The Kater Pendulum

2 weights for basic result with random error assessed; 3 weights with full assessment of systematic error.

Scientific Objective To measure the acceleration of gravity at a location in the second year laboratory with as much precision as possible (hopefully to at least 1 in 10^4), by timing the swing and measuring the length of a "Kater" pendulum.

Heuristic Objectives To show what is involved in making a high precision measurement of an unknown quantity. The experimental method uses only clearly understood, classical mechanics and measurements of length and time. Getting a good result for g involves some tricky procedures, but it is relatively straight forward. Deciding how much one should trust the result is more challenging, and can involve additional experiments.

Background Accurate scientific observation was a preoccupation of the early 1800's. Precise navigation over all the world's oceans was a vitally important need, and the invention of highly stable clocks based on temperature-compensated, torsion-spring oscillators was making it a possibility. However, it was soon realized that navigation with an accuracy of a kilometer or better on a planet of about 40,000 km circumference needed a more precise knowledge of the earth's shape, size, and gravity than was then available. Measurement of local g and how it varies with location became an important goal. Kater's invention of the adjustable double pivot pendulum was an important step forward towards achieving this. (Kater, Henry; 1818, An Account of experiments for determining the length of the pendulum vibrating seconds in the latitude of London; *Philosophical Transactions*, v 104, 109 p; Reproduction in laboratory Resource Centre).

CHECK REFERENCE

It is easy to see, in principle, that g can be measured by timing the period (T) of a simple pendulum and measuring its length (L). The theoretical period of low amplitude oscillation is just

$$T = 2\pi\,(L/g)^{1/2}, \quad \text{so} \quad g = (2\pi)^2\,(L/T^2)$$

Time and distance could be measured in the early 1800's to much better than 1 in 10^4 by straightforward methods available then. However, a real pendulum is not a simple pendulum, and the effective length L_e of a real pendulum can be difficult to define. In principle, the effective length is $L_e = k^2/r_m$ where k is the radius of gyration and r_m is the radius of the centre of mass relative to the pivot point. However, neither of these parameters is easy to determine very accurately.

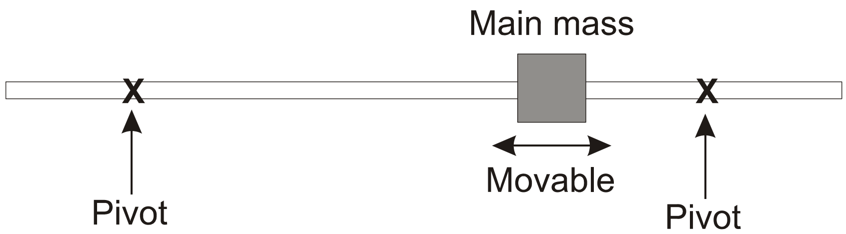

Fig.1 Schematic of a Kater pendulum. In practice, the movable mass is divided into two or more elements to enable coarse and fine adjustment.

Kater's innovation was to realize that, if one built an asymmetrical, double-ended pendulum (as is shown schematically in Fig.1), it could be adjusted so as to have exactly the same period when swung from either pivot point. And when that condition is fulfilled, the effective length of the pendulum is just the distance between the pivot points, which then can be measured by direct observation.

Our Pendulum It was designed to be transportable when packed in a wooden box, to pivot on hardened steel knife edges, and to be swung from a knife edge support clamped firmly to a massive wall. The ends of the pendulum are formed into narrow, sharpened rods so the swing can be timed precisely by lightly contacting the lower tip of the pendulum with a thin highly flexible, fixed blade, a light beam, or a mercury button to produce aural, optical or electrical "ticks". These can then be compared to the seconds ticks of a carefully rated clock. We use the pendulum in just this manner (with optical timing), except that our clock is an electronic counter-timer. The old method of comparison was stroboscopic (i.e., determining the (long) time interval between two or more successive moments of precise synchronization between the pendulum and the reference clock). For this reason, the pendulum was designed to have a period close to, but not exactly, 2 seconds.

The Experiment

A practical difficulty with the basic design shown in Fig. 1 is that it can be very hard to make a precise adjustment of the moveable mass. Our pendulum therefore has three masses in addition to the basic support structure: a fixed main mass, a coarse adjustment mass, and a fine adjustment mass.

Position of the adjustment masses can be read on an engraved scale. The idea is first to do a set of measurements in which the fine adjustment mass is fixed and the coarse adjustment mass is moved, so as to find a suitable position for the coarse adjustment mass. This must allow motions of the fine adjustment mass to bracket the desired "equal period" state. After this, the coarse adjustment is kept fixed and a series of (pairs of) period measurements are made for different positions of the fine adjustment mass.

It is not practical to reach the precise equal-period point. It is better to plot a pair of graphs of period (pendulum upright and inverted) versus position of the fine adjustment mass, and try to find the intersection point of the graphs graphically (or by least squares fitting) as precisely as possible. The period corresponding to the intersection point is the Kater period that is used to calculate *g*.

Measuring the length between pivot points. This is done using a very finely divided steel scale with a closely fitting sliding vernier scale on it. A telescope with a bubble-leveled line-of-sight is coupled to the vernier, and adjusted to align with the knife-edge pivot points. The device is called a cathetometer.

➔ **Python Requirement (PHY224/324 students only):** write a program to analyze the dependencies of Period vs. Mass position. You may use `fsolve` from `scipy.optimize` to find the intersection of the plots needed in calculating the Kater period. Your program should output the Kater period and the constant 'g'. You should pay a special attention to uncertainties (see below).

Determining the Uncertainty. This is the tricky part. Determining the possible level of random experimental error in the T and L measurements is the first step. Then one must search for all the possible sources of systematic error. There are many hypothetical possibilities, some of which can be ruled out on the basis of simple theoretical calculations; others may need small "side" experiments to test the sensitivity of the result to some possible problem.

A (not necessarily complete) list of factors that might need to be considered is the following:-

Calibration errors in timer clock and cathetometer.
Finite amplitude of pendulum
Buoyancy of air
Damping of pendulum due to friction
Imperfect knife edges
Temperature variations during measurement
Elastic variations of pendulum length
Flexibility of pendulum affecting its period.

Additional Possibilities It may be possible to calibrate the oscillator in the counter-timer (using a radio time-signal) and the cathetometer (by measuring a traceable standard of length). You can also compare your result with an estimate of gravity at the site by using the standard "Bouguer" formula for how gravity is expected to vary with latitude, land elevation, and site elevation, and with published values determined by modern gravimetric methods at certain reference sites.

Hints There is a lot of material on the web about Kater pendulums. It is a well known experiment in physics teaching labs. Ours is as accurate (or better) than most.

Appendix

Instrument details (Brief instructions for the counter-timer are pasted on the top of instrument.)

The counter-timer precisely measures the time duration of 8, 16 or 32 full periods of the pendulum. It does so by counting cycles of a 1 MHz quartz crystal oscillator.

An infra-red optical source and sensor is mounted on the black box below the pendulum. It generates a signal (a "tick") each time the lower end of the pendulum swings through it.

The tick signal is reduced to an on/off signal (square wave) making one full cycle each full pendulum period, by means of an initial binary counter in the counter-timer unit. This signal is visible on the front panel green LED.

When the reset pushbutton at the bottom of the panel is pushed and released, the counter-timer begins counting exactly n full periods of the pendulum, starting at the beginning of first new period. The numerical value of n (the count limit) may be set to 8, 16 or 32 by the front panel switch.

When the period counter is reset and waiting to start up, a yellow LED is lit on the panel. This LED turns off during the n period count interval, and it turns back on when the count finishes.

The duration of the count interval is given on the numeric display of counter timer. Displayed is the number of microseconds in the period interval, with a decimal provided so the number can be read as seconds.

NOTE re START-UP:- After initial power–up, or following any change of the *n* setting, or after a reset during a count, the pendulum must continue to operate for *n* periods before the counter-timer can accept a reset. At this time, the yellow LED comes on. The pendulum must swing for this counting to occur.

The accuracy of the 1 MHz oscillator in the timer unit has not been checked recently, but it is likely about 1:100,000.

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