The velocity of Ultrasonic Waves in Water by the Debye-Sears Effect

1. **Goal**

   This experiment features the use of an ultrasonic standing wave in water as an optical diffraction grating. You will study the diffraction of light through density fluctuations of water. You will measure the speed of sound in water and will determine the bulk modulus of water.

2. **Theoretical Background**

   (a) **The Debye-Sears Effect**

   Sound waves in liquids determine density changes, caused by the pressure nodes and antinodes. The spacing of these periodic fluctuations in density id determined by a frequency f_s and by the speed of the sound wave v_s. In a transparent medium, variations is density correspond to variations in the index of refraction. Therefore, a monochromatic parallel light beam traveling perpendicular to the sound direction is refracted as if it had passed through a diffraction grating of spacing $d = \lambda$:

   $$m\lambda = dsin\theta \tag{1}$$

   where: m is the order of diffraction, θ is the angle of diffraction and λ is the wavelength of the light source.

   The sound wave is setup by driving a piezo-electric crystal at frequency f_s of $\approx$ 1MHz. The main goal of the experiment is to deduce the velocity of sound in water, given by equation (2):

   $$v_s = f_s \lambda_s \tag{2}$$

   by measuring the corresponding d from equation (1).

   (b) **The bulk modulus of water**

   The speed of sound depends on both the inertial property of the medium (which stores kinetic energy) and the elastic property (to store potential energy):

   $$v_s = \sqrt{\frac{B}{\rho}} \tag{3}$$

   where B is called *Bulk Modulus* and accounts for the extent to which an element from the medium changes its volume when a pressure is applied: $B = -\frac{\Delta p}{\frac{\Delta V}{V}}$.

   Here $\frac{\Delta V}{V}$ is the fractional change is volume produced by a change in pressure Δp. The signs of Δp and ΔV are always opposite. The unit for B is Pascal (Pa).

   Also in equation (3) we have ρ = water density.

3. The Experiment

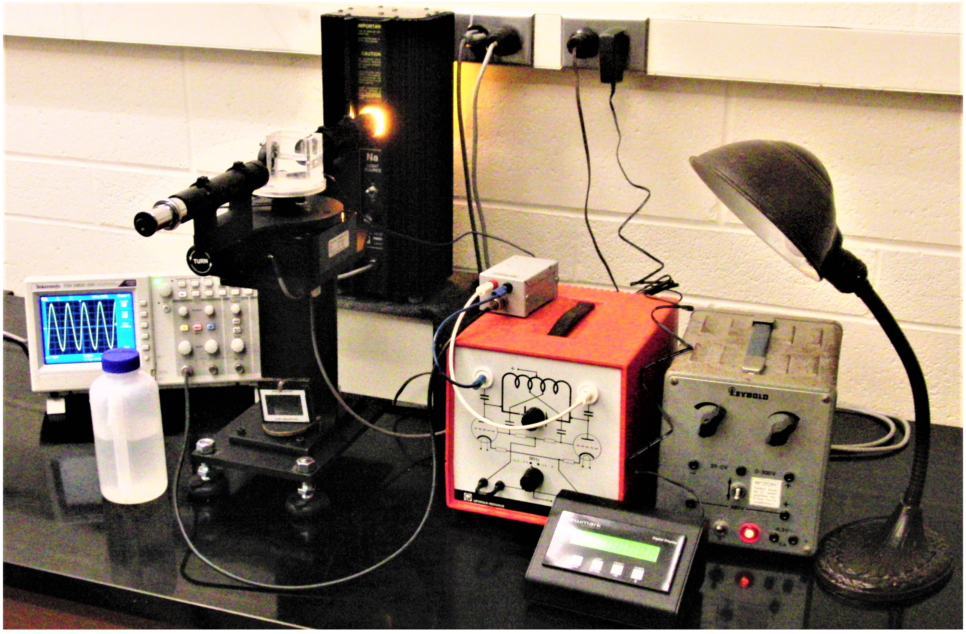

Figure 1: The experimental setup

(a) Practice and adjustments

In order to gain some familiarity with diffraction measurements, set up the spectrometer with only the standard diffraction grating provided.

Let the sodium lamp warm up for at least 10 minute (it has to emit bright yellow light). Place the diffraction grating on the rotating table in front of the spectroscope. Adjust the view until you see two thin yellow lines separated by a narrow gap when the spectrometer is aligned with the sodium lamp. Slowly rotate the viewer until you see the diffraction orders $m = \pm 1, m = \pm 2$.

Using the digital angle reader and the turn button from the rotating table, measure the angles corresponding to $m = 0, m = \pm 1, m = \pm 2$. Use the data to find the wavelength of the sodium lamp λ_L: $m\lambda_L = d sin\theta$. Here d comes from the diffraction grating: 2500 lines per inch.

Note that the digital angle reader scale reads degrees up to a resolution of 0.00225^o. You may need to press *Preset* when measuring $\Delta\theta$.

Fill the cell with water and set it on the rotating table in front of the spectrometer. Adjust the instrument slit (located at the back) to obtain a sharp image of the slit and crosshairs in the view.

(b) Connecting the instruments

Connect the ultrasound transducer to the oscillator through the *Generator* box. Connect the oscilloscope and the frequency counter. DO NOT connect the oscilloscope directly to the oscillator as its output voltage is of the order of 300V.

Switch on the ultrasonic beam and adjust the frequency to $\approx 2MHz$.

The challenge in this experiment is to measure d' corresponding to $m'\lambda_L = d' sin\theta'$. This parameter is the spacing of the *diffraction grating* created by the ultrasonic standing wave formed in the cell. $d' = \lambda_s$ where λ_s is the wavelength of the ultrasonic wave of 2 MHz.

Measure the diffraction due to the ultrasound in water for several frequencies f_s (1.80 - 2.1 MHz).

(c) Experiment outcomes

Plot λ_s vs. $1/f_s$ and output the velocity of sound v_s. Calculate the bulk modulus of water B.

Analyze the goodness of the fit χ^2. Do your results agree with a linear dependence of d vs. $1/f_s$? If not, you may have evidence of sound waves dispersion in water.

Compare your results with data from literature.

Notes:

- Velocity of sound in liquids is temperature-dependent.
- We determine *the bulk modulus for adiabatic compression* because there is no energy exchanges with the region next to the sound wave. This is different than *the isothermal bulk modulus*.

Some References:

1. M. Born, E. Wolf: Principles of Optics, Cambridge University Press, 7^{th} edition 1999.

2. Halliday, Resnick, Walker: Fundamentals of Physics, Ch. 17, Wiley, 7^{th} edition 2005.

This document was revised in 2019 by Ruxandra M. Serbanescu