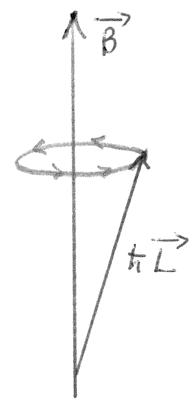
The Zeeman Effect

The *Zeeman* effect has played an important role in the development of quantum theory. It illustrates the phenomenon of space quantization, which refers to the angular momentum $\hbar \mathbf{L}$ of the atom assuming only a set of discrete orientations with respect to an external magnetic field $\mathbf{B}$.

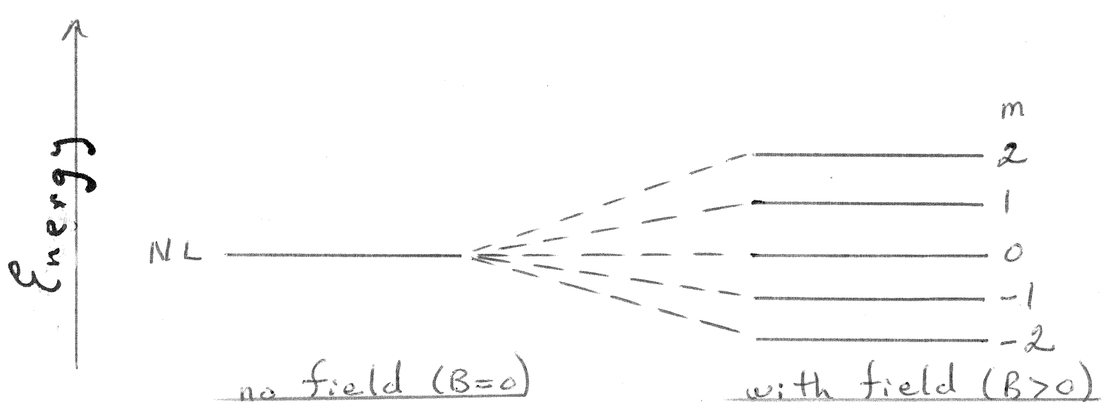


We recall that an electron in a *Bohr* circular orbit will have a magnetic dipole moment $\mu = |I|\, A$ in a direction perpendicular to the plane of the orbit given by the right hand rule for circulating current. $A = \pi R^2$ is the area of the orbit. Viz. $I = -e/T$ in coulomb per second and $pR = mvR = \hbar L$ the orbital angular momentum with $T = 2\pi R/v$ the period. $I = ev/2\pi R$ and $\mu = e\hbar L/2m = \mu_o L$. $\mu_o = e\hbar/2m$ is the *Bohr* magneton.

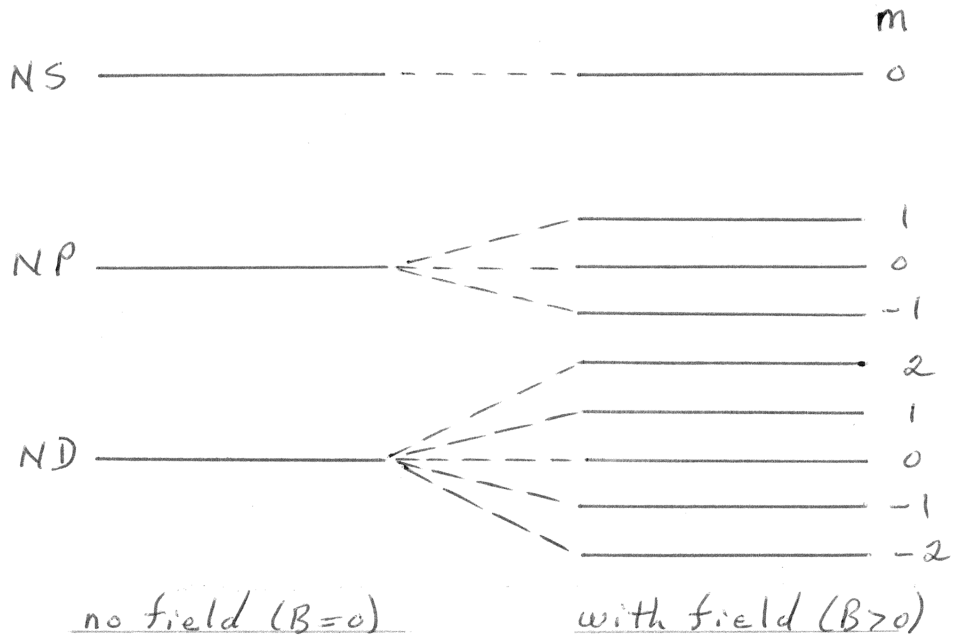
In fact we find in general that $\boldsymbol{\mu} = -\mu_o \mathbf{L}$. The torque on the atom is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} = -\mu_o \mathbf{L} \times \mathbf{B}$, which is perpendicular to both $\mathbf{L}$ and $\mathbf{B}$ and causes the tip of the orbital angular momentum vector $\hbar \mathbf{L}$ to precess in a circular orbit about $\mathbf{B}$. The interaction energy between the atom dipole moment and the field is :

$\Delta E = -\boldsymbol{\mu} \bullet \mathbf{B} = +\mu_o \mathbf{L} \bullet \mathbf{B}$ and with $\mathbf{B} = B\,\mathbf{z}$ then $\Delta E = \mu_o B L_z$

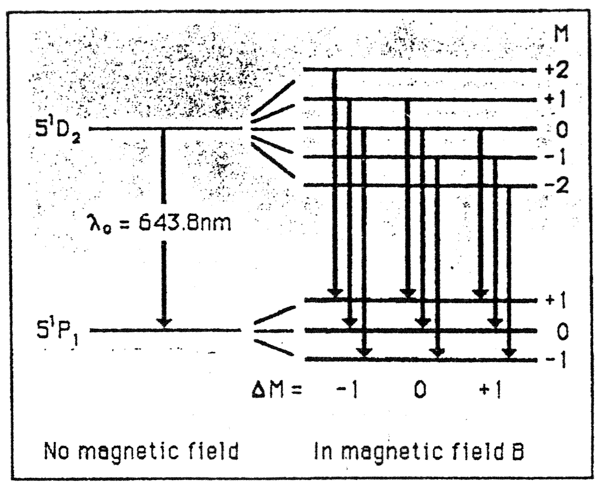
Quantum mechanics predicts that $L_z = m$ where $m = -L, -L+1, \ldots, 0, 1, 2, \ldots, +L$ only and so $\Delta E = \mu_o B m$. Therefore when an atom is placed in a magnetic field, the energy level with principle quantum number N and orbital angular momentum $\hbar L$ will split into $2L + 1$ sub levels.



The orbital angular momenta of states of the atom are given letter labels:
S (for L =0), P (for L = 1), D (for L = 2), F (for L = 3), G (for L = 4),....etc.



Cd Transition:

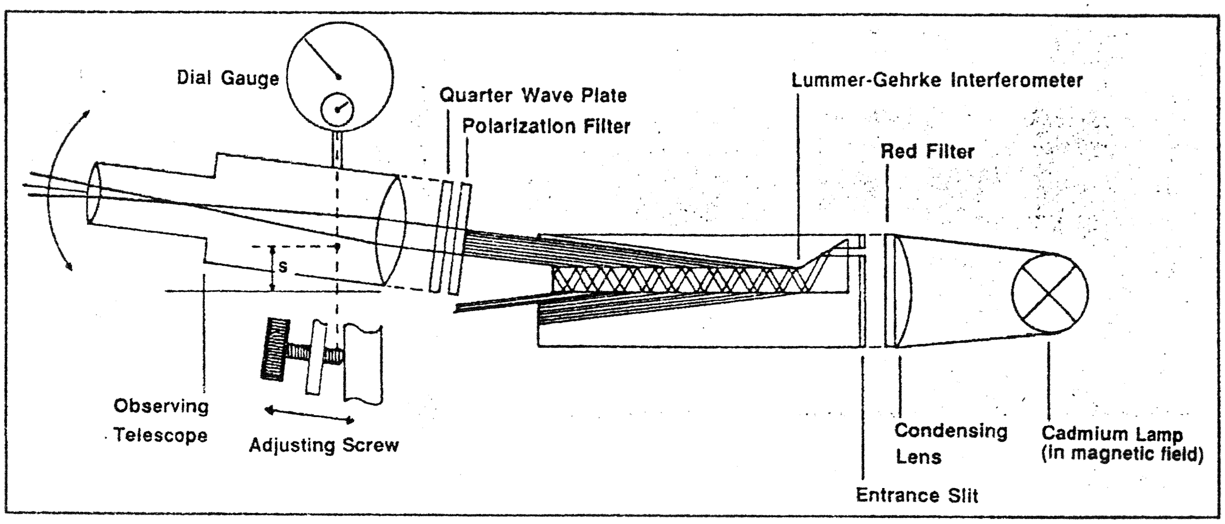


We have three allowed electric dipole transitions with the magnetic field 'on' and they have frequencies $\nu = \nu_o$, $\nu = \nu_o + \mu_o B$ and $\nu = \nu_o - \mu_o B$. $\Delta m = +1$ and -1 correspond to circularly polarized light when viewed along the **B** direction. $\Delta m = 0$ corresponds to linear polarization along the **B** direction. A propagating electromagnetic wave has **E** and **B** perpendicular to the direction of travel. Therefore when viewed along the **B** direction, we do not see the wave with frequency ν_o but we do see the $\nu = \nu_o \mp \mu_o B$ waves as right and left hand circularly polarized waves respectively. On the other hand, when viewed along a direction perpendicular to **B**, the $\nu = \nu_o$ appears as a linearly polarized wave with

E parallel to **B,** and the $\nu = \nu_o \mp \mu_o B$ appear as linearly polarized waves with **E** perpendicular to **B**.

The Experiment

In this experiment one wishes to measure the Zeeman splitting of the red line of Cadmium (Cd) in a magnetic field. The electromagnet and the power supply provided will produce uniform fields up to 0.8T. You may use a Hall probe to measure **B** at the location of the Hg-Cd discharge lamp located between the magnet pole pieces. The magnet assembly can be rotated to allow you to view the Cd red lines both parallel and perpendicular to **B**.



A red filter is used to remove other emission lines in front of the Lummer plate. When the magnetic field is turned on, each of the red interference lines splits into two or three lines. A measurement of this splitting ΔS can be made with respect to the spacing ΔA of successive interference lines with the field off. The change in wavelength is :

$$\Delta\lambda = \frac{\Delta S}{\Delta A}\,\frac{\lambda^2}{2d}\,\frac{1}{\sqrt{\eta^2-1}}$$

where λ = 643.8 nm for the red line in cadmium, d = 4.04 mm the thickness of the Lummer plate and η = 1.4567 the refractive index of quartz glass of the Lummer plate.

$$c = \nu\lambda \Rightarrow \Delta\nu = \Delta\left(\frac{c}{\lambda}\right) = -\nu\frac{\Delta\lambda}{\lambda} = -\frac{\nu^2}{c}\Delta\lambda$$

$$\Delta\nu = -\frac{\nu^2}{c}\frac{\Delta S}{\Delta A}\frac{\lambda^2}{2d}\frac{1}{\sqrt{\eta^2-1}}$$

$$|\Delta\nu| = \frac{c}{2d}\frac{\Delta S}{\Delta A}\frac{1}{\sqrt{\eta^2-1}} = \frac{\mu_0 B}{h} = \frac{e\hbar}{2m}\frac{B}{h}$$

$$|\Delta\nu| = \frac{eB}{4\pi m}$$

The discrete nature of the Cd red lines in a magnetic field is consistent with a model of the Cd atom in which the total orbital angular momentum vector can only exist in discrete orientations with respect to the direction of the applied magnetic field. This so-called *space quantization* is a major success of the Schrodinger model of the atom and of quantum mechanics as we know it today.

A further test of theory can be made by measuring the polarization of the three Cd lines in the magnetic field. Your measurements should be made both parallel and perpendicular to **B**. Use the polaroid and the quarter wave plate. Would you expect your results to be different if you use them first in front of the Lummer plate and then behind the Lummer plate?

Be *careful* with this apparatus. When in doubt ask your demonstrator to check it. The Hall probe is *fragile* and you must *tighten* down the pole pieces before turning on the magnet power supply.