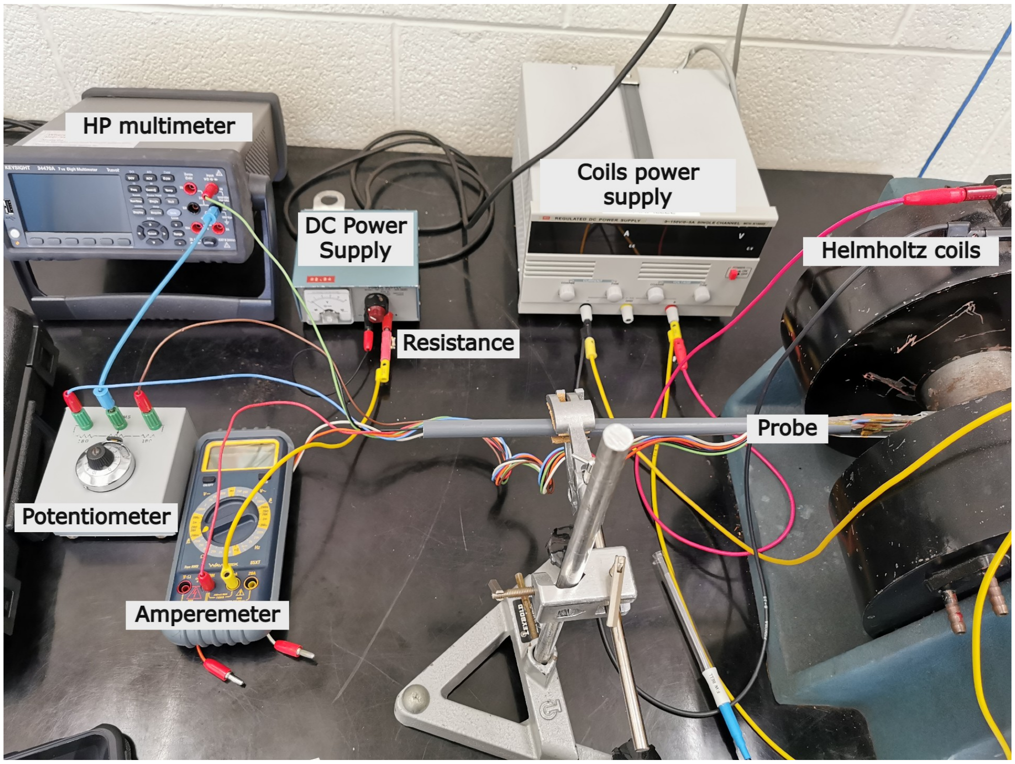
Physics
UNIVERSITY OF TORONTO

The Hall Effect



Revisions

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current revision: 63bbf38
date: August 29, 2024

Introduction

In 1879, E. H. Hall observed that when a current-carrying conductor is placed in a transverse magnetic field, the Lorentz force on the moving charges produces a potential difference perpendicular to both the magnetic field and the electric current. This effect is known as the Hall effect Purcell and Morin [2013] Measurements of the Hall voltage are used to determine the density and sign of charge carriers in a material and a method for determining magnetic fields.

Theory

Moving charges, such as electrons, which travel with velocity $\vec{v}$ through a magnetic field $\vec{B}$ experience the Lorentz force, $\vec{F} = e(\vec{v} \times \vec{B})$. If the charges are in a wire, the result of the Lorentz force is that charges build up on the sides of the wire which are perpendicular to both the magnetic field and the current. This is the Hall Effect. The unbalanced charges create a potential difference which is called the Hall Voltage. If you measure the Hall Voltage produced by a known current, you can determine the magnetic field. This is how computers read data from hard drives which are stored in the form of small regions of magnetic fields.

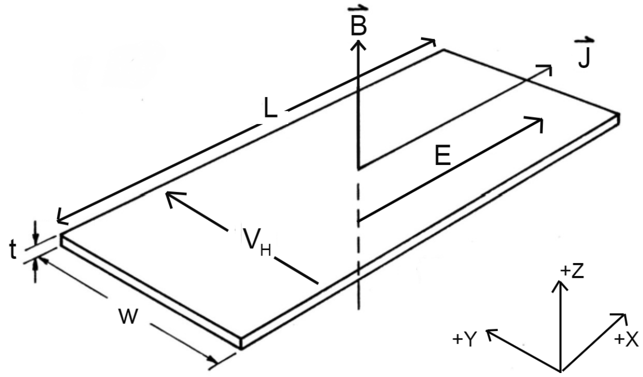


Figure 1: Geometry of the Hall probe. $\vec{B}$ - external magnetic field, $\vec{E}$ - electric field along the wire, $\vec{J}$ - current density,V_H - Hall voltage, L - length of the probe, t - thickness of the probe, w - width of the probe.

You can see in Figure 1 that a current density $\vec{J}$ going in the x-direction through a magnetic field $\vec{B}$ pointed in the z-direction gets deflected in the y-direction such that the potential difference from the Hall Effect gets created in the y-direction.

Basic circuit theory includes the following facts which are pertinent to understanding the Hall Effect. First, the resistance R of a circuit element (such as a wire) is defined as

$$R = \frac{V}{I} \tag{1}$$

where V is the potential difference across the element and I is the current going through the element. For a wire of length L and constant cross-sectional area A (in Figure 1 we have

$A = t \times w$), the resistance of the wire depends on the resistivity (ρ) of the material (often copper) as

$$R = \rho \frac{L}{A}. \tag{2}$$

The magnitude of the current density is defined as

$$|\vec{J}| = \frac{I}{A} \tag{3}$$

and the direction of $\vec{J}$ points in the direction of the current.

If we assume that the current is caused by many electrons of charge e and number density n travelling at constant average speed v_d, we find

$$|\vec{J}| = n\,e\,v_d. \tag{4}$$

Note that the assumption that electrons (with negative charge) are the what is actually moving is something you will verify in this experiment.

A detailed analysis of the Hall Effect (as in Purcell and Morin [2013]) gives us one result which is critical to this experiment. The Hall coefficient R_H, which is a constant for any given material, is found to be

$$R_H = \frac{|\vec{E}|}{|\vec{J}|\,|\vec{B}|} = \frac{1}{n\,e} \tag{5}$$

where $\vec{E}$ is the electric field created by the Hall voltage, $\vec{B}$ is the external magnetic field, and $\vec{J}$ is the current density going through the magnetic field.

Your task is to verify that the Hall voltage is proportional to B and to I as implied by equation 5, and to determine the values of n, ρ and v_d (for one of your current values) for two different materials (chromium and silver). Finally, you will show that the charge carriers (electrons) have a negative charge. You may assume you know the magnitude of the value of e.

The Experiment

The goal of this experiment is to determine the Hall constant R_H and the electric conductivity for Hall samples made of Chromium (Cr) and Silver (Ag). Figure 2 shows the two materials probes. Using these measurements you will calculate some of the microscopic properties of the two conductors: n, v_d (for a given current), and the sign of charge carriers.

Figure 2: Photograph of Chromium (Cr) and Silver (Ag) probes

Procedure

1. Measure the electric resistance between V_1 and V_2 by two methods

   There are two methods for which you can measure the resistance; the two-wire method, and the four-wire method. Although both methods are equally suited for resistance applications, one technique is better applied to a small resistance measurement as opposed to other resistance measurements. The digital multimeter used throughout this experiment employs a constant current to measure the resistance of some given circuit. The constant current (I_s) to the device is then sourced, which then measures the voltage (V_m). By calculating the resistance it displays it on the screen, given the formula:

$$R = \frac{V_m}{I_s}$$

   (a) Two-wire method

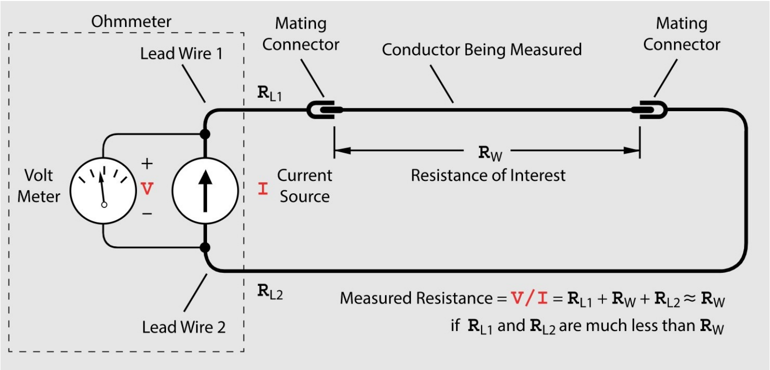
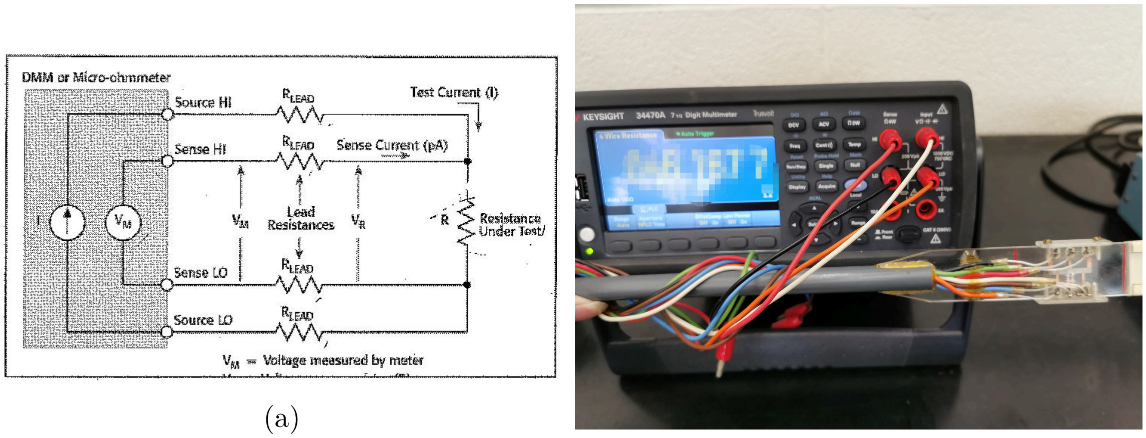


Figure 3: Diagram for the Two-wire resistance measurement CAMI Research

In general, we use a two-port multimeter (Two wires method) to measure the resistance. However, we will raise some issues here since the values of electric resistance are low for both Hall samples so the total resistance of wires (leads) is comparable to sample resistance values leading to high uncertainty. In this case, $V/I = R_{L1} + R_w + R_{L2} \neq R_w$. So it's not appropriate to use this traditional method to measure the small resistance.

(b) Four-wire (Kelvin) method
This method is a parallel connection in the wire setup and is used primarily for low-resistance systems as it eliminates much of the lead resistance. The four-wire method is used for resistances that are estimated to be below 1KΩ. We correct this problem by moving the voltage measurement points out to the endpoints of the mating pins, thus, bypassing any voltage drop that may occur in the lead wires. Refer to the Figure 4 below:



(a)

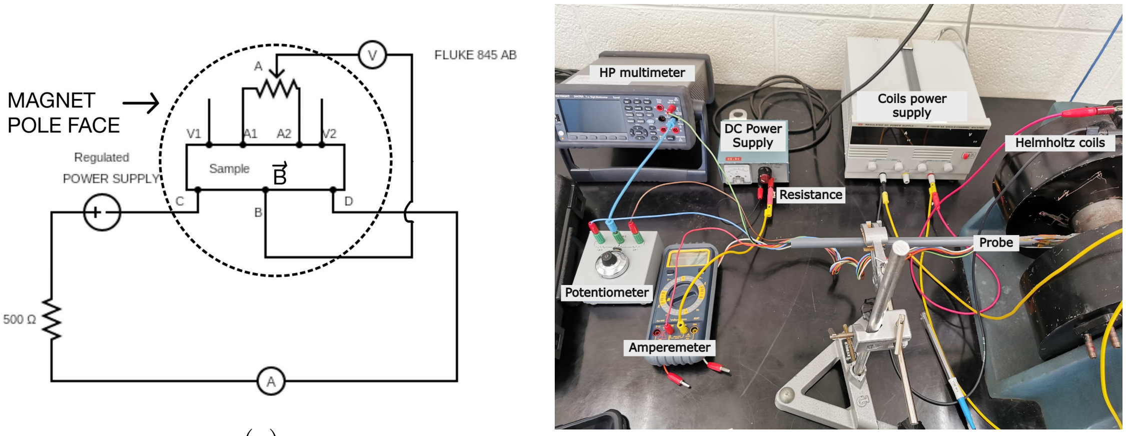
(b)

Figure 4: (a) Connection diagram for the four-wire resistance measurement. (b) Photograph of wire up for the four-wire resistance measurement.

In Figure 4a, DMM stands for digital multimeter. Our instrument is the HP multimeter. The instrument sends a test current I which is forced through the

test resistance R via one set of wires, while the voltage across V_M is measured through a second set of wires (sense leads). A very small current, in the order of pA may flow through the sense leads, but its effect is usually negligible and $V_M \approx V_R$.
(*Note: use very short wires!*)

2. Use the traveling microscope to measure the width w and the distance d between V_1 and V_2.
3. Connect the apparatus according to the schematic shown in Figure 5a. Use the photographs shown in Figure 5b for clarify. Make sure the limiting 500 Ω resistance is already connected to one of the ports of the DC power supply.



(a)

(b)

Figure 5: (a) Diagram of the experimental setup. (b) Photo of the experimental facility.

4. Turn on the DC power supply and never exceed 40 mA current through the sample, otherwise, it will be destroyed.
5. Turn on the HP multimeter and Ampermeter. Check the potentiometer setting throughout the experiment.
6. Use the potentiometer at A_1A_2 and connect the Fluke high impedance null detector between A and B. Switch the Fluke instrument to the most sensitive (microvolt) scale and adjust the null by turning the potentiometer cursor. Make sure the potentiometer can adjust the null even when high currents (30mA) are applied. (The potentiometer here is to remove the offset of the potential difference between A_1 and A_2 since there exists transverse misalignment of probes in samples and the wire across is not perfect.)
7. Turn on the power supply for Helmholtz coils. The magnetic field is measured with a Gaussmeter. The active part of the gaussmeter probe is within 0.9 mm from the tip. The flat part of the probe should be placed perpendicular to the magnetic field direction. It is recommended to measure $\vec{B}$ at every magnet current at which Hall measurements are made.

8. Place the sample in the center of the magnetic gap with its broad side perpendicular to the field. Measure the Hall Voltage in the sample, as well as the current. Collect at least three sets of data for each material with a different magnetic field. Record the values with units and uncertainty.

9. Use the traveling microscope to measure the Hall sample thickness from the interferometric photographs provided by the manufacturer. In our case:

$$t(\text{Å}) = \frac{Fringestep}{Fringeseparation} \times 2945\text{Å}$$

The sample thickness was obtained by interference measurements before the electrodes were attached. A thin semireflective layer was deposited on them and another semireflective flat was placed over, and parallel to, the substrate and sample to form a thin parallel air gap. Interference fringes were observed in light of wavelength 5890Å, as in Figure.6.

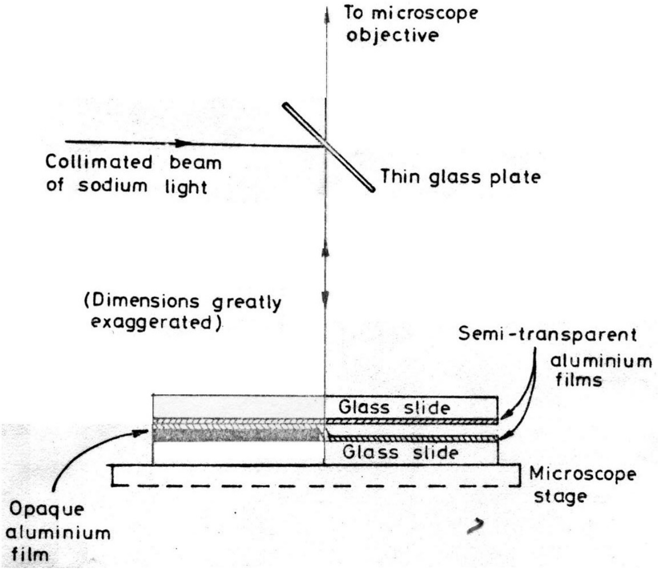


Figure 6: Arrangement of sandwiched films on a microscope stage to determine film thickness by an optical interference method

The fringes are displaced at the edge of the sample due to the change in the spacing in the air gap. It can be shown that this fringe displacement or "fringe step" is given by the relation:

$$FringeDisplacement = FringeSeparation \cdot \frac{SampleThickness}{\lambda/2}$$

provided that the fringes are observed nearly normal to the plane of the sample and $\lambda = 5890$Å is the wavelength of the light used.

Questions

1. Explain why this transverse voltage arises in the samples.

2. Using eq. (13), plot E_y vs. J_x, at constant magnetic field. Repeat at several other B-values. Create a fit function, and calculate the value for R_H, including the uncertainties in your calculation, as well as the χ^2 value. Explain any discrepancies.

3. Using the resistivity and the Hall coefficient determined for both Hall samples (Cr and Ag) calculate the values of the following:
   - Density of charge carriers n
   - The drift velocity v_d
   - The conductive mobility μ.

4. Can you explain why these values arise? Do the values you got to make sense? Be sure to include all errors in your measurements and plots.

5. Explain why the conduction of Cr is different from the conduction of Ag. Compare the Hall coefficients and explain why is one larger than the other (a diagram may be needed).

References

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