THE MILLIKAN OIL-DROP EXPERIMENT

TWO WEIGHTS

RECOMMENDED READINGS

1. R.A. Millikan, The Electron (photocopied excerpts available at the Resource Centre).
2. Instruction Manuals for Leybold - Heraeus apparatus (available at the Resource Centre).
3. https://www.aps.org/programs/outreach/history/historicsites/millikan.cfm

INTRODUCTION AND THEORY OF THE METHOD

Robert A. Millikan around 1923

The experiment is one of the most fundamental of the experiments in the undergraduate laboratory. The experimental apparatus is patterned after the original apparatus, made and used by R.A. Millikan to show that electric charge exists as integral multiples of "*e*" the charge on a single electron. Historically, this experiment ranks as one of the greatest experiments of modern physics.

This experiment first described in 1913, is based on the fact that different forces act on an electrically charged oil drop moving in the homogeneous electric field of a plate capacitor (Fig.1). Going through the capillary of the atomizer, the oil droplets acquire electric charge due to friction. The effect is known as triboelectric charging. The distance between the plates is d, the applied voltage V produces the electric field E. Taking $E = V/d$ inside the capacitor, the following forces act on a droplet of negative charge Q:

- the downward gravitational force $\boldsymbol{F}_{grav} = m_{oil}\boldsymbol{g}$, where m_{oil} is the mass of the oil droplet;

- the upward buoyant force $\boldsymbol{F}_b = - m_{air}\,\boldsymbol{g}$, where m_{air} is the mass of air displaced by the oil drop;

- electric force $\boldsymbol{F}_E = Q\boldsymbol{E}$;

- Stokes' resistance force when the spherical droplet is moving. Stokes' Law states that for a spherical object of radius r moving through a fluid of viscosity η at a speed v under the laminar flow conditions, the viscous force F on the object is given by $F_{drag} \approx 6\pi r\,\eta\,v$. The viscous force always opposes the motion and is responsible for maintaining the terminal velocity observed when a drop falls in air.

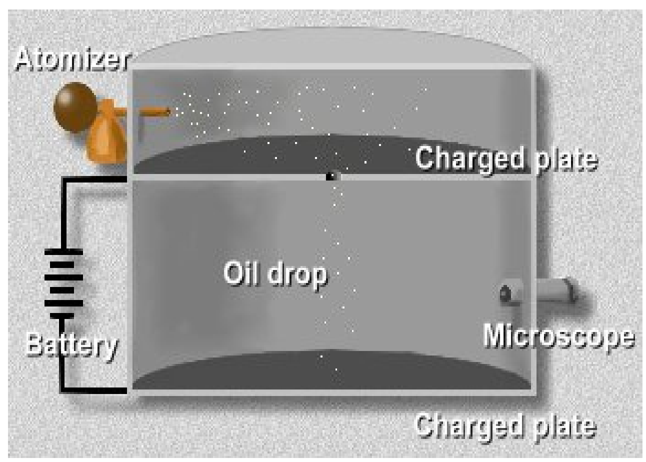

FIG.1: Sketch of a device to produce and observe motion of charged droplets.
https://sites.google.com/site/atomicmodel1017/millikan-s-experiment

Varying the applied voltage, an observer is able to watch a droplet that abbeys one of the following three equations of motion:

$g\,(m_{oil} - m_{air}) - QV_{stop}/d = 0$ (1) – the droplet is floating; the net force is zero;

$g\,(m_{oil} - m_{air}) - 6\pi r\eta v_t = 0$ (2) – the droplet is moving downward with the terminal velocity v_t; the net force is zero.

$ma = - g\,(m_{oil} - m_{air}) + QE - 6\pi r\eta v$ (3) – motion upward with acceleration.

We will use Eqs. (1) and (2) to find the elementary charge e.

Method 1.
An experimenter performs measurements in the following order:
- selects one oil-drop moving downward between the plates of the capacitor;
- applies voltage V_{stop} that can stop a moving droplet and make it floating, records this voltage;
- quickly turns voltage down to zero to release the oil-drop; the latter moves with acceleration, but in several instants of time starts uniform motion;
- measures terminal velocity of the object and substitutes the stopping voltage and the terminal velocity into the equation

$$Q = const_1 \cdot \frac{v_t^{3/2}}{V_{stop}} \qquad (4)$$

Exercise 1. Combining Eqs. (1) and (2), derive the Eq. (4) with the following values for physical quantities *η, ρ, d* and *g* in your experiment at t^o = 20°C:

the density of the oil ρ_O = 875.3 kg/m^3
the density of air ρ_a =1.204 kg/m^3 https://en.wikipedia.org/wiki/Density_of_air
the acceleration due to gravity g = 9.80 m/s^2
the dynamic viscosity of air at 1 atm η = 1.827 × 10^{-5} Pa·s https://en.wikipedia.org/wiki/Viscosity
the separation of the parallel plates d = 6.0 mm.
Find the numerical value of the coefficient $const_1$ with appropriate units.

Method 2.
For same droplet, the experimenter manages setting up voltage V_{up} that makes the droplet moving upward at a constant velocity v_2. When the droplet is close to the top of view, voltage is turned off to zero, letting the droplet falling down at a terminal velocity v_1. The charge of the droplet is calculated as follows

$$Q = const_2 \cdot (v_t + v_2)\frac{v_t^{1/2}}{V_{up}} \qquad (5)$$

Exercise 2. Combining Eqs. (1), (2) and (3) for zero acceleration, derive the Eq. (5). Find the value of the coefficient $const_2$.

APPARATUS AND MEASUREMENTS
Familiarize yourself with the apparatus before taking actual measurements. The Leybold - Heraeus apparatus in Fig. 2 allows you to determine the elementary electric charge using the digital camera image and LabVIEW software that returns the position in pixels of the selected droplet frame by frame with the frame rate of 10 Hz. We calibrated the camera to convert the outcome in pixels into the distance in millimeter, and found the calibration factor being (540 ± 1) px/mm for the left device, and (520 ± 1) px/mm for the right device. If you want to make your own calibration, request help from a TA or a lab coordinator.

Make sure that the spray nozzle of the oil atomizer is positioned before the small holes in the acrylic glass cover of the Millikan chamber. In one trial, you may combine both methods by varying applied voltage from zero to one that can accelerate the drops upward. It is expected that the lab report will contain not less than 50 measurements for reliable results.

Preparation of software

Prepare a set of 50-60 Excel files numbered according to the number of a trial and save the files in your computer.

In addition to the prepared set of Excel files for recorded position vs time data, it is recommended to prepare a table to enter the stop voltage and the title of the Excel file for each specific successful trial. This way, you will not need paper for current notes.

Find the icon of the Millikan experiment on the desktop and open the software. Twice click "OK" when required. The image of the display includes the following fields and function buttons (identify all of them before starting observations):

- window with a file name, which is by default "test.xlsx", but is recommended to be replaced by the name of your file; there is a small button to the right of this window to open your file;
- large central window to observe the internal space of the chamber close to its axis;
- a smaller window to the right that displays a position of a trapped droplet vs number of frame;
- Save Data button to start recording the data in the open Excel file;
- Track Image button to locate the chosen droplet;
- four small ROI windows with coordinates of the corners of the frame you used to trap the droplet;
- three columns with sliders for adjusting brightness and contrast of the field of view;
- STOP button at the upper right corner to reload the software for new series of data acquisition.

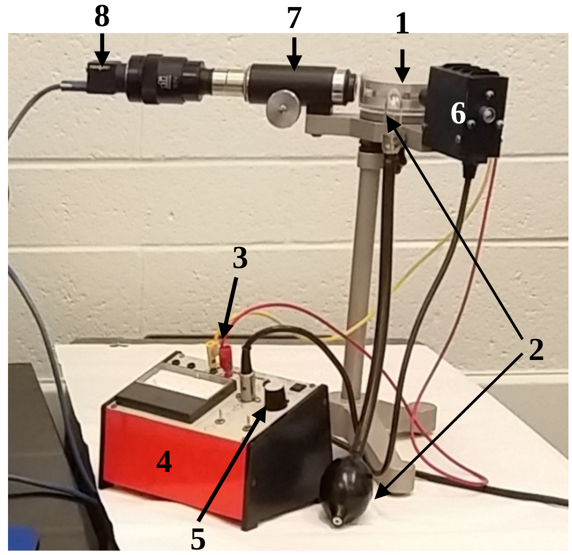

FIG. 2: Leybold - Heraeus apparatus:
1 – chamber (plate capacitor);
2 – oil atomizer with rubber bulb;
3 – socket pair for charging the plate capacitor connected to DC power supply 4 with adjusting knob 5;
6 – light source;
7 – microscope connected to the CCD camera 8.

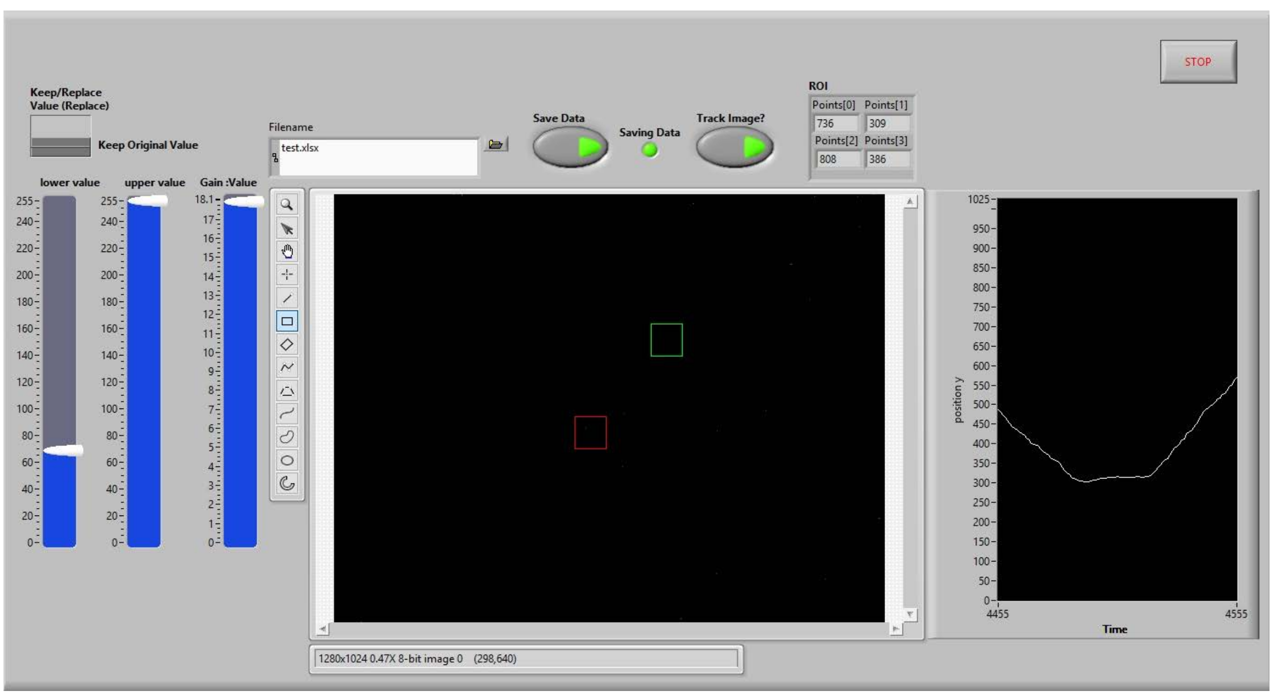

FIG. 3: Software display image. Right image corresponds to motion upward (left), rest and fall

Procedure

Turn off the room lights; this experiment is performed in a dark room. Check the socket pair (3) to be sure that the positive terminal is connected to the upper socket of the chamber. Turn on the power supply (4); a switch is at the back.

- Slide “Upper Gain” to its top position as in Fig.3. Manage the display illumination by moving the “Lower Gain” slowly up to increase brightness to dark grey.
- Set the voltage close to maximum position. This will simplify selecting a droplet.
- Squeeze the rubber bulb (2) two-three times to let a bunch of droplets inside the chamber (1). The main display will show a “snowfall” of dots of different sizes and brightness. At the high voltage applied, it is recommended that you begin observing the droplets that are slowly moving upward. To locate and track a droplet, you will have to slightly adjust brightness by the “Lower Gain” and focus on a specific droplet by tuning a knob mounted on the microscope (7).
- Exercise several times in selecting a bright spot, adjusting contrast of view and stop voltage. The good brightness of the droplet on a completely dark background is the crucial condition for successful application of the software. Only a very bright spot on a black background will be tracked by the software.
- In the window for the Excel file name open the file you are going to record your data.
- Make a trial with recording the data. Squeeze the bulb, quickly find a good droplet, stop it by adjusting applied voltage, adjust brightness and quality of the image.
- Record the stop voltage.
- Use mouse to locate the droplet: draw a green frame around the droplet. Wait a couple of seconds to be sure the droplet is suspended. If it moves, adjust voltage and write the new value.
- Click on the button Track Image. There will appear a red frame coinciding with the green frame. The red frame must follow the droplet thus providing recording its coordinates.
- Click the button Save Data to turn on the program of recording the droplet coordinates.
- Quickly turn off voltage with the knob (5). The droplet will start moving downwards still been enclosed in the red frame. If the droplet comes out of the red frame, turn off Save Data, and begin looking for another drop. Update software for your next trial.
- While the droplet is moving, you can watch its position as a function of the number of the frame on the right dark screen. For the terminal motion, the straight line is expected.
- Figure 3 shows three different modes of motion of the same trapped oil drop. You are recommended to try a similar experiment, which combines measuring of (a) the stop voltage, (b) terminal speed in the free fall, and (c) voltage V_{up} and the terminal speed of moving upward under the voltage V_{up}. Thus, each trapped oil drop can be used for both Method 1 and Method 2.
- Click Save Button – the data recording stops. Click Track Image – the trial is finished.
- Open the Excel file you used for recording the data. The data is obtained as one column with the values of position of the droplet in each frame. For numbering the frames, include a column to the left with the integers of the positions.
- Highlight the two columns to insert a diagram. The “XY Scatter” is recommended. Save this file on your USB drive. If your graph is a straight line, use it to find the terminal velocity v_1 and v_1 by converting pixels in millimeters and frame numbers into time.
- Create a table of raw data for stop voltage, V_{up} and terminal velocities for the two methods and the calculated charge of the drop according to the Eqs. (4) and (5). At least 50 trials are recommended.

Quantification of the elementary charge

Using the table of Q_i, calculated in many trials, you may apply two different methods to find the elementary charge: create a Python code to calculate the greatest common divisor for the series of Q_i; or you may plot the number of measurements vs Q_i within the range of uncertainty in a histogram. For the X-axis, we recommend a smallest division of $1x10^{-20}$ C. For Y-axis, you will use the number of outcomes within each bin.

The data analysis section must include an uncertainty of the found elementary charge and explanation of how you calculated the uncertainty. Discuss the accuracy of the result and compare the difference between the experiment and expected values. Compare this difference with the uncertainty of the technique.

QUESTIONS

1. Estimate the radius of typical droplets in your experiment.
2. Is the buoyant force significant in this experiment?
3. Does the experiment work better with smaller or larger radii? Why?

Software produced by Josiah Sinclair and Larry Avramidis, August 2017. Upgrade by Larry Avramidis in August 2018.

Write-up is written by N.Krasnopolskaia in January 2018 and last updated in Oct 2018.