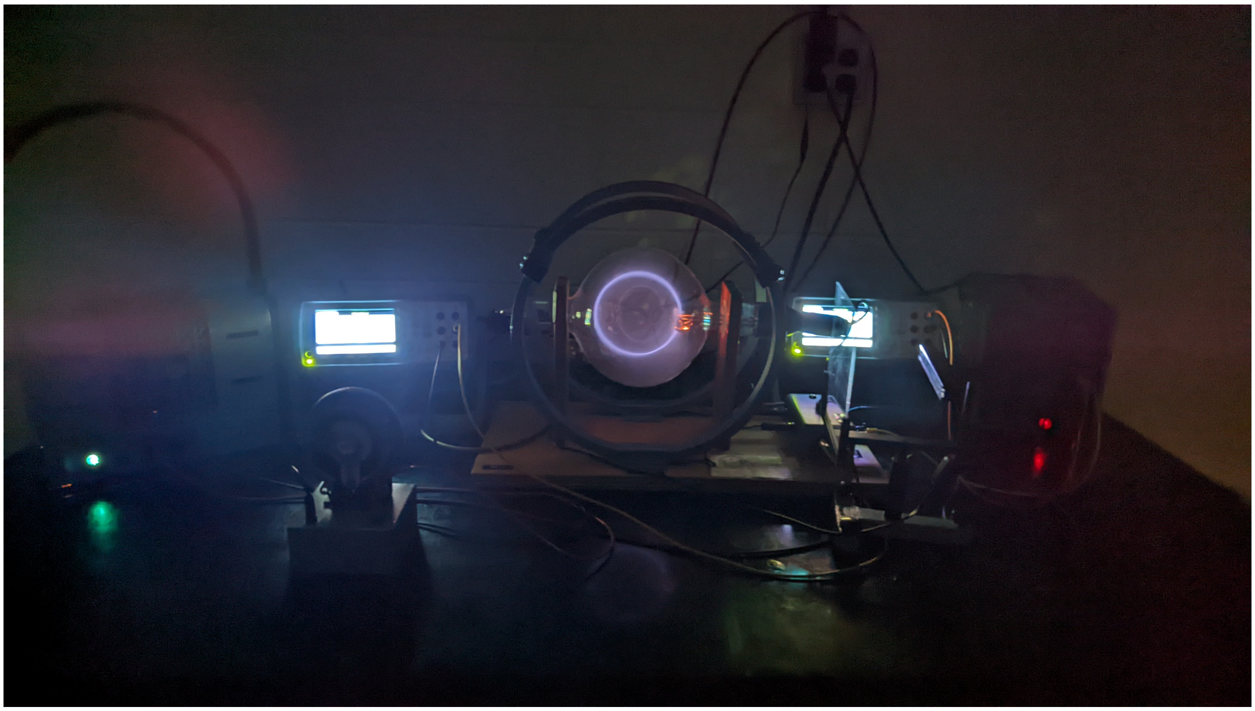
Physics
UNIVERSITY OF TORONTO

Charge-to-mass ratio for the electron



Revisions

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Introduction

This is a variation of the original experiment carried out by J. J. Thomson in 1895. The deflection of a charge moving in a magnetic field is clearly demonstrated.
A particle of mass m and charge e moving in a magnetic field $\vec{B}$ with velocity $\vec{v}$ will experience a force $\vec{F}$ given by

$$\vec{F} = e\vec{v} \times \vec{B}. \tag{1}$$

The vector cross product means the force $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. If $\vec{B}$ is constant and $\vec{v}$ is perpendicular to $\vec{B}$, the particle will move in a closed circular orbit of radius r, with magnetic force providing the centripetal acceleration according to Newton's second law:

$$evB = m\frac{v^2}{r}. \tag{2}$$

Now in this experiment, the particle is accelerated through a potential difference ΔV in order to reach the speed v. Thus in the non-relativistic approximation,

$$e\Delta V = \frac{1}{2}mv^2 \tag{3}$$

Combining Equations 2 and 3, gives a curvature of the electron orbit of:

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{B}{\sqrt{\Delta V}} \tag{4}$$

For more details on the derivation presented here, see Griffiths [2017].

Theory

In this experiment, the magnetic field $\vec{B}$ is generated by the current flowing through a pair of Helmholtz coils. The sketch showing the geometry of Helmholtz coils is shown in Figure 1.
Note that the radius of each coil, R, and the coil separation distance are the same. This configuration gives the minimum variation of the magnetic field B near the center of the pair of coils. The magnitude of the magnetic field B due to a single current-carrying coil of radius R along its axis, distance z away can be expressed as:

$$B = \frac{\mu_0 I R^2}{2\left(R^2 + z^2\right)^{3/2}}, \tag{5}$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ T m A^{-1} is a permeability of free space, I is the current in the coil, and the direction of the magnetic field is along the coil axis.

Over a volume containing the geometrical center of the configuration, the magnetic field due to the coils is directed along the coil axis and is approximately uniform with the value:

$$B_c = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n I}{R} \tag{6}$$

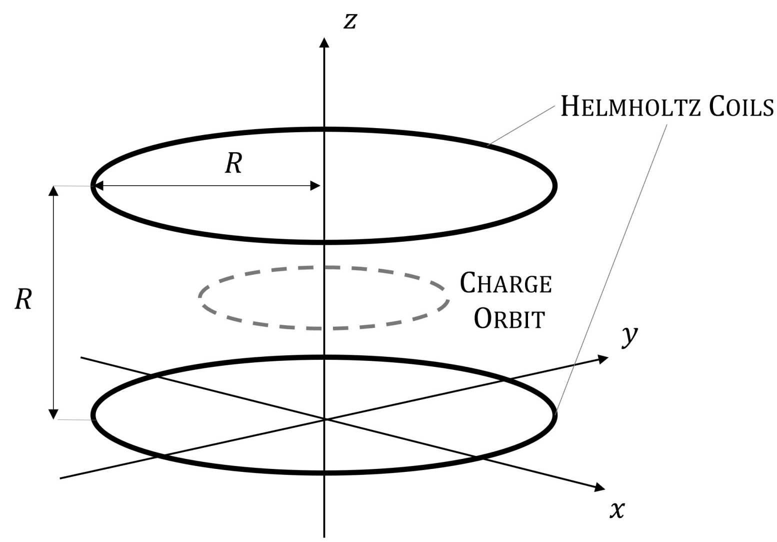


Figure 1: Schematic geometry showing Helmholtz coils (thick solid line) of radius R with a potential orbit of a charged particle indicated with dashed grey line.

where n is the number of turns in each coil.
The total axial magnetic field in the region of the electron beam is the field from the coils, B_c, plus the external field from the earth, building, and other instruments/devices in the laboratory in that direction, B_e. Thus,

$$\vec{B} = \vec{B_c} + \vec{B_e} \tag{7}$$

Substituting Eq. 7 to 4 leads to the expression:

$$\frac{1}{r} = \sqrt{\frac{e}{2m}} \frac{1}{\sqrt{\Delta V}} \left[\left(\frac{4}{5} \right)^{3/2} \frac{\mu_0 n I}{R} + B_e \right]. \tag{8}$$

Defining:

$$k = \frac{1}{\sqrt{2}} \left(\frac{4}{5} \right)^{3/2} \frac{\mu_0 n}{R}$$

and

$$I_0 = B_e / k$$

we can rewrite Equation 4 as:

$$\frac{1}{r} = \sqrt{\frac{e}{m}} k \frac{I + \frac{1}{\sqrt{2}} I_0}{\sqrt{\Delta V}} \tag{9}$$

or, alternatively

$$\frac{\sqrt{\Delta V}}{r} = \sqrt{\frac{e}{m}} k \left(I + \frac{1}{\sqrt{2}} I_0 \right), \tag{10}$$

where k is the characteristic of the coil dimensions, and I_0 is a constant, proportional to the external magnetic field.

Apparatus

The apparatus, shown in Figure 2, consists of a stand with a mounted glass bulb (GB) placed between two Helmholtz coils (HC). The stand also includes a box (PB) where all the power comes into the system. The glass bulb, filled with hydrogen gas at low pressure, also contains the electron gun (EG).

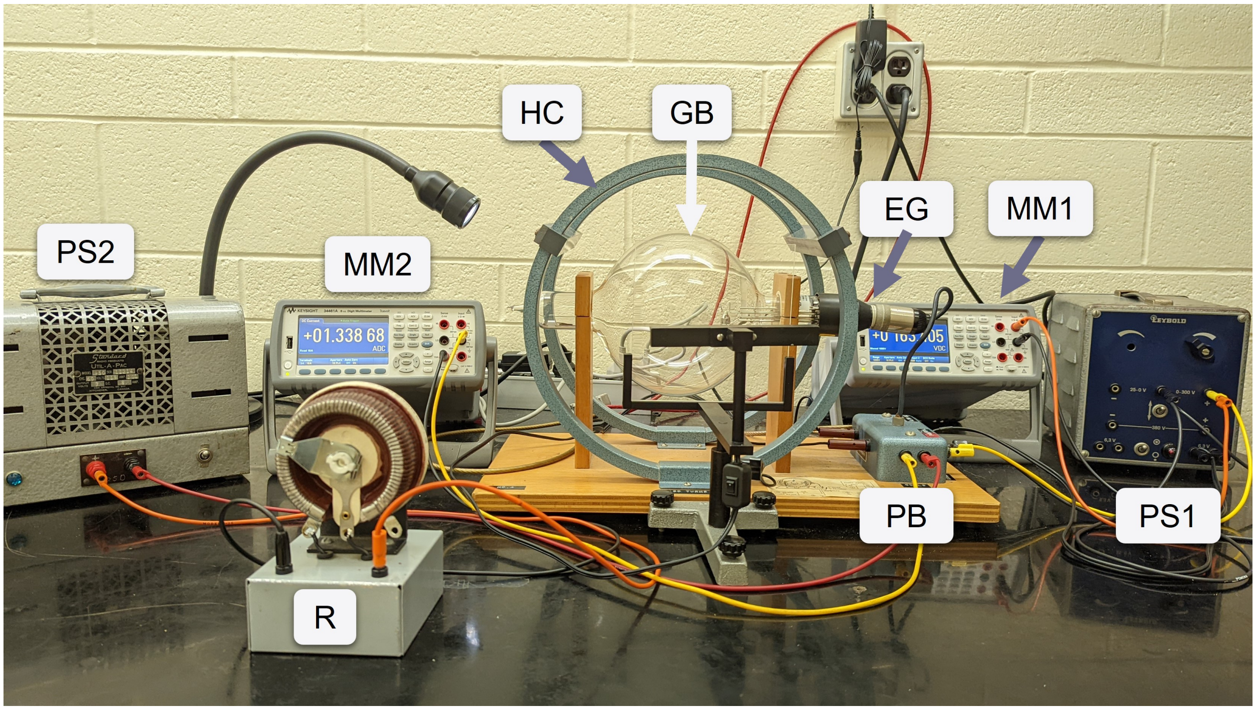


Figure 2: Experimental apparatus. EG - electron gun, GB - glass bulb, HC - Helmholtz coils, MM1 - anode voltage meter, MM2 - coil current meter, PB - power box, PS1 - anode and filament power supply, PS2 - coils power supply, R - rheostat

Electrons emitted by a hot filament within the gun are shaped into a beam by accelerating them through a specially shaped anode. The anode voltage, V_{anode}, is provided by a 0 to 300 V power supply (PS1), connected to the "ANODE" input of the box (PB). The voltmeter (MM1) is connected in parallel with the power supply. The same power supply is a source of 6.3 V AC voltage that connects to the "FILAMENT" input. Figure 4a shows the power box with all the connections labeled and Figure 4b includes a close up of PS1 and its connection to both inputs and the voltmeter (MM1). The coil current is provided by an 8 V D.C. power supply (PS2) (around 10 V under load) connected in series with a rheostat (R) and an ammeter(MM2) to the "COILS" input.

Experimental Procedure

The schematic diagram of the experimental setup is shown in Figure 3. The three circuits powering the apparatus are indicated with the same symbols used in Figure 2 for clarity.

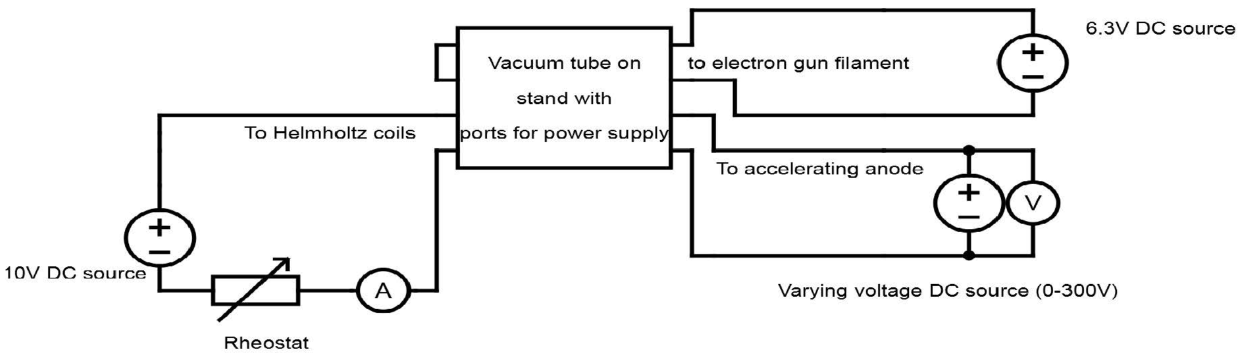
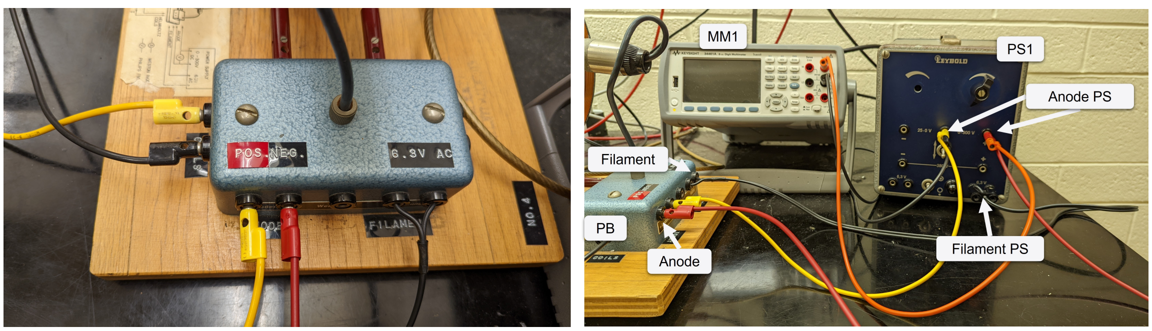


Figure 3: Schematic diagram of the apparatus set up with the three power circuits.



(a) (b)

Figure 4: (a) Close ups of the power supply box. (b) Close up of the connection of the anode and filament power supply (PS1) and its voltmeter (MM1) to the apparatus.

NOTE: the voltmeter used in the experiment has been found to accumulate charge. Hence it is advisable to turn on the devices and conduct measurements promptly before the voltmeter has time to charge up, which could result in voltage output value fluctuations.

Procedure

1. Connect the apparatus according to the schematic shown in Figure 3. Use the photographs shown in Figure 4 for clarification.

2. Turn on the power supply for the filament of the electron gun. Leave it on for approximately 30 s to ensure the filament heats up. **Note: the filament should never be turned off unless the anode voltage is also off, otherwise the tube may be damaged**.

3. Turn on the anode voltage.

4. Turn on the power supply for Helmholtz coils.

   The beam of electrons becomes visible when the electrons have sufficient kinetic energy to excite the gas during the collision. The collisions, however, are sufficiently rare that the beam is scarcely affected.

5. Gently rotates the bulb to ensure the beam follows closed paths.

   (a) If the beam curves in such a way that it hits the glass you can either rotate the tube in its mount or you can switch the polarity of the leads to reverse the direction of the magnetic field.

   (b) Make sure that the trajectory of the electrons is a vertical circle and not a helix-like shape.

6. The diameters of the paths can be measured with the self-illuminated scale and plastic reflector provided. The illuminated scale, if well positioned, eliminates problems of parallax in the measurement. You may need to spend some time figuring out how to use the scale before you start your readings.

7. As you can see in Equation 9, the radius of the electron path depends on two variables: accelerating potential ΔV and the coil current I.

   Collect at least two sets of data, one for constant accelerating potential and one for constant current. Record the values with units and uncertainties.

Questions

Incorporate the answers to these questions into your laboratory report. Keep in mind, that while the answers should be obviously present, they should fit into the narrative of the report.

1. Explain how you used the self-illuminated scale and a plastic reflector to eliminate problems of parallax.

2. Investigate the anomalous behavior of the electron trajectory in the case of low accelerating voltage ΔV and high current in the coils I (resulting in a strong magnetic field B).

   (a) Are all of the parts of the trajectory equally affected?

   (b) Does this introduce an error in the measurements? If so, consider the ways it can be eliminated/reduced.

   (c) When considering corrections to the values you obtain for e/m and B_e, it may be useful to take into account the fact that although the field generated by the coils is nearly constant along the axis, it decreases away from the axis. It can be shown that for off-axis distances ρ, which are less than $0.2R$, the z-component (the only one we have been considering) of the field, $B(\rho)$, is smaller than the axial field $B(0)$ by less than 0.075%. For $0.2R < \rho < 0.5R$ the ratio $B(\rho)/B(0)$ is given approximately by

$$\frac{B(\rho)}{B(0)} = 1 - \frac{\rho^4}{R^4\left(0.6583 + 0.29\frac{\rho^2}{R^2}\right)^2}$$

There is some thought that needs to go into designating the ρ value for each measurement. The electrons are generated at a fixed point and as such the radial electron path is not centered at the coil axis, causing ρ to vary throughout the path. As an approximation for large radii, using the corresponding radius measurement as ρ should be close enough to achieve acceptable results. This correction factor can be applied directly when calculating both B_c and I from the collected data.

3. Calculate, from Equation 8, the "extra" field, B_e, NOT produced by the coils.
Hint: You may want proceed as follows: rewrite Equation 8 in the form

$$B = (B_c + B_e) = \sqrt{\frac{2m}{e}\Delta V}\frac{1}{r} \rightarrow B_c = \alpha\frac{1}{r} - B_e, \tag{11}$$

where $\alpha = \sqrt{\frac{2m}{e}\Delta V}$ is a slope of the graph of B_c as a function of $1/r$. then compute B_c from Equation 6 for every known current. Apply correction determined in Question 2c to your computed values and plot B_c as a function of $1/r$ - the y-intercept should give the value of B_e. Comment on the magnitude of this field (i.e. how does it compare with the average value of the Earth's magnetic field, etc.) and its uncertainty.

Feel free to explore other ways to determine the value.

4. Evaluate the influence of nearby ferromagnetic materials and other sources of magnetic fields on the electron trajectory (for example, bring a cell phone near the glass bulb). Is it significant enough to affect the measurements?

5. Obtain the value of the charge-to-mass ration, e/m, using the value B_e estimated above and relationships described in Equations 9 and 10.

References

D. J. Griffiths. *Magnetostatics*, chapter 5, pages 210–265. Cambridge University Press, 4th edition, 2017. ISBN 978-1108420419. doi: https://doi.org/10.1017/9781108333511. URL https://www.cambridge.org/highereducation/books/introduction-to-electrodynamics/3AB220820DBB628E5A43D52C4B011ED4#overview.