Physics
UNIVERSITY OF TORONTO

Kater Pendulum

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2023 C. Lee
2005 G. F. West

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Introduction

In principle it is easy to measure the acceleration due to gravity (g) by measuring period T of a simple pendulum of known length T. The theoretical period of low amplitude oscillation is just

$$T = 2\pi \left(\frac{L}{g}\right)^{\frac{1}{2}} \tag{1}$$

so

$$g = (2\pi)^2 \left(\frac{L}{T^2}\right) \tag{2}$$

Time and distance could be measured in the early 1800's to better than 1 part in 10^4. However, the distance L is defined assuming a simple pendulum with a point mass. Real pendulum have an effective length $L_e$that can be difficult to define and measure.

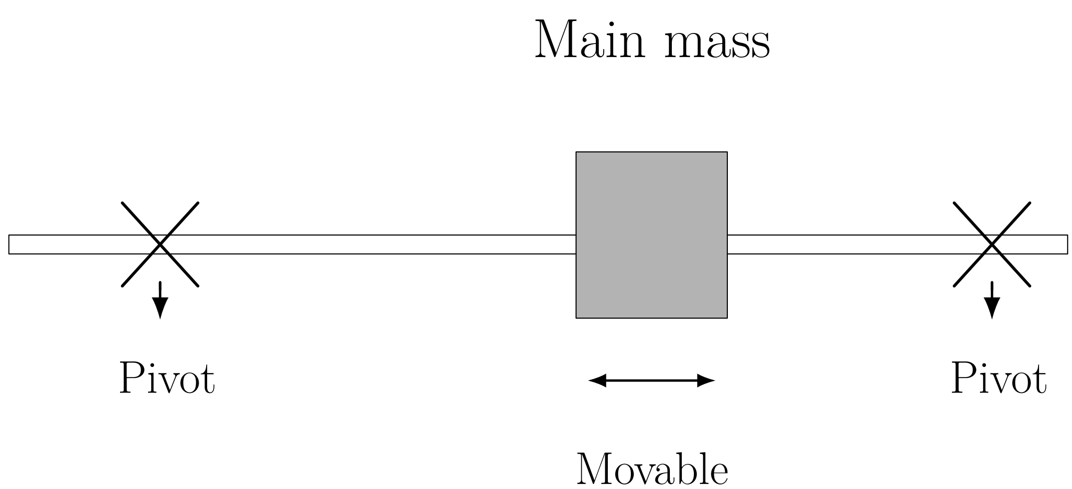

Figure 1: Schematic of a Kater [1818] pendulum. In practice, the movable mass is divided into two or more elements to enable coarse and fine adjustment.

In principle, the effective length is $L_e = \frac{k^2}{r_m}$ where k is the radius of gyration and r_m is the radius of the centre of mass relative to the pivot point. However, neither of these parameters is easy to determine very accurately. Kater's innovation was to realize that and asymmetrical double-ended pendulum (figure 1) could be adjusted to have identical periods from either pivot point. Further, Kater realized that when this condition is fulfilled the effective length of the pendulum is just the distance between the pivot points, which then can be measured by direct observation.

Our Pendulum was designed to be transportable when packed in a wooden box and to swing from a knife edge support clamped to another surface. The ends of the pendulum are formed into narrow rods so the swing can be timed precisely by the rod breaking a light-beam sensor to trigger an electronic switch.

The original method relied on measuring the time between synchronizing of the Kater pendulum and a known, accurate, clock. Counting the ticks of the Kater pendulum and the reference clock between synchronization would give an accurate measure of the Kater pendulum's period. For this reason the pendulum was designed to have a period close to, but not exactly, 2 seconds.

With more accurate reference clocks, we now measure the period of the pendulum directly by measuring the time taken to break the light-beam sensor a pre-defined number of times.

A practical difficulty with the basic design shown in figure 1 is that it can be very hard to make a precise adjustment of the moveable mass. Our pendulum therefore has three masses in addition to the basic support structure: a *fixed* main mass, a *coarse* adjustment mass, and a *fine* adjustment mass. The position of the adjustment mass can be read from the scale engraved on the pendulum.

1 The Experiment

With two masses to adjust the procedure is more complex than a single mass, but better precision can be achieved by using the coarse mass to adjust the period by (relatively) large amounts and the fine mass to adjust the period by (relatively) small amounts.

It is not practical to position the masses such that both pivot points produce precisely equal periods. It is much easier to generate datasets that, when plotted, make a pair of graphs of period (pendulum upright and inverted) versus position of the fine adjustment mass that intersect. This intersection point is the point of equal-periods and period can be used to give a precise measurement of g.

Procedure

One possible procedure is to first keep the fine mass fixed (at any value) and adjust the coarse mass such that both pivot points produce approximately equal periods. After fixing the location of the fine mass on the pendulum choose a location for coarse mass and measure the period from both pivot points. The coarse mass is in a good location if the fine mass can be adjusted to bracket a potential location where both periods are identical. A possible procedure for this step is

1. Place the fine mass near one extreme of its scale.
2. Place the coarse mass on its scale.
3. Measure the period from both pivot points
4. Move the fine mass to the other extreme and measure the periods again.

After completing these four measurements if the period range for the upright and inverted pendulum overlap the coarse mass is well places. If the periods do not overlap the coarse mass should be moved away from the pivot that gives the shorter period.

With a fixed coarse mass the next step could be to adjust the fine mass to find the identical periods from both pivot points. Since this method is impractical it is simpler to measure the periods for a range of fine mass locations and infer (graphically or mathematically) the position that would give an equal period.

The position of the masses are not critical to experiment, but the period of oscillation them produce is critical.

1.1 Measuring the Pendulum

The Kater pendulum is designed to give a period of about 2 seconds, so its length is *about*1 metre. More precise measurements can be made using a device called a Cathetometer. The Cathetometer is a large steel scale that uses a Vernier scale and a telescope to measure locations on an object. The entire device is mounted on adjustable screws and a bubble level to ensure it measures distance vertically.

For any fine measurements in this experiment, the Cathetometer can be used by measure the location on the pendulum relative to a fixed location(e.g. an accessible pivot point).

1.2 Analysis

Find the value of gravity as precisely as possible. This can be done by manually finding the intersection of the pendulum periods. More practically the period versus length from both pivot points can be used to find the intersection numerically. In Python (PHY224, PHY324) the 'scipy.optimize' module contains a function 'fsolve' that will find the intersection of two lines. You should calculate the period of the Kater pendulum, the acceleration due to gravity, and characterize the uncertainties in this measurement

1.3 Determining the Uncertainty

There are many sources of uncertainty in this experiment. Random experimental error arises from measurements of period and length, and systematic errors *might* by caused by some of the following causes.

- Calibration errors in timer clock and cathetometer.
- Finite amplitude of pendulum
- Buoyancy of air
- Damping of pendulum due to friction
- Imperfect knife edges
- Temperature variations during measurement
- Elastic variations of pendulum length
- Flexibility of pendulum affecting its period.

Some of these errors can be ruled out without further experiments, other might require additional measurements such as repeating the procedure entirely or other methods. It might also be possible to calibrate the Cathetometer and timer with other accurate sources of time or distance. Finally, you can compare your measure of gravity with the expected value at the location of the Kater pendulum on Earth using the @Bouger formula or with modern gravimetric methods at nearby reference sites.

2 Appendix

The counter-timer precisely measures the time duration of 8, 16 or 32 full periods of the pendulum. It does so by counting cycles of a 1 MHz quartz crystal oscillator. Brief instructions for the counter-timer are pasted on the top of instrument.

An infra-red optical source and sensor is mounted on the box below the pendulum. It generates a signal (a "tick") each time the lower end of the pendulum swings through it. Two "tick" signals make one oscillation of the pendulum and individual ticks are marked on the timer front panel green LED.

The timer is started by pressing and releasing the 'reset' button on the panel. After release n full oscillations are measured starting from the first beam-break. The numerical value of n (the count limit) may be set to 8, 16 or 32 by the front panel switch. The duration of the count interval is given on the numeric display of counter timer in seconds.

When the period counter is reset and waiting to start up, a yellow LED is lit on the panel. This LED turns off during the n period count interval, and it turns back on when the count finishes. After initial power–up, or following any change of the n setting, or after a reset during a count, the pendulum must continue to operate for n periods before the counter-timer can accept a reset. At this time, the yellow LED comes on. The pendulum must swing for this counting to occur.

The accuracy of the 1 MHz oscillator in the timer unit has not been checked recently, but it is likely about 1 in 100,000.

References

H. Kater. An account of experiments for determining the length of the pendulum vibrating seconds in the latitude of london. *Philosophical Transactions*, 104:33–103, 1818.