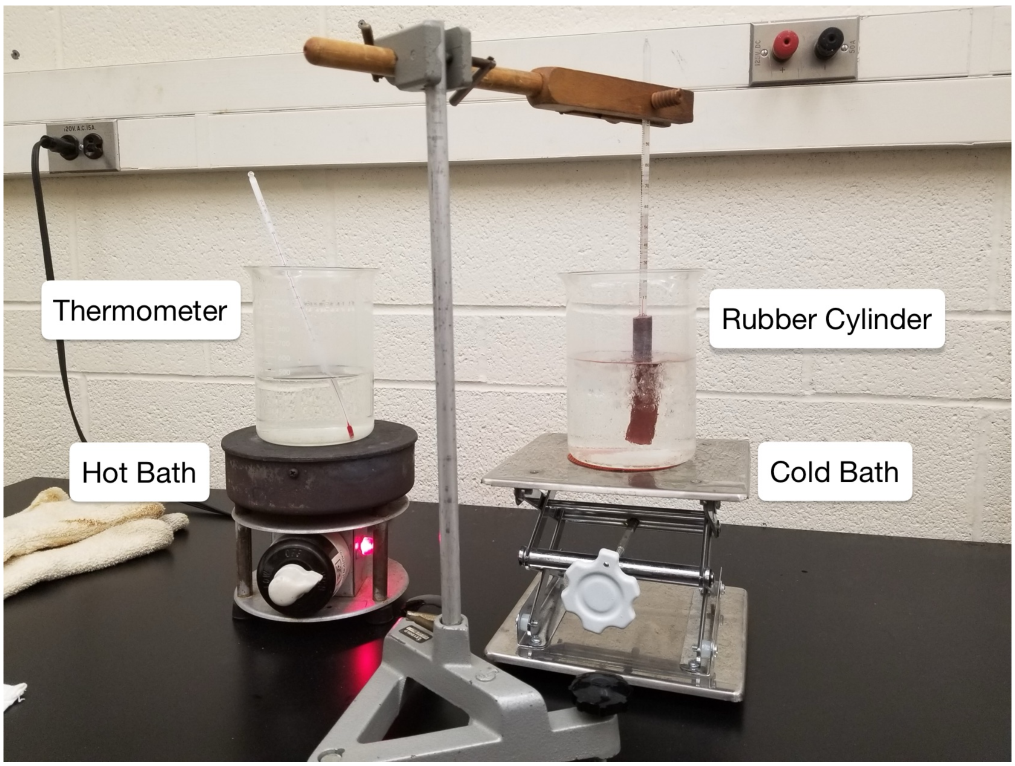
Physics
UNIVERSITY OF TORONTO

Thermal Diffusivity



Revisions

2023 B. Wilson

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Overview

You will place a thermometer with a rubber cylinder wrapped around its bottom into alternating hot and cold baths. By measuring the time delay of the thermometer's response to the changing external temperature you will measure the thermal diffusivity of the rubber cylinder. You will repeat the experiment with several different periods of alternation between hot and cold. You will also need to measure the inner and outer radii of the rubber cylinder which covers the thermometer.

Boiling water, and the hot plate, are dangerous. Please handle with care!

Introduction

The motion of thermal energy through an object (given certain assumptions) is described by the equation of thermal diffusion

$$\frac{\partial T}{\partial t} = m\nabla^2 T \tag{1}$$

where T is the temperature (which depends on both time and position) and m is the thermal diffusivity which is a constant for any given substance. Typical values of m are around 10^{-4} m^2/s for good heat conductors, and 10^{-7} m^2/s for heat insulators.

The rubber you will be using is in the shape of a long ($z \gg r$) cylinder. This allows us to assume axial symmetry, which reduces equation (1) to the simpler form

$$\frac{\partial T}{\partial t} = m\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right). \tag{2}$$

Equation (2) is conveniently solved by the method of separation of variables. We write $T(r,t) = R(r)\exp(i\omega t)$, where ω is the angular frequency of the temperature applied to the surface of the rubber. If we substitute this trial solution into equation (2) we get

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \lambda^2 R \tag{3}$$

where $\lambda^2 = -i\omega/m$. This differential equation is known as Bessel's Equation of order zero. It was first studied by Daniel Bernouilli in 1738 when he looked at the oscillation of heavy chains! Fourier followed the development above for heat flow in 1822 and obtained Bessel's equation too.

A solution to equation (3) relevant to our situation (our periodically changing boundary conditions) is J_0, a Bessel function of order zero. Setting $z = \lambda r$ we get

$$J_0(z) = 1 - \frac{z^2}{2^2} + \frac{z^4}{2^2 4^2} - \frac{z^6}{2^2 4^2 6^2} + \ldots \tag{4}$$

Remember that z is complex.

What we care about is the phase ϕ of our solution. This can be found to be

$$\tan\phi = \frac{\frac{x^2}{2^2} - \frac{x^6}{2^2 4^2 6^2} + \ldots}{1 - \frac{x^4}{2^2 4^2} + \ldots} \tag{5}$$

where $x^2 = r^2\omega/m$. Note that we're back to using real numbers now. At $r = 0$ we have $\phi = 0$, so ϕ is a measure of the phase difference from the centre of the rubber cylinder to any other location a distance r from the centre.

We can assume that the thermometer measures T at the inner radius of the rubber cylinder. The outer radius of the cylinder is in contact with the bath water which is alternatively 100° or 0°. The time delay Δt between when you move the thermometer from one bath to the other and when the thermometer reaches a maximum or minimum temperature is a measure of $\phi(r_1) - \phi(r_2) = \Delta\phi = \omega\Delta t$. See Figure 1 for sample data.

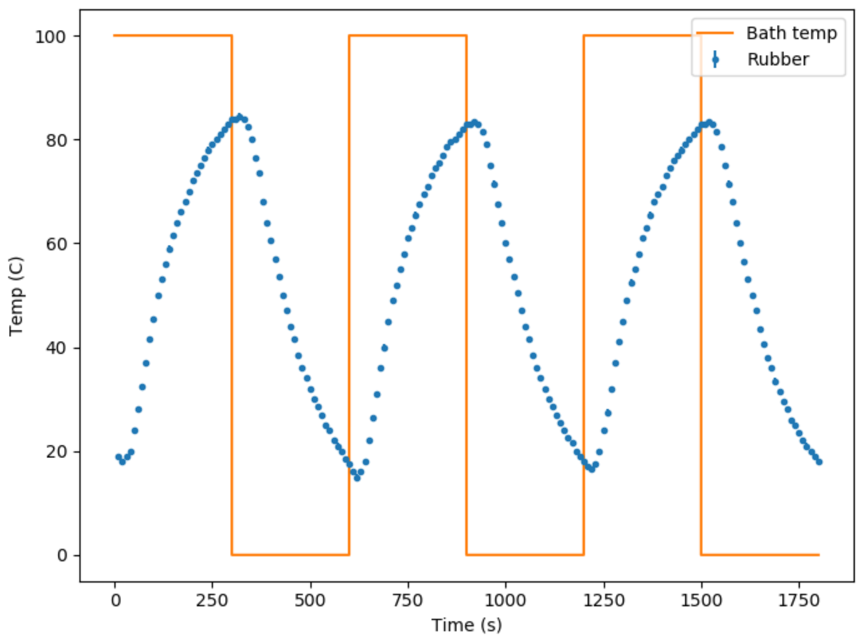

Figure 1: Sample time series data of the reading of the thermometer (blue) alternating between the two baths (bath temperature in red). Note that the errorbars are too small to be seen.

Experiment

Fill one glass jar with water and put it on the hot plate. Turn the hot plate to high. Wait for the water to boil. Meanwhile, fill the other glass jar most of the way with ice, then add some water. Ideally, the heights of the water in both jars will be about equal, and the ice-water will always have some ice but not so much ice that the ice reaches the bottom of the jar once the experiment starts.

Gently tighten the clamp on one end of the thermometer with the rubber cylinder in it so that the rubber is mostly in the water but does not touch the bottom of the jar. Adjust the height of the cold water jar using the knob on the metal scissor lift so that both jars are the same height. You want to be able to quickly move the thermometer from one jar to the other, but not have to hold it by hand, and make sure the thermometer never touches glass.

Measure the two r values (inner radius and outer radius) of the rubber cylinder. Measure Δt for at least six different bath angular frequencies ω. Plot $\Delta t(\omega)$ and fit it appropriately to find m, which is your only free parameter. This is the important graph, you should not include all your graphs like Figure 1 except in the appendix or, perhaps, including one as a guide to explain what the time delay is. Depending on how you decide to take the needed data, you might not even have graphs like Figure 1 for most of your angular frequencies.

References

[1] Carslaw, H.S. and Jaeger, J.C. Conduction of Heat in Solids. Clarendon Press, Oxford, 1959.

[2] Abramowitz, M. and Stegun, I. A. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. Dover Books on Advanced Mathematics, Dover Publications, New York, 1965.