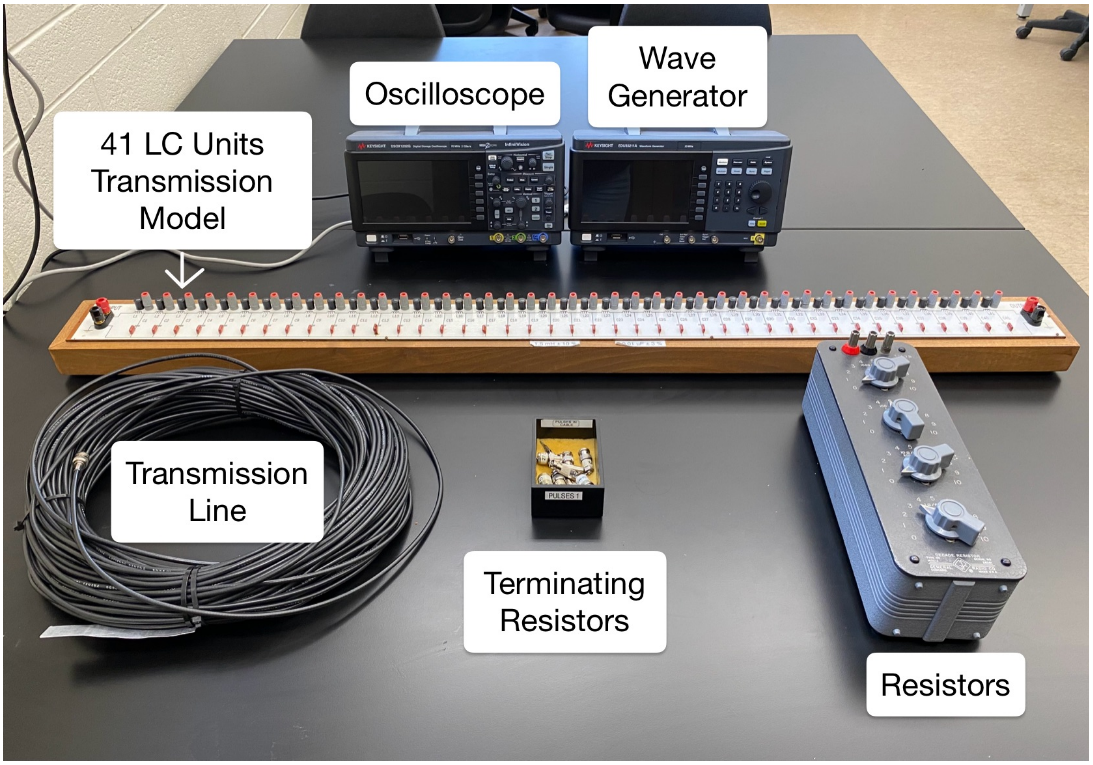
Physics
UNIVERSITY OF TORONTO

Pulses in cables



Revisions

2023 C. Lee, T. Vahabi
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2007 R. Serbanescu & L Helt, N Edwards.

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Introduction

Standard AC and DC circuit theory assumes that the speed of propagation of information in a circuit is infinite. If a change occurs anywhere in the circuit, then the rest of the circuit instantaneously reacts to that change. In reality pulses do not actually travel even as fast as speed of light c in cables, but are reduced by a factor which is related to the physical properties of the cable insulator.

The ideal transmission line

Transmission lines are discrete ladder-like networks of inductors (L) and capacitors (C) capable of storing and transmitting electric and magnetic energy. The basic unit of an ideal transmission line is an LC element of length dx and zero electric resistance (see Figure 1).

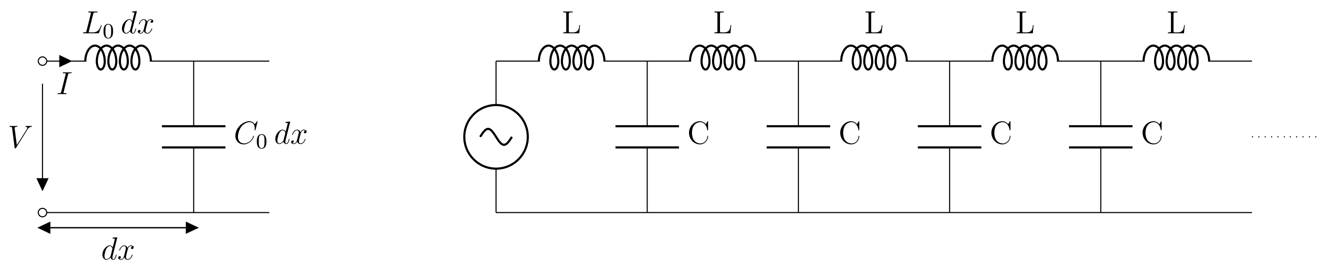

Figure 1: (left) basic unit of an ideal transmission line, (right) an ideal transmission line

Analyses of the rates of change for both current I and voltage V take into account the self inductance of the element (L_0dx) and its capacitance (C_0dx)

$$\frac{\partial V}{\partial x} = -L_0\frac{\partial I}{\partial t} \tag{1}$$
$$\frac{\partial I}{\partial x} = -C_0\frac{\partial V}{\partial t} \tag{2}$$
$$\tag{3}$$

This leads to the *wave* equations for voltage and current:

$$\frac{\partial^2 V}{\partial x^2} = L_0C_0\frac{\partial^2 V}{\partial t^2} \tag{4}$$
$$\frac{\partial^2 I}{\partial x^2} = L_0C_0\frac{\partial^2 I}{\partial t^2} \tag{5}$$
$$\tag{6}$$

The velocity of wave propagation is defined as

$$\nu^2 = \frac{1}{L_0C_0}. \tag{7}$$

We can integrate the voltage drop along an infinitesimal unit of length (equation 1) with a solution of

$$V_+ = \nu\, L_0 I_+ \tag{8}$$

where the subscript + means the positive direction of wave propagation.

Then the ratio of voltage to current is called the characteristic impedance of the line Z_0:

$$\frac{V_+}{I_+} = Z_0 = \nu\, L_0 = \sqrt{\frac{L_0}{C_)}}. \tag{9}$$

Coaxial cables

Consider two conductor cables, separated by a dielectric material (Figure 2) with a continuous distribution of LC elements. Inductance per unit length can be written as:

$$L_0 = \frac{\mu}{2\pi} \ln \frac{R_2}{R_1}, \tag{10}$$

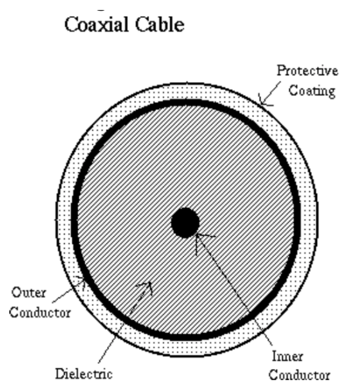

Figure 2: Cross section of a coaxial cable.

where R_1 and R_2 are the radii of the inner and outer conductors, respectively; μ is the magnetic permeability of the dielectric.

Capacitance per unit length is given by:

$$C_0 = \frac{2\pi\varepsilon}{\ln \frac{R_2}{R_1}}, \tag{11}$$

where ε is the permittivity of the dielectric.

Taking the product of equations 10 and 11 it can be verified that

$$\frac{1}{L_0 C_0} = \frac{1}{\varepsilon\mu} = \nu^2 \tag{12}$$

Knowing the velocity of signal travel in a certain coaxial cable, we can accurately calculate the travel time of an electrical signal along a given length.

A useful visual analogy of a pulse traveling in the cable is that of a wave moving along a rope. We shall see that the action of the pulse at the end of the cable has a strong analogy with what happens at the fixed end of an oscillating rope, suggesting that the math might be similar.

In the laboratory we shall see these effects by using pulses of duration $\sim 10^{-8}$ seconds = 10ns, since this corresponds to a "length" of ∼3m.

"Pulse length" is the real physical length of the pulse in the cable, so we can use the "length" of the pulse in either time units or length units.

The load effect

To determine the effect of a load with impedance Z_L at the end of the cable we can solve the boundary conditions that must be satisfied by a wave travelling in the positive + and negative − at the boundary.

Inside the cable the impedance remains the same

$$\frac{V_+}{I_+} = Z_0 = -\frac{V_-}{I_-}, \tag{13}$$

at the end of the cable the load Z_L sees the combined voltage $V = V_+ + V_-$ and current $I = I_+ + I_-$ to give

$$Z_L = \frac{V}{I}. \tag{14}$$

These equations can be used to define a reflection coefficient $r = \frac{V_-}{V_+}$ and a transmission coefficient $t = \frac{V}{V_+}$

$$r = \frac{Z_L - Z_0}{Z_L + Z_0}, \tag{15}$$

$$t = \frac{2Z_0}{Z_L + Z_0} \tag{16}$$

We can solve this easily for the extreme values of Z_L. An **open circuit** is effectively $Z_L = \infty$, leading to complete reflection $r = 1$, $t = 0$ shown in figure 3. A **short circuit** is $Z_L = 0$ and $r = -1$,(t is undefined as there is nothing to transmit into) - the reflection is inverted in a short circuit as shown in figure 3

Given these two extreme cases, it is plausible to assume that for some intermediate value of resistance at the end, the reflection coefficient $r = 0$. In that case, there is no difference between putting that value of impedance and extending the length of the cable to infinity. In either case, the pulse never returns to the first piece of cable. This value of impedance needed for no reflection is referred to as the characteristic impedance of the cable, defined in equation 9 as Z_0. For our cable $Z_0 = 50\Omega$. [1]

[1] This *impedance matching* is important when designing connectors for analog signals and passive electronic components like speakers. Matching the cable and speaker reduces the reflections in the cable improving the signal quality.

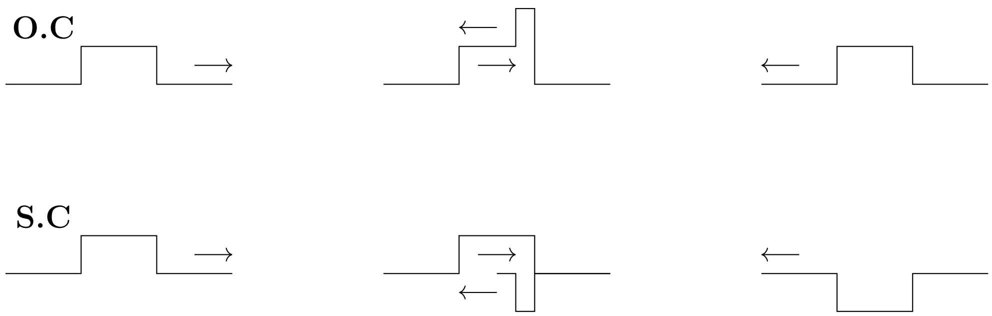

Figure 3: O.C: Pulse reflections from an open ended conductor. S.C: Pulse reflections from an short circuited conductor.

Assuming only resistive impedance (R_L) terminating the cable, it is reasonable to use the simplified expression:

$$r = \frac{R_L - Z_0}{R_L + Z_0}. \tag{17}$$

Voltage and current are $\pi/2$ out of phase in space and time. A standing wave is formed in a short-circuited transmission line of length when the signal is a sinusoidal wave. At the end of the line voltage will be zero and current will be maximum. The total energy propagation will be zero.

Signals are not propagated without losses. Attenuation is the most visible defect of any cable:

$$\text{Attenuation(dB)} = 10 \log_{10} \frac{\text{OutputIntensity(W)}}{\text{InputIntensity(W)}} = 10 \log_{10} \left(\frac{V_{\text{reflected}}}{V_{\text{initial}}} \right)^2 \tag{18}$$

Attenuation is also a function of signal frequency and cable length l:

Apparatus notes

You will be using a KEYSIGHT EDU33211A waveform generator for coaxial cable studies. You'll use a digital oscilloscope to display and measure the pulses.

The common setup for all of the experiments is shown in figure 4. Connect the waveform generator to the oscilloscope through a TEE connector, and connect the transmission line to the open end of the TEE connector.

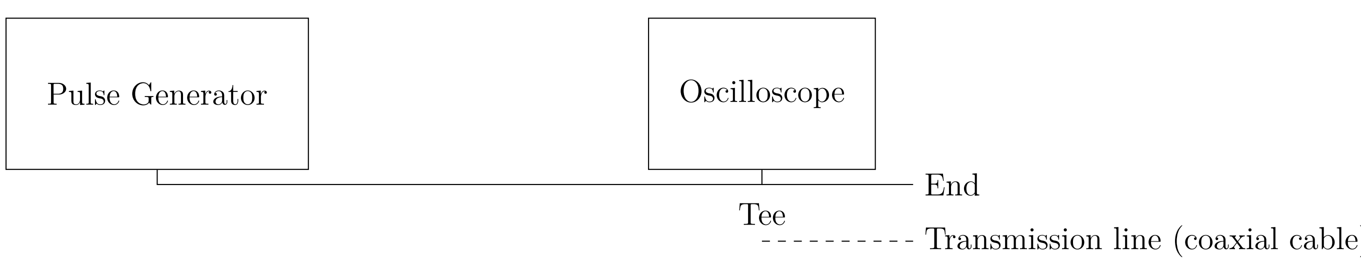

Figure 4: Circuit used to study transmission lines and coaxial cables

The TEE piece is symmetrical in all three arms. The pulse generator has an impedance of Z0 and the oscilloscope has very large impedance. Can we safely ignore the presence of the oscilloscope as far as reflections go?

You should now be able to determine what comes back to the oscilloscope for any termination of the long cable, particularly for short-circuit and open-circuit cases. As a check consider that any result must also be true as we increase the length of the pulses so that they overlap and eventually go to the DC levels on the cable. For example, if the cable is short-circuited in the DC case, there can never be any potential difference in the circuit. What will it be the case for an open circuit?

Exercise 1: A transmission line

Use the 41 LC units transmission line (**TL**), and KEYSIGHT EDU33211A waveform generator. Connect the waveform generator using the TEE to channel 1 and through a TEE connector to the transmission line to the TEE connector. Connect the output of the transmission line to channel 2 of the oscilloscope.

Start with a pulse width of around $75\mu s$ and a frequency of $300Hz$. Vary the pulse duration and frequency until you observe a returned pulse on both Ch1 and Ch2. The pulse width and the frequency can be altered through entering the pulses waveform. Explain what each channel measures.

Insert a resistive load R across the end of the **TL**. Vary the resistance R until there is no returned pulse. This will be the impedance match between the source and the load. Use the Oscilloscopes **MATH** menu to combine channel 1 and 2 if necessary.

The transmission line behaves like a long cable with a known impedance by using inductors-capacitors (LC) units to behave like a large impedance. Every LC unit also has a connector to measure the signal. Determine the speed of pulse propagation along the line and compare with the calculated value from equation 7. One way to determine the speed is to measure the delay times of the transmitted and reflected pulses along the **TL**. Plot the delay time verses the number of LC units and fit a suitable equation to the data to find the speed of propagation (probably in units of LC-units/second). In the **TK** the capacitors have a value $0.01\mu F$ and inductors have a value of $1.5mH$.

Exercise 2: Coaxial Cables

Use the KEYSIGHT EDU33211A waveform generator with various cable lengths, terminations and pulse widths. The pulse width should be adjusted to 50ns - the frequency is less important and can be set to around 15kHz. You should be able to investigate the open circuit, short circuit and several resistive terminations.

Measure the delay time of the pulse and compare it to the result using the speed of light. Pulses do not actually travel as fast as light in the cable, but are reduced by a factor which is related to the dielectric constant of the insulator.

Measure the attenuation factor of the cable. Express your answer in dB/m.

Hint: use an open circuit so that the entire pulse is reflected.

Find the value of Z_0 for the cable.

Some cable constants: $\varepsilon = 2.25$; $\mu = 1$ (polyethylene)

Geometrical characteristics (radii) can be found at: https://www.belden.com/products/cable/coax-triax-cable/50-ohm-coax-cable.