The Hall Effect

1 Introduction

In 1879, E. H. Hall observed that when a current-carrying conductor is placed in a transverse magnetic field, the Lorentz force on the moving charges produces a potential difference perpendicular to both the magnetic field and the electric current. This effect is known as the Hall effect [1]. Measurements of the Hall voltage are used to determine the density and sign of charge carriers in a material, as well as a method for determining magnetic fields.

2 Theory [1], [2]

In a conductor, the flow of electric current is the movement of charges due to the presence of an electric field. If a magnetic field is applied in a direction perpendicular to the direction of motion of the charges, the moving charges accumulate such that opposite charges lie on opposite faces of the conductor. This distribution of charges produce a potential difference across the material, that opposes the migration of further charge. This creates a steady electrical potential as long as the charges are flowing in the material and the magnetic field is on. This is the Hall effect.
Consider for example, a thin flat uniform ribbon of conducting material, which is oriented so that its flat side is perpendicular to a uniform magnetic field as seen in Figure 1. The electrons are the charge carriers in this model. Assuming the conductor is arranged with length in the x-direction, width w in the y-direction and thickness t in the z-direction as seen below:

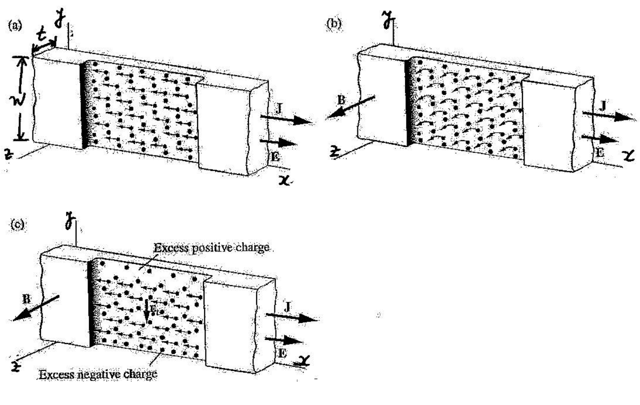

Figure 1: Geometry of the Hall probe [1]

The microscopic form of Ohm's Law relates the current density $J = \frac{I}{A}$ defined as current I per cross-sectional area A of conductor and the microscopic properies of the conducting material: drift velocity v_d, number of charge carriers per unit volume n and the average time τ:

$$\vec{J} = -ne\vec{v_d} = ne^2\tau\frac{\vec{E}}{m} \tag{1}$$

where e is the electronic charge: $e = 1.6 \times 10^{-19}$, and m is the mass of the electron. The drift velocity v_d is the average velocity of the charge carriers over the volume of the conductor. Each charge carrier moves in a random way, undergoing collisions with the lattice. The average time between collisions is τ. Only under the influence of an applied electric field $\vec{E}$ there will be a net transport of carriers along the conductor. Ohm's Law in microscopic form also relates the current density $\vec{J}$ with the electric field applied $\vec{E}$:

$$\vec{J} = \sigma\vec{E} \tag{2}$$

where σ is the electric conductivity.
Using Newton's second law, we can also write:

$$m\vec{a} = \vec{F} = -e\vec{E} \tag{3}$$

and:

$$\vec{a} = \frac{\vec{v_d}}{\tau} \tag{4}$$

In Equation (3), $\vec{a}$ is the average acceleration over a time τ.

Using Eqs. (1)-(3), we obtain:

$$J = \frac{I}{A} = \frac{Q}{At} = \frac{nALe}{A\frac{L}{v_d}} = nev_d \tag{5}$$

When placing the current-carrying conductor in a magnetic field B, the total force has to include the Lorentz force:

$$\frac{m\vec{v_d}}{\tau} = -e(\vec{E} + \vec{v} \times \vec{B}) \tag{6}$$

Assuming the magnetic field in the z-direction, as in Fig.1 and defining the cyclotron frequency $\omega_c = \frac{eB}{m}$, we can write the three components of the drift velocity:

$$v_{d_x} = -\frac{e\tau}{m}E_x - \omega_c\tau v_{d_y} \tag{7}$$

$$v_{d_y} = -\frac{e\tau}{m}E_y + \omega_c\tau v_{d_x} \tag{8}$$

$$v_{d_z} = -\frac{e\tau}{m}E_z \tag{9}$$

With current density J in the x-direction, the magnetic field will drift electrons from the conductor along the negative y-direction, leading to a charge buildup and therefore to an electric field E_y within the conductor. When steady state is reached, the drift down along the negative y-direction stops, electric force due to E_y will cancel the action of magnetic force and the current density will be strictly along the x-direction. Using Equation (7) and steady-state condition: $v_{d_x} = 0$, we obtain the Hall field:

$$E_y = v_{d_x}B_z \tag{10}$$

The Hall voltage is the potential difference across the sample: the quantity you'll measure in this experiment. It is related to the Hall field by:

$$V_H = -\int_0^w E_y dy = -E_y w \tag{11}$$

From equations (5), (7) and (8), we obtain:

$$E_y = -\frac{eB\tau}{m}E_x \tag{12}$$

A convenient experimental quantity is the Hall coefficient, R_H defined as:

$$R_H = \frac{E_y}{J_x B} = \frac{eB\tau E_x}{m}\frac{m}{ne^2\tau E_x B} = \frac{1}{ne} \tag{13}$$

The Hall coefficient is positive if the charge carriers are positive and negative if the charge carriers are negative. The SI units of the Hall coefficient are: m^3/C.
Also related to the drift velocity v_d is the *electric mobility* μ, defined as:

$$\mu = \frac{v_d}{E} \tag{14}$$

Using Eq.(5) and Ohm's Law in microscopic form (2), we obtain a very useful relation between the Hall coefficient R_H and the electric mobility μ:

$$\mu = \sigma R_H \tag{15}$$

The quantity measured in this experiment is not conductivity σ but resistivity $\rho = \frac{1}{\sigma}$.

3 Experimental Methods

The goal of this experiment is to determine the Hall constant R_H and the electric conductivity for Hall samples made of Chromium (Cr) and Silver (Ag). Using these measurements you will calculate some of the microscopic properties of the two conductors: n, v_d and the sign of charge carriers.

The four-wire (Kelvin) method

The electric resistance measurements needed to determine the Hall samples resistivity ρ raise some issues: the values are low for both samples so that the total resistance of wires (leads) is comparable to sample resistance values leading to a high uncertainty.

The common method of using a two-port multimeter to measure a very low resistance is not appropriate in this case. Instead, we shall use a four-wire connection method that reduces the effect of the leads. Figure 2 shows the connection diagram:

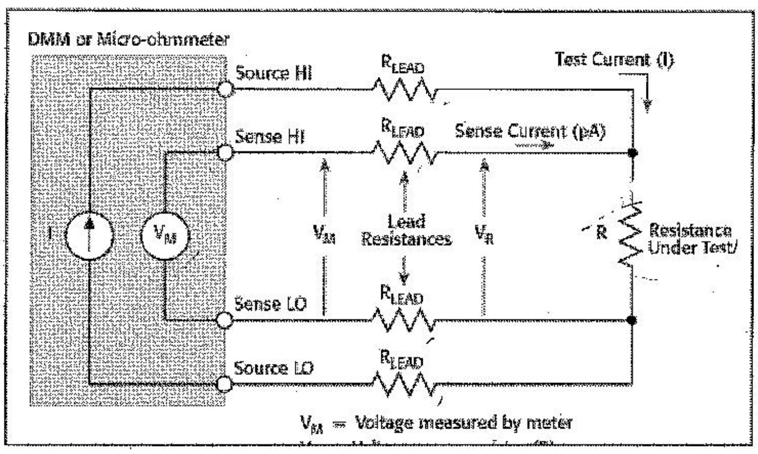

Figure 2: Connection diagram for the four-wire resistance measurement

In Figure 2, DMM stands for digital multimeter. Our instrument is the HP multimeter. The instrument sends a test current I which is forced through the test resistance R via one set of wires, while the voltage across V_M is measured through a second set of wires (sense leads). A very small current, in the order of pA may flow through the sense leads, but its effect is usually negligible and $V_M \approx V_R$.
Note: use very short wires!

4 The experiment

The set up for measuring R_H is sketched in Figure 3.

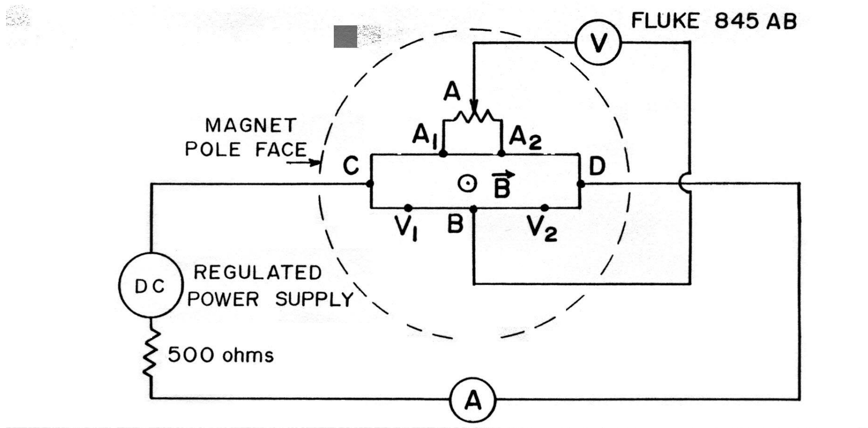

Fig. 2 Hall Chip and Circuit

Figure 3: Diagram of the experimental setup

The limiting 500Ω resistance is already connected to one of the ports of the DC power supply. Using the diagram, identify the wires from the Hall probe (start with Chromium).

The magnetic field is measured with a Gaussmeter. The active part of the gaussmeter probe is within 0.9 mm from the tip. The flat part of the probe should be placed perpendicular to the magnetic field direction. It is recommended to measure $\vec{B}$ at every magnet current at which Hall measurements are made.

Use the potentiometer at A_1A_2 and connect the Fluke high impedance null detector between A and B. The Hall sample has to be in zero $\vec{B}$-field but connected to the power supply. Switch the Fluke instrument to the most sensitive (microvolt) scale and adjust the null by turning the potentiometer cursor. Make sure the potentiometer can adjust the null even when high currents (30mA) are applied .

$\Longrightarrow$ *Question*: Explain why this transverse voltage arises.

Always use the limiting resistor on the power supply and never exceed 40 mA current through the sample, otherwise it will be destroyed. Check the potentiometer setting throughout the experiment.

Place the sample in the centre of the magnetic gap with its broad side perpendicular to the field. Measure the Hall Voltage in the sample, as well as the current.

$\Longrightarrow$ Using eq. (13), plot E_y vs. J_x, at constant magnetic field. Repeat at several other B-values. Create a fit function, and calculate the value for R_H, including the uncertainties in your calculation, as well as the χ^2 value. Explain any discrepancies.

In order to determine the electric conductivity σ needed in eq. (15), you have to measure the electric resistance between V_1 and V_2 using the four-wire method at zero $\vec{B}$. Calculate resistivity ρ. The width w and the distance between V_1 and V_2 can be measured using the traveling microscope provided.

Hall sample thickness measurements

The thickness t of the Hall samples can be determined from the interferometric photographs provided by the manufacturer. In our case:

$$\frac{fringestep}{fringeseparation} \times 2945\mathring{A} = t(\mathring{A})$$

$\Longrightarrow$ Using the resistivity and the Hall coefficient determined for both Hall samples (Cr and Ag), calculate the values of the following:
- density of charge carriers n
- the drift velocity v_d
- the conductive mobility μ.

$\Longrightarrow$ Can you explain why these values arise? Do the values you got make sense? Be sure to include all errors in your measurements and plots.
$\Longrightarrow$ Explain why the conduction of Cr is different from the conduction of Ag. Compare the Hall coefficients and explain why is one larger then the other (a diagram may be needed).
What to submit: all items marked with $\Longrightarrow$, all plots, Python output: χ^2, fit coefficients.

References

[1] E.M. Purcell, D. Morin: Electricity and Magnetism, 3rd ed. Cambridge Univ. Press, 2013 Ch. 6 (6.9).

[2] A. Melissinos, J. Napolitano: Experiments in Modern Physics, 2nd ed. AP 2003 Ch. 2 (2.3)

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