Blackbody Radiation

References
1) R.A. Serway, R.J. Beichner: Physics for Scientists and Engineers with Modern Physics, 5th Edition, Vol. 2, Ch.40, Saunders College Publishing (A Division of Harcourt College Publishers), 2000.
2) PASCO Instruction Manual and Experiment Guide for the Blackbody Radiation Experiment, PASCO Scientific Model OS-8542 (Guide 012-07105B).
3) Hyper Physics, Georgia State University, Department of Physics and Astronomy, written by Dr. R. Nave: http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html

INTRODUCTION

Thermal radiation is emitted by heated objects. At the beginning of the 20th century, there was a large understanding of the fact that heating a solid object resulted into vibrations of its atoms and molecules. As Isaac Newton explained in *Opticks* in 1704: "*Do not all fix'd Bodies, when heated beyond a certain degree, emit Light and shine; and is not this Emission perform'd by the vibrating motion of its parts?*"

All matter emits electromagnetic radiation when it has a temperature above absolute zero. The radiation represents a conversion of a body's thermal energy into electromagnetic energy, and is therefore called *thermal radiation*. Conversely all matter absorbs electromagnetic radiation to some degree.

An object that absorbs all radiation falling on it, at all wavelengths, is called *a blackbody*.

It is well known that when an object, such as a lump of metal, is heated, it glows; first a dull red, then as it becomes hotter, a brighter red, then bright orange, then a brilliant white. Although the brightness varies from one material to another, the colour (strictly spectral distribution) of the glow is essentially universal for all materials, and depends only on the temperature. In the idealised case, this is known as *blackbody* or *cavity radiation.*

At low temperatures, the wavelengths of thermal radiation are mainly in infrared. As temperature increases, objects begin to glow red.

The concept of the blackbody is an idealization, as perfect blackbodies do not exist in nature. Experimentally, blackbody radiation may be measured as the steady state equilibrium radiation from a cavity in a rigid body. A good approximation of black body absorption is a small hole leading to the inside of a hollow object. A closed oven of graphite walls at a constant temperature with a small hole on one side produces a good approximation to ideal blackbody radiation emanating from the opening.

Blackbody radiation becomes a visible glow of light if the temperature of the object is high enough. At 1000 K, the opening in the graphite oven looks red; at 6000 K, it looks white. No matter how the oven is constructed, or of what material, as long as it is built such that almost all light entering is absorbed, it will be a good approximation to a blackbody, so the spectrum, and therefore color, of the light that comes out will be a function of the cavity temperature alone.

A graph of the amount of energy inside the oven per unit volume and per unit frequency interval plotted versus frequency is called the *blackbody curve*. Different curves are obtained by varying the temperature:
- The emitted radiation does not depend on the incident radiation, but only on the objects' characteristics.
- The emitted energy is produced by resonant modes of the radiating cavity.
- The amount of radiation emitted in a given frequency range should be proportional to the number of modes in that range.

Classical physics suggested that all modes had an equal chance of being produced, and that the number of modes went up proportional to the square of the frequency. The predicted

continual increase in radiated energy with frequency was named "the ultraviolet catastrophe" (Fig.1):
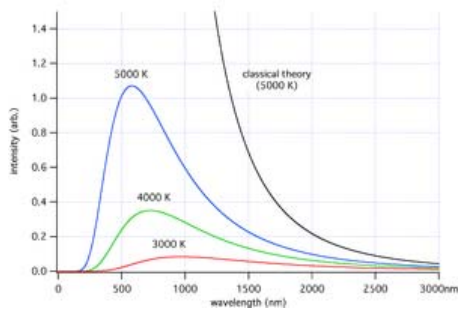

Figure 1: The amount of radiation as a function of frequency: classical and quantum theories (From Wikipedia)

THEORY

The energy of blackbody radiation varies with temperature and wavelength. As the temperature increases, the peak of the intensity of blackbody radiation shifts to lower wavelengths. This explains why heated objects change appearance from not appearing at all (low temperature, infrared emission) to glowing red (peak in visible at the red end of the spectrum). The shift was found to obey the **Wien's displacement law**:

$$\lambda_{max}T = 2.898 \times 10^{-3} m \cdot K \qquad (1)$$

Where: λ_{max} is the wavelength of the peak and T is the absolute temperature of the emitting object.

Another experimental fact observed from thermal radiation studies is that the amount of energy the heated object emits increases with temperature. **Stefan's Law** can be put in the form:

$$I = \sigma T^4 \qquad (2)$$

Where: **I** is the intensity of radiation at the object surface and **σ** is a constant.

Blackbody radiation is emitted in the form of a spectrum. Power per unit area emitted in the wavelength interval $d\lambda$ can be conveniently defined as: $I(\lambda, T)d\lambda$.

In the classical model of blackbody radiation, the **Rayleigh-Jean Law** takes into account that cavity atoms are modeled as oscillators emitting electromagnetic waves of all wavelengths:

$$I(\lambda, T) = \frac{2\pi ckT}{\lambda^4} \qquad (3)$$

Where: k is Boltzmann's constant; c is speed of light in a vacuum.

Only at long wavelengths, the Rayleigh-Jean Law is in good agreement with experimental data. We can see from (2) that intensity approaches infinity as λ approaches zero. This is the "ultraviolet catastrophe" from Fig. 1.

In 1900 Planck derived the formula for blackbody radiation which is in total agreement with experimental data. The intensity (I) of radiation emitted by a body is given by **Planck's Radiation Law**:

$$I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \qquad (3')$$

Where: $h = 60626 \times 10^{-34} J \cdot s$ is Planck's constant, *k* is Boltzmann's constant, *T* is the absolute

temperature of the body, and λ is the wavelength of the radiation. At long wavelengths, (3) reduces to the Rayleigh-Jean expression (2) and at short wavelengths is predicts an exponential decrease in intensity with decreasing λ.

The total intensity radiated within all wavelengths is given by the **Stefan-Boltzmann Law**:

$$P = \int_0^\infty I(\lambda, T)d\lambda = 2\pi hc^2 \int_0^\infty \frac{d\lambda}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}}-1} = \sigma T^4 \qquad (4)$$

The assumption that Planck made consisted of two strong ideas that marked the birth of quantum theory:

- Molecules have only discrete energy values: $E_n = nhf$ where *n* is the quantum number and *f* is frequency;
- Molecules emit and absorb energy in discrete "packets", by jumping from one quantum state to another.

Many commonly encountered light sources, including the sun and incandescent light bulbs, are closely modeled as "blackbody" emitters.

METHOD

The spectrum of an incandescent light bulb is scanned by hand using a prism spectrophotometer that measures relative light intensity as a function of angle. A Broad Spectrum Light Sensor is used with a prism so the entire spectrum from approximately 400 nm to 2500 nm can be scanned without the overlapping orders caused by a grating.

The wavelengths corresponding to the angles are calculated using the equations for a prism spectrophotometer. The relative light intensity can then be plotted as a function of wavelength as the spectrum is scanned, resulting in the characteristic blackbody curve. The intensity of the light bulb is reduced, reducing the temperature, and the scan is repeated to show how the curves shift in the peak wavelength.

Determination of wavelengths from angle measurements

The 60 degree prism is mounted so that the back face is perpendicular to the incident light as can be seen in Figure 2:
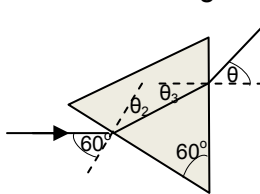

Figure 2: Schematics of the prism

Applying Snell's Law at each of the two refracting faces of the prism, we can obtain an expression for the index of refraction of the prism:

$$n = \sqrt{\left(\frac{2}{\sqrt{3}}sin\theta + \frac{1}{2}\right)^2 + \frac{3}{4}} \qquad (5)$$

➔**Demonstrate (5)**

The index of refraction varies with wavelength as it can be seen from Table 1:

Index of Refraction	Wavelength (nm)
1.68	2325.40
1.69	1970.10
1.69	1529.60

1.70	1060.00
1.71	852.10
1.72	706.50
1.72	643.00
1.73	589.30
1.75	486.10
1.76	435.80
1.78	404.70

Table 1: Variation of the prism index of refraction with wavelength

The Cauchy equation gives the relationship between the index of refraction and wavelength:

$$n(\lambda) = \frac{A}{\lambda^2} + B \tag{6}$$

Where: A and B are constants depending on the prism material. Our prism's coefficients are: A = 13900 and B = 1.689

The final expression for the wavelength: $\lambda = \lambda(\theta)$ can be obtained by substituting (5) into (6).

Determining the temperature of the black body source

The temperature can be estimated indirectly by determining the blackbody electric resistance from the measured voltage and current.

The resistance varies with temperature according to:

$$R = R_0[1 + \alpha_0(T - T_0)] \tag{7}$$

where α_0 is the thermal coefficient at room temperature T_0. Our blackbody is the tungsten filament of a light bulb, which has a coefficient of $\alpha_0 = 4.5 \times 10^{-3} \, K^{-1}$.

Solving (7) for temperature, we obtain:

$$T = 300K + \frac{\frac{V/I}{R_0} - 1}{\alpha_0} \tag{8}$$

Voltage (V) and current (I) are displayed by the acquisition software. R_0 is the bulb resistance at room temperature. Its approximate value is 1.1 Ω.

SETUP

Optical: The Prism Spectrophotometer is setup for you. Check that the prism is oriented with the apex facing the light source (Figure 3). Verify that the base piece of the prism is aligned with the zero mark on the spectrophotometer table when the prism apex points toward the light source..

The collimating lens must be ~10 cm from the collimating slits.

Set the collimating slits on Slit #4. Set the light sensor mask on Slit #3.

Acquisition Software: Open the Blackbody Radiation Labview application using the desktop shortcut. Do not turn the acquisition ON yet! Turn on the Signal Generator (switch is at the back).

Figure 3: Prism Orientation

Several buttons are placed at the top of the acquisition window: Clear Data erases the current file; Export Data allows you to save the data (not the plot) as a .txt format; Clean Up Data applies a filter to reduce noise; Integrate Area Under Curve does what it says between two cursor positions setup by the user; Automatic Data Limit Switch allows you to scan the emitted spectrum over 30 degrees or 80 degrees (remember that the initial angle is 75.7 degrees).
At the bottom, there are display windows for average Voltage and Current as well as for the two pairs of cursors.
Caution: If 10 volts is applied to the blackbody light source for an extended amount of time, the life of the bulb will be reduced. Only turn the bulb on when taking measurements.

PROCEDURE
Note: The light detector has a systematic drift in the output. In order to minimize it, do not bring your hands close to it (you emit infrared radiation, too!), do not touch it and use only the sides of the light sensor mask to move it during the scan.

Rotate the scanning arm counterclockwise until it touches the stop. This will be the starting position for all the scans.

Set the Broad Spectrum Light Sensor gain switch to "x10" and press the tare button.

Choose an 80 degree acquisition and setup a voltage of 6V.

Look at the light coming from the Blackbody Light Source. Observe the color.

Look at the spectrum on the Light Sensor screen. Are all the colors (from red to violet) present?

Slowly rotate the scanning arm through the spectrum and continue all the way until it turns off by itself

There will be a main peak on the intensity vs. angle and also a smaller blip where the light sensor is aligned with the light source because some light passes by the prism instead of going through the prism. This blip enables the *initial angle* to be exactly determined. We determined it for you in many experimental runs. Use the value of 75.7 degrees for this angle called "Init".

Switch to the 30 degree scan; bring the arm back to the starting position.

1. **Wien's Displacement Law**
Using voltages between 10V and 4V, take data of: voltage, current and determine the angle of the main peak by using the cursors on a 30-degree scan. You may need to switch the gain to 100 at lower voltages. After each gain change you have to press the Tare button on the light sensor several times.
➜Calculate the experimental peak wavelength values λ_{exp}. To calculate wavelength, assume that θ = Init-(Peak Angle).
➜Calculate temperature values T using voltage, current and expression (8)

2. **Stefan-Boltzmann Law**
The light sensor records the relative intensity and returns a voltage value directly related to power per unit area that falls upon it.
For each of the voltage runs from 10V to 4V, limit the peak area to integrate by placing the two vertical cursors at 10 degree and 26 degree (you may choose other values, if you wish) and use the Integrate button to obtain the area under curve. Notice that the intensity does not fall to the baseline after the peak due to ambient light. Delimit the "tail" using cursors and re-integrate.
➜Use the "tail" area value as correction to the peak area value. You also need to calculate the temperatures or use the values calculated before for Wien's Law.
Note: Keep the gain constant from one voltage run to another in this part of the experiment!

ANALYSIS

➔Use data from Wien's Law and calculate the average value of $T \times \lambda_{exp}$. How does it compare with Wien's Displacement Law (1)?

    ➔ *Optional: Integrate Eq..(4) to prove Stefan-Boltzmann Law*

➔*Python requirement (PHY224/324)*: Use data from Stefan-Boltzmann Law and plot: Area vs. T^4. How well your (qualitative) graph compares with equation (4)?

QUESTIONS

➔How did the color of the bulb change with temperature? How did the color composition of the spectrum change with temperature? Considering the peak wavelengths, why is a bulb's filament red at low temperatures and white at high temperatures?

➔For the highest temperature, is more of the intensity (area of the intensity vs. wavelength graph) in the visible part of the spectrum or in the infrared part of the spectrum? How could a light bulb be made more efficient so it puts out more light in the visible?

Submit answers to all questions marked by➔. Students from PHY224/324 have to write all the Python programs and hand in a hard copy to the teaching assistant.

This guide sheet was written by Ruxandra M. Serbanescu (v.2 2013).
Larry Avramidis built the Data Acquisition Device and wrote the Labview acquisition program.