Interference and Diffraction of Light

References:
[3] PHYWE Series of Publications, Laboratory Experiments, Physics LEP 2.3.01, PHYWE SYSTEME GMBH

Introduction
Observations of light passing through narrow openings show that light spreads out behind the opening and forms a distinct pattern on a distant screen. By scanning the pattern with a light sensor and plotting light intensity versus distance, differences and similarities between interference and diffraction are examined.

Observation of diffraction intensity can be used in a simple quantum mechanical treatment to confirm the Heisenberg’s uncertainty principle.

Diffraction
When diffraction of light occurs as it passes through a slit, the angle to the minima (dark spots) in the diffraction pattern is given by

$$a \sin \theta = m' \lambda \quad (m' = 1, 2, 3, \ldots) \quad (1)$$

where "a" is the slit width, \( \theta \) is the angle from the center of the pattern to the \( m' \)th minimum, \( \lambda \) is the wavelength of the light, and \( m' \) is the order of diffraction (1 for the first minimum, 2 for the second minimum, ...counting from the center out).

In Figure 1, the laser light pattern is shown just below the computer intensity versus position graph. The angle theta is measured from the center of the single slit to the first minimum, so \( m' = 1 \) for the situation shown in the diagram.

Double-Slit Interference
When interference of light occurs as it passes through two slits, the angle from the central maximum (bright spot) to the side maxima in the interference pattern is given by

$$d \sin \theta = m \lambda \quad (m = 0, 1, 2, 3, \ldots) \quad (2)$$

where "d" is the slit separation, \( \theta \) is the angle from the center of the pattern to the \( m \)th maximum, \( \lambda \) is the wavelength of the light, and \( m \) is the order (0 for the central maximum, 1 for the first side maximum, 2 for the second side maximum, counting from the center out). In Figure 2, the laser light pattern is shown just below the computer intensity versus position graph.

The angle theta is measured from the midway between the double slit to the second side maximum, so \( m \) equals two for the situation shown in the diagram.
Intensity in the diffraction pattern
Light intensity is proportional to the square amplitude of the wave producing it ($E^2$, where $E$ is the electric field amplitude). By using the concepts from superposition of waves [1], we obtain:

$$I(\phi) = I(0)\left(\frac{\sin \phi}{\phi}\right)^2 \text{ where: } \phi = \frac{\pi a}{\lambda} \sin \theta$$

(3)

The first intensity minimum is located at $\theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$ according to Equation (1)

Setup
- Mount the single slit disk to the optics bench. Each of the slit disks is mounted on a ring that snaps into an empty lens holder. The ring should be rotated in the lens holder so the slit at the center of the ring is vertical in the holder (see Figure 3). To select the desired slit(s), just rotate the disk until it clicks into place with the desired slit at the center of the holder.

NOTE: All slits are vertical EXCEPT the comparison slits, located on the multiple slit set, that are horizontal. The comparison slits are purposely horizontal because the wide laser diode beam will cover both slits to be compared. If you try to rotate these slits to the vertical position, the laser beam may not be large enough to illuminate both slits at the same time.

- The rotary motion sensor, aperture bracket (set on slit #6) and linear translator are mounted at the end of the optics track (see Figure 4).

- Place the diode laser on the bench at one of the ends. Put the slit holder on the optics bench a few centimeters from the laser, with the disk-side of the holder closest to the laser. Plug in the diode laser and turn it on. CAUTION: Never look into the laser beam.

- Adjust the position of the laser beam (left-right and up-down until the beam is centered on the slit. Once this position is set, it is not necessary to make any further adjustments of the laser beam when viewing any of the slits on the disk. When you rotate the disk to a new slit, the laser beam will be already aligned. Since the slits click into place, you can easily change from one slit to the next, even in the dark. When the laser beam is

Figure 3: Mounting the Slits

Figure 4: Scanner with Light Sensor
properly aligned, the diffraction pattern should be centered on the slits in front of the light sensor (see Figure 5). Begin with the light sensor gain switch set on x10 and if the intensity goes off scale, turn it down to x1.

Familiarization with the patterns
- Start with the single slit wheel setup to 0.16 mm. Attach a small piece of paper to the sensor aperture bracket. Look at the pattern.
- Select the combination slits from the double slit wheel. Look at the pattern produced by each of the four combinations. Draw a diagram of each slit combination and the corresponding diffraction pattern. Note that the single slit from the combination is always 0.04 mm

Single slit exercise
- Mount the single slit wheel with the 0.04 mm slit.
- The low rpm driving motor drives the light sensor along the linear translator between two switch stops (~14 cm travel distance). It can be operated at low speed for data collection or at a higher speed for moving the sensor to a desired position on the linear translator.
- Before starting the recording, move the light sensor to one side of the laser pattern.
- Turn out the room lights and click on the Interference and Diffraction shortcut on the desktop. Click on the ‘Start’ arrow button, located at the upper left corner of the screen, turn on the motor, turn ON the acquisition to scan the pattern.
- Turn OFF the acquisition when you have finished the scan. The acquisition turns OFF by itself when the light sensor reaches the end of travel (switch stop).
- The program allows you to use two pairs of cursors to get the intensity and position along the pattern. Cursors usually reside at the left of the graph. You may save the numerical file in a temporary computer directory and/or print the graph.
- You may have to change the gain setting on the light sensor (1x, 10x, 100x) depending on the intensity of the pattern. You should try to use slit #4 on the mask on the front of the light sensor.
- Determine the slit width using Equation (1). Measure at least two different minima and average your answers. Estimate the errors.

Double slit exercise
- Replace the single slit disk with the multiple slit wheel. Set it on slit separation 0.25 mm (d) and slit width 0.04 mm (a).
- Set the light sensor aperture bracket to slit #4.
- Apply the same steps as before (single slit procedure)
- Zoom in to enlarge the central maximum and the first side maxima. Use the cursors to measure the distance between the central maximum and the first side maxima.
- Measure the distance between the central maximum and the second and third side maxima. Also, measure the distance from the central maximum to the first minimum in the DIFFRACTION (not interference) pattern.
- Determine the slit separation using Equation (2) and the first, second, and third maxima, and find the average "d". Find the percent difference between your average and the stated slit separation on the wheel.
- Determine the slit width using Equation (1) and the distance between the central maximum and the first minimum in the diffraction pattern (not interference pattern). Is this the slit width given on the wheel?
- Repeat the steps above for the interference patterns for two other double slits. Estimate the errors.

Quantum mechanical interpretation
The Heisenberg uncertainty principle (http://www.aip.org/history/heisenberg/p08.htm) states that the simultaneous measurements of the momentum and position (or the energy and time) for a moving particle entails a limitation on the precision (standard deviation) of each measurement. Namely: the more precise the measurement of position, the more imprecise the measurement of momentum, and vice versa. In the most extreme case, absolute precision of one variable would entail absolute imprecision regarding the other.

If we consider a pack of photons characterized by position uncertainty $\Delta y$ and momentum uncertainty $\Delta p$, we can express Heisenberg’s relation as:

$$\Delta y \cdot \Delta p \geq \frac{h}{4\pi} \quad (4)$$

Where $h = 6.6262 \times 10^{-34} \, J \cdot s$ is Planck’s constant.

For a pack of photons passing through a slit of width a we have $\Delta y = a$.

In order to estimate $\Delta p$, we assume that photons reaching the slit move only in the direction perpendicular to the slit (x-direction), but after passing through the slit, they will have velocity components in both directions (x and y). The $v_y$ component of photon velocity is given by the intensity distribution in the diffraction pattern.

Defining by $\theta_1$ the angle of the first minimum of diffraction, we can express the uncertainty of velocity and momentum as:

$$\Delta v_y = c \sin \theta_1 \quad (c \text{ is speed of light}) \quad (5)$$

$$\Delta p_y = \frac{h}{\lambda} \sin \theta_1 \quad (6)$$

(using the de Broglie relationship: $\frac{h}{\lambda} = p$)

The angle $\theta_1$, of the first diffraction minimum can be determined from the experimental setup geometry (see Figure 1 and Equation 1) and the momentum uncertainty results as:

$$\Delta p_y = \frac{h}{a} \quad (7)$$

Combining $\Delta y$ with $\Delta p_y$ in (3), we obtain the uncertainty relation:

$$\Delta p_y \cdot \Delta y = h \geq \frac{h}{4\pi} \quad (8)$$
Experimentally, we can obtain the angle $\theta_1$ from the position of the first minimum:

$$\tan \theta_1 = \frac{l}{b} \quad \text{(see Figure 1)}$$

By substituting this relation into (6), we obtain

$$\Delta p_y = \frac{h}{\lambda} \sin \left( \tan^{-1} \frac{l}{b} \right).$$

Equation (8) becomes

$$\frac{ah}{\lambda} \sin \left( \tan^{-1} \frac{l}{b} \right) = h \geq \frac{h}{4\pi} \quad \text{(9)}$$

so then

$$\frac{a}{\lambda} \sin \left( \tan^{-1} \frac{l}{b} \right) = 1.$$

Experimental verification:
- Measure the half width of the central maximum ($l$) for three different single slit widths ($a$). Measure the distance between the light sensor aperture and the laser aperture ($b$).
- Verify Equation (9). Carefully estimate the errors and discuss about confirming or not Heisenberg’s uncertainty principle by using this experimental verification.

**Diffraction pattern analysis**

Setup the diffraction experiment with a slit of 0.16mm and a laser-detector distance of about 1.1m. Record the diffraction pattern.

Using the cursors, measure the pattern intensity corresponding to the central maximum $I(0)$ (Equation (3)). Measure the heights of three other secondary maxima $I(\theta_i)$; calculate the relative heights of secondary maxima $\frac{I(\theta_i)}{I(0)}$. From the setup geometry, calculate the corresponding angles $\theta_i$ of secondary intensity maxima.

**Python Requirement**

Using Equation (3) verify the intensity formula by performing a nonlinear fit on the data collected by the detector.

Output the ratio $a : \lambda$ and comment on goodness of fit. Comment on error sources.

**QUESTIONS**

1. What physical quantity is the same for the single slit and the double slit?
2. How does the distance from the central maximum to the first minimum in the single-slit pattern compare to the distance from the central maximum to the first diffraction minimum in the double-slit pattern?
3. What physical quantity determines where the amplitude of the interference peaks goes to zero?
4. In theory, how many interference maxima should be in the central envelope for a double slit with $d = 0.25$ mm and $a = 0.04$ mm?
5. How many interference maxima are actually in the central envelope?

*The National Instruments interface was setup and programmed by Larry Avramidis. Larry also built the low rpm motor driving the linear translator.*

*This experimental guide sheet was written by Ruxandra M. Serbanescu in 2008 (updated 2009).*