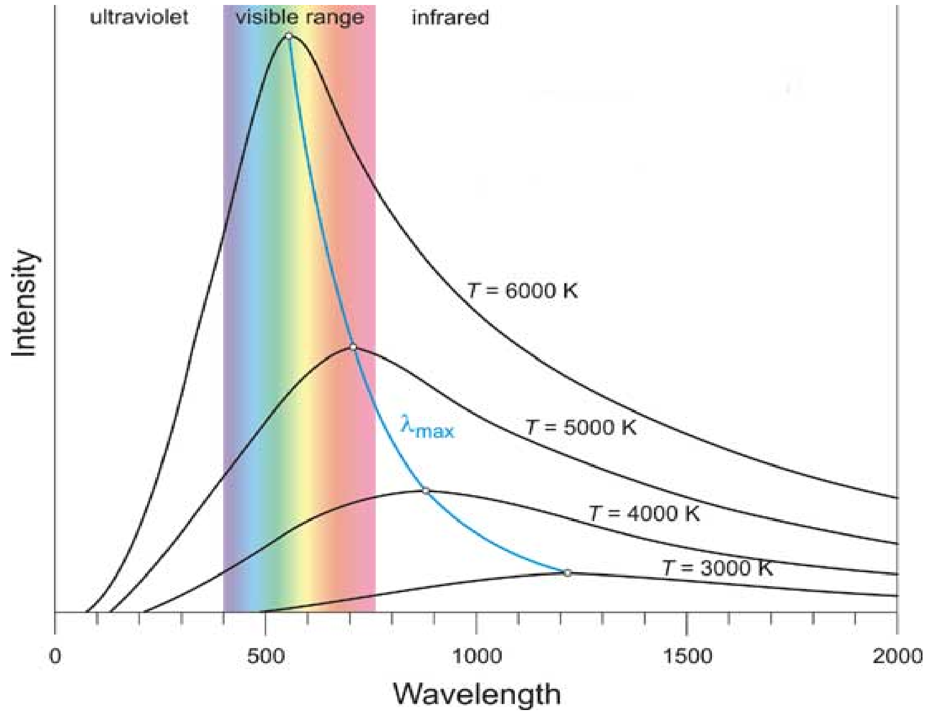


Figure 2: Wien's displacement law (Image from http://www.electrical4u.com/black-body-radiation/)

In 1900, Planck derived a formula for blackbody radiation which is in total agreement with experimental data **(Fig. 1)**. The intensity of radiation emitted by a body is given by **Planck's Radiation Law:**

$$I(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \tag{4}$$

where $h = 6.62607 \times 10^{-34}$ J $\cdot$ s is Planck's constant, $k = 1.3806 \times 10^{-23} J \cdot K^{-1}$ is the Boltzmann's constant, and T is the absolute temperature of the body, and λ is the wavelength of the radiation. At long wavelengths, (4) reduces to the Rayleigh-Jean expression (3) and at short wavelengths it predicts an exponential decrease in intensity with decreasing λ.

Planck's main assumption was based on the idea that marked the birth of quantum theory, namely that energy is emitted only in discrete energy values, or quants: $E_n = nh\nu = nhc/\lambda$, where n is an integer, h is a constant (later called Planck's constant), c is the speed of light, ν is the frequency of radiation and λ is the wavelength.

The total intensity radiated within all wavelengths in all directions can be found by integrating equation 4 over the total solid angle and all wavelengths.

Many commonly encountered light sources, including the Sun and incandescent light bulbs, are closely modeled as "blackbody" emitters.

Experiment

The energy emitted by an incandescent light bulb can be scanned by separating the light in wavelength using a prism, and using a spectrophotometer to measure the light intensity as a function of an incident angle. Due to chromatic dispersion of the material of the prism, different wavelengths have different indices of refraction in the prism and refract by different amounts. A Broad Spectrum Light Sensor is used with a prism so the entire spectrum from approximately 400 nm to 2500 nm can be scanned.

The wavelengths corresponding to the angles are calculated using the equations for a prism spectrophotometer. The relative light intensity can then be plotted as a function of wavelength as the spectrum is scanned, resulting in the characteristic blackbody curve. As the power input to the light bulb is reduced, the temperature of the bulb and intensity of the light is reduced. Repeating the measurement for a range of temperatures will to show how the curves shift in the peak wavelength with changing temperature.

Determining wavelengths from angle measurements

The 60 degree prism is mounted so that the back face is perpendicular to the incident light as can be seen in Fig.3:

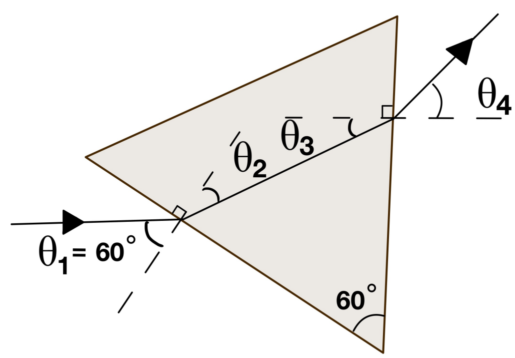


Figure 3: Schematics of the prism

Applying Snell's Law at each of the two refracting faces of the prism, we can obtain an expression for the index of refraction of the prism:

$$n = \sqrt{\left(\frac{2}{\sqrt{3}}sin\theta + \frac{1}{2}\right)^2 + \frac{3}{4}} \tag{5}$$

The index of refraction varies with wavelength as it can be seen from the table below:

Index of Refraction n	Wavelength λ (nm)
1.703	1014.00
1.709	852.10
1.717	706.50
1.721	656.30
1.722	643.80
1.728	587.60
1.734	546.10
1.746	486.10
1.748	480.00
1.762	435.80
1.776	404.70

Table 1: Variation of the prism index of refraction with wavelength

The Cauchy equation gives the relationship between the index of refraction and wavelength:

$$n(\lambda) = \frac{A}{\lambda^2} + B \tag{6}$$

where A and B are constants depending on the prism material. Our prism's coefficients are: A = 13900nm^2 and B = 1.689.

You will need to **derive the final expression** for the wavelength: $\lambda = \lambda(\theta)$ combining (5) and (6). Use this equation in your analysis to calculate wavelength from angles, and to propagate the uncertainty in the measurement to the uncertainties in your wavelength values.

Determining the temperature of the black body source

The temperature can be estimated indirectly by determining the blackbody's electrical resistance from the measured voltage and current. The resistance varies with temperature according to:

$$R = R_0[1 + \alpha_0(T - T_0)] \tag{7}$$

where α_0 is the temperature coefficient at the room temperature T_0. Our blackbody is the tungsten filament of a light bulb, which has the thermal coefficient $\alpha_0 = 4.5 \times 10^{-3}$ K^{-1} at $T_0 = 293K$, and $R_0 = 1.1\Omega$. Solving (7) for temperature, one can obtain:

$$T = T_0 + \frac{\frac{V/I}{R_0} - 1}{\alpha_0} \tag{8}$$

Voltage (V) and current (I) are displayed by the acquisition software. If the room temperature is significantly different from 20°C you will be able to improve the measurement using a better value temperature coefficient of Tungsten at the actual room temperature. Find a source for the temperature coefficient of tungsten for the real room temperature and substitute the real temperature in the Eq.(8).

Apparatus

The Prism Spectrophotometer is set up for you. Check that the prism is oriented with the apex facing the light source **(Fig. 4)**. Verify that the base piece of the prism is aligned with the zero mark on the spectrophotometer table when the prism apex points toward the light source.

The collimating lens must be ∼10 cm from the collimating slits.

To start, set the collimating slits on Slit #4, and the light sensor mask on Slit #3. You might find better results on other settings but these values provide a reasonable starting point.

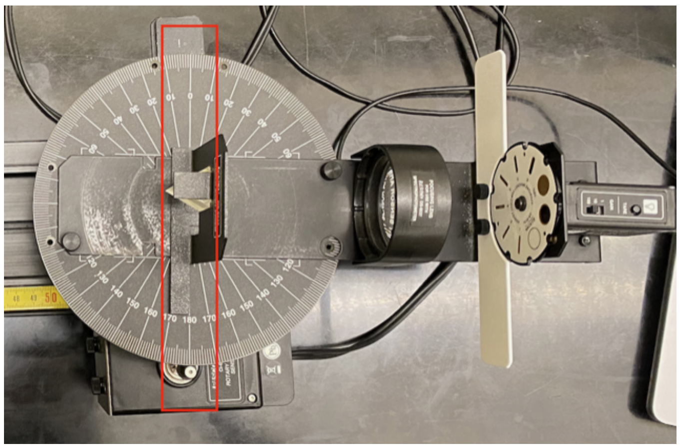


Figure 4: Prism Orientation.

Acquisition Software

Open the Blackbody Radiation LabVIEW application using the desktop shortcut. Do not turn the acquisition ON yet! Turn on the Signal Generator (a switch is at the back).

Several buttons are placed at the top of the application window:

`Clear Data`: erases the current file;

`Export Data`: allows you to save the data (not the plot) as a .txt format;

`Clean Up Data`: applies a filter to reduce noise;

`Integrate Area Under Curve`: attempts to integrate the total area under the curve (including vertical offset) between two cursor positions set up by the user;

`Automatic Data Limit Switch`: allows you to scan the emitted spectrum over 30 degrees or 90 degrees (the initial angle θ_{init} is between 74 and 79 degrees depending on the prism position).

At the bottom, there are display windows for average Voltage and Current as well as for the two pairs of cursors.

If 10 volts is applied to the blackbody light source for an extended amount of time, the life of the bulb will be reduced. To prevent over saturation, only turn the bulb on when recorded measurements.
The light detector has a systematic drift in the output. In order to minimize it, do not bring your hands close to it (you emit infrared radiation, too!), do not touch it and use only the sides of the light sensor mask to move it during the scan.

Procedure

Rotate the scanning arm counterclockwise until it touches the stop. This will be the starting position for all the scans.

Set the Broad Spectrum Light Sensor gain switch to “x10” and press the tare button. Choose an 90 degree acquisition and select voltage at 6 V.

Look at the light coming from the Blackbody Light Source. **Observe and record** the color.

Question 1: Are all the colors from red to violet present in the spectrum on the Light Sensor screen?

Slowly rotate the scanning arm through the spectrum and continue all the way until it turns off by itself. There will be a main peak on the intensity vs. angle diagram and also a smaller blip where the light sensor is aligned with the light source because some light passes by the prism instead of going through the prism. This blip makes it possible to find the initial angle θ_{init} more precisely. Due to small displacements of the prism, the initial angle can slightly alter from experiment to experiment. You should experimentally determine the current initial angle θ_{init} for your device performing 3 - 5 runs to find the position of the small peak in the light intensity plot.

Record the value of θ_{init} with its uncertainty; its accuracy will greatly affect your result! Your ability to convert the angles into wavelength values depend on this measurement. In the following exercise you will need the difference between the initial angle and the peak angle.

You can either take measurements in the 30 degree scan mode which potentially gives you more accurate angle measurements, or measure with the 90 degree scan mode to get the initial angle for each measurement at the same time as the angle of the peak intensity.

At the start of each lab session you should also retake the measurement of θ_{init}. Other students will use the equipment in between your sessions and might change the setup.

Wien's Displacement Law

Using voltages between 4 V and 10 V, measure the voltage and current, and determine the angle of the main peak θ_{peak} by using the cursors on a 30-degree scan. You may need to switch the gain to "x100" at lower voltages. After each gain change you have to press the "Tare" button on the light sensor to remove biases.

To calculate wavelength, first find the angular separation from the original small peak to the larger peak you find now. Using the equation you derived from equations 5 and 6 to convert angle to wavelength. Calculate the temperature using equation 8 and your voltage and current measurements.

Using your calculated values of temperature and wavelength, find the constant in your version of equation 1. You can average the data using the uncertainties to calculate a weighted average and uncertainty, or you can fit a suitable model to find the coefficient you need. Compare your value of the constant with Wien's displacement law given by equation 1. Discuss any difference you find between the two values.

> Before you finish the in-person component of the lab, calculate a few (or all) values of wavelength from your angle data. Make sure that the angles you calculate make sense and can be converted to physically reasonable wavelengths. If not, you may need to repeat the experiment to measure the angle between the two light peaks more accurately.

Stefan-Boltzmann Law

The light sensor records the relative intensity and returns a voltage value directly related to power per unit area that falls upon it. Make measurements for not less than 7 different values of voltage in the range between 4 V and 10 V. The greater number of runs will increase quality of the result.

For each of the voltage runs from 10 V to 4 V, use vertical cursors to limit the peak area you integrate to avoid the wings (e.g., from 10 degrees either side of the peak) and use the `Integrate` button to obtain the area under the curve. Notice that the intensity does not fall to the baseline after the peak due to ambient light. Delimit the "tail" using cursors and re-integrate.

With the corrected areas under the blackbody curve, and your calculated temperature values, determine whether your data agrees with the Stefan-Boltzmann law. You could plot area against T^4, or fit the exponent explicitly. Discuss the causes and effects of any systematic uncertainties in your calculations.

> Note: The relative nature of the measurement means that the absolute value of area is not important here. However, you will need to keep the gain constant from one voltage run to another in this part of the experiment!

Question 2: The wavelength of the peak intensity should change as a function of temperature. Do you see a similar change in color of the light bulb with changing temperature? Can you explain the change in apparent color considering the change in peak wavelength from low to high temperature?

Question 3: As the temperature increases, you should see the fraction of light in the visible spectrum increase compared to the infrared part of the spectrum. At your highest temperature setting, was more of the intensity (the area of the intensity vs. wavelength graph) in the visible part of the spectrum or in the infrared part of the spectrum? How would you use this information to make a (visible spectrum) light bulb more efficient?