# POLARIZATION OF LIGHT

# Introduction

Light, viewed classically, is a transverse electromagnetic wave. Namely, the underlying oscillation (in this case oscillating electric and magnetic fields) is along directions perpendicular to the direction of propagation. This is in contrast to longitudinal waves, such as sound waves, in which the oscillation is confined to the direction of propagation. Light is said to be linearly polarized if its oscillation is confined to one direction (the direction of the oscillation of the electric field is defined as the direction of polarization). Most light sources in nature emit unpolarized light i.e., light consists of many wave trains whose directions of oscillation are completely random.

Light may be polarized by passing it through a sheet of commercial material called Polaroid, invented by E.H. Land in 1938. A sheet of Polaroid transmits only the component of light polarized along a particular direction and absorbs the component perpendicular to that direction.

Consider a light beam in the z direction incident on a Polaroid which has its transmission axis in the y direction. On the average, half of the incident light has its polarization axis in the y direction and half in the x direction. Thus half the intensity is transmitted, and the transmitted light is linearly polarized in the y direction.

## Malus' law

Suppose we have a second piece of Polaroid whose transmission axis makes an angle  $\theta$  with that of the first one. The *E* vector of the light between the Polaroids can be resolved into two components, one parallel and one perpendicular to the transmission axis of the second Polaroid (see Figure 1). If we call the direction of transmission of the second Polaroid y',

$$E_{x'} = E \sin \theta$$
 and  $E_{y'} = E \cos \theta$ 

Only the second, y' component is transmitted by the second Polaroid.

The intensity of light is proportional to the square of the electric field amplitude. Thus the intensity transmitted by both Polaroids can be expressed as:

$$I(\theta) \cong E_{u'}^2 = E^2 \cos^2 \theta$$

If  $I_0 \cong E^2$  is the intensity between the two Polaroids, the intensity transmitted by both of them would be:

$$I\left(\theta\right) = I_0 \cos^2\theta \tag{1}$$

This equation is known as Malus Law after its discoverer, E.L. Malus (1775-1812). It applies to any two polarizing elements whose transmission directions make an angle  $\theta$  with each other. When two polarizing elements are placed in succession in a beam of light as described here, the first is called **polarizer** and the second is called **analyzer**. As you will see, no light reaches the photocell when the polarizer and analyzer are crossed  $(\theta = 90^{\circ})$ .





### Theory for three polarizers

Unpolarized light passes through 3 polarizers (see Figure 2):



Figure 2: The electric field transmitted through three polarizers.

The first and last polarizers are oriented at  $90^{\circ}$  with respect to each other. The second

polarizer has its polarization axis rotated an angle  $\phi$  from the first polarizer. Therefore, the third polarizer is rotated an angle  $\frac{\pi}{2} - \phi$  from the second polarizer. The intensity after passing through the first polarizer is  $I_1$  and the intensity after passing through the second polarizer,  $I_2$ , is given by:

$$I_2 = I_1 \cos^2 \phi$$

The intensity after the third polarizer,  $I_3$ , is given by:

$$I_3 = I_2 \cos^2\left(\frac{\pi}{2} - \phi\right) = I_1 \cos^2\phi \cos^2\left(\frac{\pi}{2} - \phi\right) \tag{2}$$

Rearranging Equation 2, we obtain:

$$I_3 = \frac{I_1}{4}\sin^2(2\phi)$$
 (3)

Because the data acquisition begins when the transmitted intensity through Polarizer 3 is a maximum, the angle  $\Theta$  measured in the experiment is zero when the second polarizer is 45° from the angle  $\phi$ . Thus the angle  $\phi$  is related to the measured angle  $\Theta$  by:

$$\phi = 45^{\circ} + \Theta \tag{4}$$

#### Reflectance

Consider unpolarized light incident on a surface separating air and glass or air and water. Define the plane of incidence, containing the incident, reflected and refracted rays as well as the normal to the surface.

When light is reflected from a flat surface, the reflected light is partially polarized. This is due to the fact that the reflectance of light R = (Reflected Intensity)/(Incident Intensity) depends on the polarization itself. The degree of polarization depends on the angle of incidence and the indices of refraction of the two media.

For reflection at an air-glass interface (indices of refraction  $n_1$  for air and  $n_2$  for glass), Fresnel equations give the reflection coefficients  $r_{\parallel}$ ,  $r_{\perp}$ :

$$r_{\perp} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \tag{5a}$$

$$r_{\parallel} = \frac{n_1 \cos \theta_2 - n_2 \cos \theta_1}{n_1 \cos \theta_2 + n_2 \cos \theta_1} \tag{5b}$$

where  $\theta_1$  is the angle of incidence, and  $\theta_2$  is the angle of refraction.

Above  $r_{\parallel}$  and  $r_{\perp}$  refer to the reflection coefficients for polarized light whose direction of polarization lie in the plane of incidence and perpendicular to the plane of incidence, respectively.

Reflectance R (parallel or perpendicular) is defined as the square of the corresponding reflection coefficient:  $R_{\parallel} = r_{\parallel}^2$ ,  $R_{\perp} = r_{\perp}^2$ 

Note that  $\theta_2$  is not measured in this experiment and must be inferred from Snells law of refraction:

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{n_1}{n_2} \tag{6}$$

Figure 3 shows initially unpolarized light incident at the polarizing angle  $\theta_p$ , for which the reflected light is completely polarized with its electric field vector perpendicular to the plane of incidence. The electric field vector E of the incident wave can be resolved into two components: parallel to the plane of incidence and perpendicular to the plane of incidence.

Note that  $R_{\parallel} = 0$  when  $n_1 \cos \theta_1 = n_2 \cos \theta_2$ , which leads to the definition of the Brewster angle (or polarizing angle):

$$\tan \theta_p = \frac{n_2}{n_1} \tag{7}$$

Here  $\theta_p$  is the angle of incidence of unpolarized light which makes the reflected light completely polarized in the perpendicular direction to the plane of incidence (Sir David Brewster, 1812).

When the angle of incidence of the initially unpolarized light is  $\theta_p$ , the reflected and refracted rays are perpendicular to each other.



Figure 3: Unpolarized light incident at the polarizing angle.

Note: If the incident light has no component of E perpendicular to the plane of incidence, there is no reflected light.

# Apparatus, Experiment and Procedure

# Malus' Law: Apparatus Notes

Be careful not to leave your fingerprints on optical surfaces.

- Rotate the aperture disk so the translucent mask covers the opening to the light sensor.
- Verify that the Rotary motion sensor is mounted on the polarizer bracket and connected to the polarizer pulley with the plastic belt.
- Verify that the Rotary motion sensor is plugged into the National Instruments (NI) interface.
- Place all the components on the Malus Law Optics Track in the order shown in Fig. 4. Space components apart along the optics track (largest distance should be between the detector and the polarizer).



Figure 4: Experiment Components.

To open up the software necessary for this part of the experiment, click on the desktop shortcut Polarization of Light. Click on the little arrow located in the upper left corner of the screen to start the acquisition. The program is self-explanatory.

## Exercise 1: Two Polarizers, verify Malus' law

In the first two procedure steps, polarizers are aligned to allow the maximum amount of light through.

• Since the laser electromagnetic wave is already polarized, the first polarizer must be aligned with the polarization axis of light. Remove the holder with the polarizer and Rotary motion sensor from the track. Slide all the other components on the track close together and dim the room lights. Click ON/OFF and rotate the polarizer that <u>does not</u> have the Rotary motion sensor until the light intensity on the graph is at its maximum.

- To allow the maximum intensity of light through both polarizers, bring back the holder with the polarizer and Rotary motion sensor on the track, and rotate the polarizer until light intensity on the graph is maximum. Before you begin a new scan, you may clear previous data. *Note: If the maximum exceeds* 4.5 *V, decrease the gain on the light sensor. If the maximum is less than* 0.5 *V, increase it.*
- To scan the light intensity versus angle press ON/OFF and rotate the polarizer <u>which has</u> the Rotary motion sensor through 180 degrees. Rotation should be constant. Do not rotate back! Acquisition stops by itself at the end of 180 degrees. Practise until you get the best (smooth) recording of Intensity of light vs. angle.

## Analysis

Students not from PHY224/324: Export data, analyze dependencies: Intensity vs.  $\cos \theta$ , and Intensity vs.  $\cos^2 \theta$ 

 $\Rightarrow$  PHY224/324 students: Python Requirement 1: write a fitting program for Intensity vs.  $\cos^2 \theta$ . You have to include experimental errors for both light intensity and angle readings. The data set includes about 300 values (Intensity,  $\theta$ ). Compare with Malus Law prediction, Eq. 1.

### **Exercise 2:** Three Polarizers

- Repeat the experiment with 3 polarizers (see setup in Figure 6). Place Polarizer 1 on the track and rotate it until the transmitted light is a maximum.
- Place Polarizer 2 on the track and rotate it until the light transmitted through both polarizers is a minimum.
- Place a Polarizer 3 on the track between the first and second polarizers. Rotate it until the light transmitted through all three polarizers is a maximum (see Fig. 5).



Figure 5: Experiment components for the three polarizer part of the experiment.

- Press ON/OFF and scan Intensity vs. Angle for 360 degrees as you rotate the third polarizer that has the Rotary motion sensor. Try the qualitative fitter.
- Select your data from 2 polarizers and from 3 polarizers. What two things are different for the Intensity vs. Angle graph for 3 polarizers compared to 2 polarizers? Export data and find the best fit matching Equation (3). Include experimental errors.

 $\Rightarrow$  Python Requirement 2 (PHY224/324 students only): write a fitting program using your data and the intensity function from Equation 3.

#### Questions

- 1. For 3 polarizers, what is the angle between the middle polarizer and the first polarizer to get the maximum transmission through all 3 polarizers? Remember: in the experiment, the angle of the middle polarizer automatically reads zero when you start taking data but that doesn't mean the middle polarizer is aligned with the first polarizer.
- 2. For 3 polarizers, what is the angle between the middle polarizer and the first polarizer to get the minimum transmission through all 3 polarizers?

## Brewster's Angle: Apparatus Notes

Light from a diode laser is reflected off the flat side of an acrylic semi-circular lens. The reflected light passes through a polarizer and is detected by a light sensor. The angle of reflection is measured by a rotary motion sensor mounted on the spectrophotometer table. The intensity of the reflected polarized light versus reflected angle is graphed to determine the angle at which the light intensity is a minimum. This is Brewster's Angle, which is used to calculate the index of refraction of acrylic (Eq. 7).

Since the index of refraction depends on the wavelength of incident light, using a monochromatic light source (diode laser) should render the interpretation of results relatively simple.

We now describe the preliminary setup:

- Move the diode laser to the optics bench marked Brewsters angle.
- Attach 2 polarizers to the same holder, and place the collimating slits on the track as shown in Figure 6. Make sure the rotary motion sensor is mounted on the spectrophotometer table with the bigger diameter of spindle against the table (see Figure 6). Attach the spectrophotometer table base to ground as instructed by your TA.



Figure 6: Complete setup for Exercise 3. Note that the computer interface is different in your experiment.

- Two round polarizers are used on the holder. To read the polarizer angle, use the brass pointer at the top of the holder. Note that these angle readings differ from the real orientation of the polarizer by 180 degrees.
- Rotate the second polarizer (second from laser) to 225 degrees and lock it in place by tightening the brass screw. (In reality, this is a 45 degree polarizer, used to solve the problem that the laser light is already polarized). The first polarizer (closest to the laser) is used throughout the experiment to adjust the light level.
- The square analyzing polarizer (see Figure 7) has its transmission axis marked, and for normal use the label should be on top with its axis horizontal and thus 90 degrees from the polarization axis of the reflected light. In this way, it is used to find the variation in the parallel component of the reflected light and to determine Brewsters angle. By placing the analyzing polarizer with its transmission axis vertical, you can also look at the variation in the perpendicular component of the reflected light.



Figure 7: The analyzing segment of the setup.

- The small metal Brewster's angle accessory disk should be zeroed so that the mark at the top of the label above the N in the word ANGLE is aligned with the zero angle mark on the spectrophotometer disk.
- The step base has two marks. For reflected light, use the mark on the higher step side. Align it to zero on the disk. The D lens is placed on the lower surface, flush against the step when data is being collected.
- To align the laser beam, remove all polarizers, collimating slits, and D lens. Set the spectrophotometer arm near 180 degrees. Use the *x-y* adjustment on the laser diode to get the beam at the center of the light sensor slit. The light sensor bracket slit should be set on the unobstructed opening, without any slits or translucent films. Place the collimating slits on track and adjust the slit position so the laser beam passes through the opening on the detector.

Note about reflection angle measurements: The angle is calculated by dividing the actual angle (recorded by the computer) by two, with a correction with respect to the spindle diameter. The markings on Brewster's disk are there only for convenience (in this experiment) and are not used directly. To get the laser beam exactly on to the slit, you must make fine adjustments while watching the digits display on computer for the maximum light intensity. You can adjust either the Brewsters disk or the spectrophotometer arm until the intensity is maximized.

#### Exercise 3: Polarization by Reflection and Brewster's angle

Dim the room lights. Click on the Brewsters Angle application tab. Make sure the Light Sensor is set to the unobstructed opening setting and not to any slit.

Place the double polarizer on the track and the D lens on the step. Rotate the Brewsters disk to the 120 mark located counter-clockwise on the spectrophotometer table. Click ON/OFF and, while watching the digits display of light intensity, rotate the spectrophotometer arm to get into the beam. Rotate the first polarizer (nearest to the laser) to adjust the intensity level to be as high as possible. The Light Sensor should be on gain of 1 or 10. This will set up the starting point of the rotational motion sensor.

Start the acquisition without the square polarizer: slowly rotate the spectrophotometer arm clockwise, while **at the same time** rotating the Brewster's disk to keep the reflected beam aligned with the opening at the Light Sensor. Use the cursors to read the intensity  $(I_0)$  and angle values of graph maxima. Note: another way to obtain this data is to slowly rotate the spectrophotometer arm while quickly spinning the Brewster's disk back and forth, so that the reflected beam passes across the opening at the Light Sensor. The data obtained this way will be messy, so the first method is advisable: however, arguably this approach easier on the experimenter's back.

Place the square analyzing polarizer (axis horizontal) on the spectrophotometer arm in front of the slits. Note: The square analyzing polarizer must sit level, flat on the arm.

Do an acquisition with square polarizer (horizontal) using the same procedure as before. Read intensity (I) and angle values of graph maxima.

#### Analysis

Take the ratio  $I/I_0$ , graph it versus angle. Include errors in reading intensity and angle. Determine the Brewsters angle.

Use Brewster's angle to calculate the index of refraction of acrylic using Equation 7. Use  $n_1 = 1$ .

Calculate the parallel and perpendicular reflectances using Fresnel equations (5a and 5b).

 $\Rightarrow$  Python Requirement 3 (PHY224/324 students only): do the three analysis steps above by writing a program to fit the data and to output Brewsters angle, index of refraction of acrylic and the two reflectances from (5a) and (5b).

#### Questions

- 1. Would Brewster's angle be more or less for light in air reflecting off water?
- 2. How would data look like for an arrangement with vertical square polarizer?
- 3. How do polarized sunglasses reduce glare? Which direction is the axis of polarization in a pair of polarized sunglasses? How could you check this?

# References

- 1. Halliday/Resnick/Walker Fundamentals of Physics, Chapter 33, 7th ed. Wiley 2005
- 2. PASCO EX9917A and EX9919 guide sheets (written by Ann Hanks)

This lab manual was written by Ruxandra Serbanescu in 2009 and revised in 2015. The acquisition DAQ board was built and programmed by Larry Avramidis. The lab manual was revised in 2014 by P. Albanelli and S. Fomichev.