

Thermal Noise

Introduction

One might naively believe that if all sources of electrical power are removed from a circuit that there will be no voltage across any of the components, a resistor for example. On average this is correct but a close look at the rms voltage would reveal that a "noise" voltage is present. This intrinsic noise is due to thermal fluctuations and can be calculated as may be done in your second year thermal physics course! The main goal of this experiment is to measure and characterize this noise: Johnson noise.

In order to measure the intrinsic noise of a component one must first reduce the extrinsic sources of noise, i.e. *interference*. You have probably noticed that if you touch the input lead to an oscilloscope a large signal appears. Try this now and characterize the signal you see. Note that you are acting as an antenna! Make sure you look at both long time scales, say 10 ms, and shorter time scales, say 1 μ s. What are the likely sources of the signals you see? You may recall seeing this before in the First Year Laboratory.

This interference is characterized by two features. First, the noise voltage is characterized by a *spectrum*, i.e. the noise voltage $V_n(f)$ is a function of frequency. Since noise usually has a time average of zero, the power spectrum $\langle V_n^2(f) \rangle$ is specified in each frequency interval df . Second, the measuring instrument is also characterized by a spectral response or *bandwidth*. In our case the bandwidth of the oscilloscope is from $f_L=0$ (when input is DC coupled) to an upper frequency f_H usually noted on the scope (beware of bandwidth limiting switches). In this case the scope is a 'low pass' instrument since all frequencies up to $\approx f_H$ are treated equally. When the scope is AC coupled $f_L > 0$ so that it rejects signal components with frequencies less than f_L . In this case the scope is in a 'band pass' configuration where $f_L < f < f_H$ is the pass band.

In this light one obvious way to reduce noise is to limit the bandwidth of the system. This is done in audio amplifiers which have their gain rolled off at about 20 KHz which happens to be the upper limit of hearing. Radio receivers make use of a pass band which just allows the signals of interest through (perhaps from one particular cell phone channel) and not other signals or noise.

Interference can also be reduced by shielding. Compare the signal seen on a scope with a banana plug connected vs. a coaxial cable. Wherever possible in this experiment you should use coaxial cables to effectively shield out the interference you observed earlier.

Now that extrinsic sources of noise have been dealt with let us consider intrinsic sources of noise. There are different kinds of noise, some of which we will discuss here. As current is carried by discrete units, electrons, it is not continuous. This gives rise to *shot noise* whenever current flows. This noise is 'white' as there is equal power in each frequency interval df . The noise current spectrum is given by $dI_n = 2edf$ where e is the charge of an electron. Shot noise is most easily seen in semiconductor devices, i.e. diodes. Another type of noise in driven systems is $1/f$ noise. As the name implies the power spectrum varies as $1/f$. You might suppose that DC would therefore give infinite noise power but you must remember that **true** DC requires an infinite observation time! In fact, the longest set of observations available are the Egyptian records of the level of the Nile, spanning ≈ 3500 years. The fluctuations fit a $1/f$ power law very well! Although $1/f$ noise is not very well understood it does seem to be ubiquitous. A typical combination of white and $1/f$ noise is displayed in figure 1.

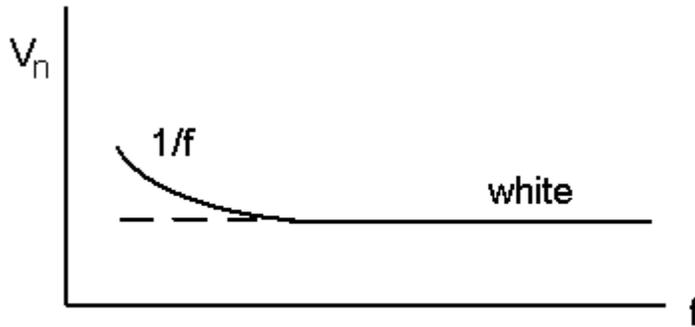


Figure 1 Typical noise spectrum with white and $1/f$ noise.

Fluctuations in a circuit in equilibrium can be treated thermodynamically. The thermal noise of a resistor R was first observed and explained in the early days of radio by Johnson and Nyquist. This 'Johnson noise' is white with

$$d \langle V_R^2(f) \rangle = 4k_B TRdf \quad (1)$$

where k_B is Boltzmann's constant and T is the temperature of the circuit. As mentioned above one might expect the total noise would diverge over infinite bandwidth. We have already discussed the impact of the bandwidth limitations of the measuring instruments but there is also the limiting features of the actual component. In this case a real resistor has leads and their associated shunting capacitance. Figure 2 illustrates the equivalent circuit of a resistor with its lead capacitance C and noise source. If V_R is the noise voltage of an ideal resistor R then the measured noise spectrum is given by

$$d \langle V_{meas}^2(f) \rangle = \frac{d \langle V_R^2(f) \rangle}{(1 + (2\pi fRC)^2)} \quad (2)$$

Integrating over all frequencies to get the total mean square noise voltage now yields a finite result

$$\langle V_{meas}^2 \rangle = \int_0^\infty \frac{4k_B TR}{(1 + (2\pi fRC)^2)} df = \frac{k_B TR}{RC} = \frac{k_B T}{C} \quad (3)$$

This result is equivalent to

$$\frac{1}{2} C \langle V^2 \rangle = \frac{1}{2} k_B T \quad (4)$$

where the energy in the capacitor is associated with one thermodynamic degree of freedom by the equipartition theorem (you will encounter this in the second year thermodynamics course). In the derivation of eqn. 1 one actually starts with the equipartition theorem and then develops the expression for the noise power spectrum.

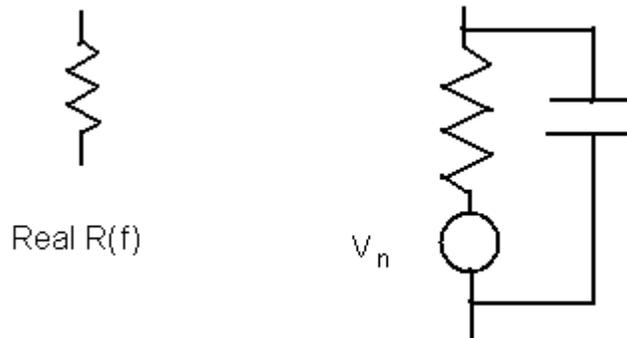


Figure 2 Equivalent circuit of a real resistor.

Experiment

The formal part of this experiment involves measuring the noise of resistors allowing one to extract Boltzmann's constant k_B . Before beginning the quantitative measurements you will first make some qualitative observations. Start by looking at the noise of a 33 K Ω resistor.

Connect this resistor, in the metallic shielded box, to the Princeton Applied Research differential amplifier with two coaxial cables. Note that the outer conductors of

the cable, the metal box and the BNC connectors of the amplifier, form a continuous EM shield. It is important to utilize this shielding in order to screen out unwanted EM signals. This type of shielding is effective for screening out electric fields but not for magnetic fields. In principle one could use high permeability materials but a more reasonable, and simpler, solution is to twist the coaxial cables together - as you would two wires to make a twisted pair. How does this technique manage to reduce magnetic pickup of external signals?

Connect the output of the amplifier to the scope and the loudspeaker. In this way you can "see" and "listen" to the noise. Examine the scope signal to confirm that extrinsic signals have been eliminated. Note the scope waveform as you change the timebase. What is this telling you about the noise spectrum? Change the high and low pass filter settings on the amplifier. How does this affect the sound? See how the noise changes with different values of resistance R. Is the noise zero when you set R=0?

Observe the noise that results from flexing the coaxial cables. This is referred to as a triboelectric effect. Clearly you would rather not be moving your cables etc. while performing quantitative measurements!

To make a quantitative measurement of the noise, the amplifier output voltage needs to be measured with a voltmeter of sufficient bandwidth. Here we use the HP3400A, a true rms voltmeter with a bandwidth of ≈ 10 MHz. Your measurements of $\langle V_{meas}^2 \rangle$ depend on the bandpass filter limits f_L and f_H of the amplifier. You can measure the frequency response of the amplifier by connecting an oscillator to the amplifier input and plotting the output signal vs. frequency. A simple RC filter gives a high frequency rolloff $\approx \frac{1}{\sqrt{1 + f^2 / f_H^2}}$. If f_L is set to just suppress $1/f$ noise can it be

ignored relative to f_H ? Assuming other bandwidth limiting factors are negligible the measured voltage would be

$$\langle V_{meas}^2 \rangle = 2\pi k_B T R G^2 f_H + V_n^2 G^2 f_H \quad (5)$$

where G is the amplifier gain and V_n is the amplifier input noise voltage (*rms* Volts/ \sqrt{Hz}). Thus a suitable plot of data will allow you to extract Boltzmann's constant. You must collect sufficient data to verify the veracity of equation (5). Think about what kinds of plots would be useful. How will you go about determining V_n ? Take a close look at equation (5) and explain why the V_n^2 term contains the factor $G^2 f_H$ and what assumptions accompany this form. Note that equation (5) gives an incomplete description of amplifier noise but is sufficient for this experiment.

You can test the temperature dependence of equation (5) by measuring the noise of a carbon resistor at room temperature and liquid nitrogen temperature. First compare

the noise voltage of the 10 k Ω carbon resistor with the 8.2 k Ω in the shielded box. Now immerse the resistor in liquid nitrogen. Recalling triboelectric effects you might want to raise the liquid nitrogen about the resistor rather than moving the wires. Can this device be used as a thermometer?

You can also try and measure the extra noise when a DC current is passed through the resistor. You can compare this to the noise of a forward-biased diode as well.

Some questions to consider.

- 1) Johnson noise is based on the equipartition of energy which is a classical concept. At what temperature would you expect this result to fail?
- 2) A real resistor also has an inductance. What is the effect of also shunting the ideal resistor with an inductance?
- 3) The radiation resistance of free space is $\sqrt{\mu_0 / \epsilon_0}$. What is the rms noise voltage at the terminals of an antenna which will compete, say, with an FM radio signal?

References

- J.B. Johnson, Phys. Rev. **32**, 97 (1928).
- H. Nyquist, Phys. Rev. **32**, 110 (1928).
- F. Robinson, *Noise in Electrical Circuits*.
- Kittel and Kroemer, *Thermal Physics*.

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