Measuring ”g” via precession of an air gyroscope

1. Experiment

In this experiment, the gyroscope (Figure 1) is a precisely machined sphere sitting on a thin cushion of air produced by a jet underneath it. The sphere is truncated, meaning it has been sliced to remove a spherical cap. The gyroscope is rotated with its axis of rotational symmetry horizontal. It is then driven at a constant angular velocity by an alternating magnetic field produced by a coil connected to the A.C. mains.

Thus, the gyroscope and coil form a synchronous motor with the rotor not fixed to an axis, wherein the angular momentum $I\vec{\omega}$ can change direction freely within the horizontal plane.

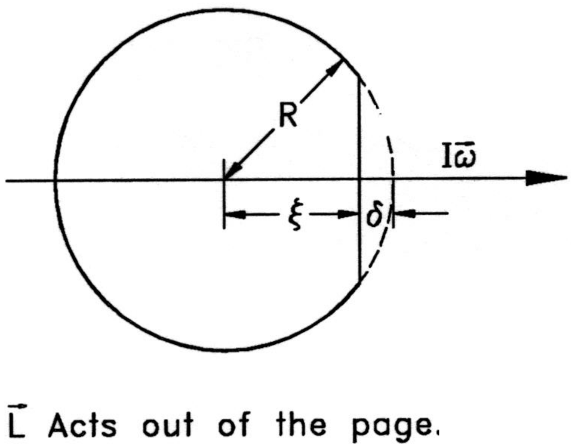

Figure 1: The gyroscope

The truncation in the sphere results in an imbalance, meaning the Earth's gravity will produce a torque $\vec{L}$ acting in the horizontal plane and perpendicular to the rotational symmetry axis. This causes a pure precession of the sphere around the vertical axis with angular velocity $\Omega = \frac{L}{I\omega}$.

Assuming a uniform density of the sphere, L and I are functions of: radius R, density ρ, and the distance from the centre of the flat face to the centre of the sphere, ξ.

L and I take the form of double integrals; as part of your experimental writeup, you must evaluate these integrals yielding, equations (1) and (2) below. We define the quantity $\epsilon = 1 - \frac{\xi}{R}$ for convenience.

$$L = g\rho\frac{\pi R^4}{4}\epsilon^2(2-\epsilon)^2 \tag{1}$$

$$I = \rho \frac{\pi R^5}{10} (2-\epsilon)^3 \left(\epsilon^2 + \epsilon + \frac{2}{3} \right) \tag{2}$$

Thus:

$$\frac{L}{I} = g \frac{5}{2R} \frac{\epsilon^2}{\left(2-\epsilon\right)\left(\epsilon^2 + \epsilon + \frac{2}{3}\right)} \tag{3}$$

2. Procedure

Begin by measuring the sphere with calipers provided. Measure both the nominal diameter $2R$ and the truncated diameter $R + \xi$.

Note: You should find that the errors for these measurements contribute dominantly to the error in your final result for g, so take extra care to measure precisely.

Place the gyroscope in the base and open the bottom jet. For best results, aim for a pressure of 0.6 psi. Avoid opening it too far or you risk popping the hoses. Open the side jet all the way while holding a pencil or other thin, blunt object against the centre of the flat of the rotor such that the axis of rotational symmetry is held horizontal. Once the sphere's rotation is stable (but accelerating), use a strobe light to track the spheres angular speed, adjusting the strobe frequency so that the image of the sphere under the light remains still. Ensure the light is not strobing at some sub-multiple of the selected frequency; in this case, the small divot cut off-centre on the flat face will appear in multiple places. Be sure also to calibrate the strobe against the built-in 120Hz vibrating reed. Stabilize the angular speed at just over 60Hz by gently adjusting the side air jet pressure once the gyroscope reaches this point. Turn the magnet on to the max setting, then as slowly as possible, close the side jet. The sphere's rotation should lock to the field of the coil. This may require several attempts.

Once the driving jet is cut off, the sphere precesses freely with a period $T \approx$ 10min. You may wish to reduce the bottom air jet somewhat (until the noise changes pitch) to reduce any torque from it. The period is easily measured by reflecting a laser off the flat side of the sphere onto a screen and timing passages of the beamspot when the beam is reflecting again. The first few precessions will be slightly unstable, and thus give you inaccurate period measurements.

3. Analysis

Knowing ω and Ω and the geometry of the gyroscope, obtain as precise a value for g as you can. This will necessitate measuring the precession in both directions (i.e. for both directions of $\vec{\omega}$) to correct for torque imparted by the air suspension due to departure from level.

$\rightarrow$ **Requirement (PHY224/324 only): write a Python program to calculate g, including the error.**

Revised in 2020 by Ruxandra M. Serbanescu. Previous versions: A.Liblong, 2017, J Vise, 1971