The Zeeman effect has played an important role in the development of quantum theory. It illustrates the phenomenon of space quantization, which refers to the angular momentum $L$ of the atom assuming only a set of discrete orientations with respect to an external magnetic field $B$.

We recall that an electron in a Bohr circular orbit will have a magnetic dipole moment $\mu = IA$, where $A$ is the area enclosed by the orbit of radius $R$, $A = \pi R^2$, and $I$ is the current associated with the electron due to its motion (we assume the speed of this motion is so great that we can treat the current $I$ as steady and hence magnetostatics applies), namely $I = \frac{dq}{dt} = -\frac{e}{T}$. Here $T$ is the orbital period, $T = \frac{2\pi R}{v}$. The direction of the magnetic dipole moment will be perpendicular to the plane of the orbit — it is given by the right hand rule for circulating current. The magnitude of orbital angular momentum $L$ of the electron can be estimated as follows: $L = pR = mvR = \hbar l$, where $l$ is the so-called azimuthal quantum number (in chemistry it is usually associated with the subshell that a given electron occupies). Thus $v = \hbar l mR$. Hence we can write for the current $I = \frac{ev}{2\pi R}$ and hence the magnetic moment generated by the motion of the electron is

$$\mu = IA = \frac{ev}{2\pi R} \pi R^2 = \frac{e\hbar}{2m\pi R^2} \pi R^2 = e\hbar/2m = \mu_0 l$$  \hspace{1cm} (1)

(here $\mu_0 = e\hbar/2m$ is the Bohr magneton).

Now the above expression for the magnetic dipole moment is only true when the total spin of all electrons in an atom is zero. In general, electron spin has to be accounted for: following an argument too long to be described here (see [1] for details), we can write, for a spin angular momentum $S$

$$\mu = -\frac{\mu_0}{\hbar} (L + 2S)$$
whereas the total angular momentum $J$ is simply the vector sum

$$J = L + S$$

The factor of 2 appearing in the expression for the magnetic dipole moment means that the magnetic dipole moment vector is not, in general, collinear with the total angular momentum: this makes the analysis more complicated. If, however, the total electron spin couples to zero, i.e. $S = 0$ \(^1\), then the expression reduces to that in equation (1). Historically, the case with zero spin was the first one discovered (this was the version of the Zeeman effect observed and described by Zeeman himself in 1896): it has to come to be referred to as “normal” Zeeman splitting, as opposed to “anomalous” Zeeman splitting, which is the case of non-zero spin (for further discussion of this we again point the reader to [1]).

Assume we are dealing with regular Zeeman splitting, so that (1) holds (consequently, $J = L$ and we shall continue to use $L$ for angular momentum). We begin by assuming that the magnetic field is a weak one. The choice of a weak field stems from the fact that if the field is too strong, it destroys the coupling between the orbital and spin angular momenta (sometimes referred to as LS coupling, after the vectors of angular momentum, $L$ and $S$), and results in a different effect, called the Paschen-Back effect, in which the lines split disproportionally and one can no longer assume the splitting distance linearly proportional to the strength of the field. It remains to be specified what exactly one might take to be “strong” and “weak”: the clue is provided by the fact that the field has to destroy the LS coupling for Zeeman effect to evolve into Paschen-Back effect. The typical strength of the internal magnetic field of the atom acting on an electron is on the order of 1 Tesla [1]: hence we designate fields “weak” if they are about 0.8 Tesla and lower, and “strong”, if they are 1.5 Tesla and higher. In this experiment, the range of fields accessible to us is only up to about 0.8 Tesla, hence we will not be observing the Paschen-Back effect \(^2\).

The torque on the atom due to the external field is

$$\tau = \mu \times B = -\frac{\mu_0}{\hbar} L \times B,$$

\(^1\)An example of this is $^{198}$Hg, whose overall spin is zero — that is, the sum of the spins of all the electrons is zero.

\(^2\)It should of course be noted that the numbers given above vary significantly when one uses different atoms: for example, observing the Paschen-Back effect with a lithium lamp requires significantly lower magnetic field magnitudes.
which is perpendicular to both \( \mathbf{L} \) and \( \mathbf{B} \) and causes the tip of the orbital angular momentum vector \( \mathbf{L} \) to precess in a circular orbit about \( \mathbf{B} \) (see fig. 1). The interaction energy between the atom’s magnetic dipole moment and the field is:

\[
\Delta E = -\mu \cdot \mathbf{B} = +\frac{\mu_0}{\hbar} \mathbf{L} \cdot \mathbf{B}
\]  

Figure 1: Precession of the orbital angular momentum vector \( \mathbf{L} \) about the direction of the magnetic field \( \mathbf{B} \).

Take the direction of the field to be the z-direction, i.e. let \( \mathbf{B} = B\mathbf{z} \). Then we may write \( \Delta E = \frac{\mu_0}{\hbar} B L_z \), where \( L_z \) is the projection of the orbital momentum on the z-axis. According to quantum mechanics, the magnitude of the projection will also be quantized, with the magnetic quantum number \( m_l \) as the index: \( L_z = m_l \hbar \), where \( m_l = -l, -l + 1, ..., +l - 1, +l \). Hence we obtain for the energy levels

\[
\Delta E = \mu_0 B m_l
\]  

Therefore, when an atom is placed in a magnetic field, the energy level with principal quantum number \( n \) and orbital angular momentum indexed by azimuthal quantum number \( l \) will split into \( 2l + 1 \) sub levels (see fig. 2).
Figure 2: Energy level splitting corresponding to different projections of orbital angular momentum onto the axis of the magnetic field.

By convention, the orbital angular momenta of states of the atom are given letter labels: S (for \(l = 0\)), P (for \(l = 1\)), D (for \(l = 2\)), F (for \(l = 3\)), G (for \(l = 4\)), H, etc (hereafter it proceeds in alphabetical order). See fig. 3 for energy level splitting of different orbitals.

Figure 3: Energy level splitting for different orbitals (i.e. different angular momentum magnitudes).
We have three allowed electric dipole transitions with the magnetic field “on” (see fig. 4): they have frequencies $\nu = \nu_0$, $\nu = \nu_0 + \mu_0 B$ and $\nu = \nu_0 - \mu_0 B$. The last two, with $\Delta m = +1$ and $-1$, correspond to circularly polarized light when viewed along the $B$ field direction (i.e. parallel to the field lines). The first one, with $\Delta m = 0$, corresponds to linear polarization along the $B$ field direction. A propagating electromagnetic wave has electric and magnetic fields, $E_w$ and $B_w$, perpendicular to the direction of travel. Therefore, when viewed along the $B$ field direction (of the field of electromagnets, not of the wave), we do not see the wave with frequency $\nu_0$, but we do see the $\nu = \nu_0 \pm \mu_0 B$ waves as right and left hand circularly polarized waves, respectively. On the other hand, when viewed along a direction perpendicular to $B$ (this is the straight-on orientation of the apparatus, with the electromagnets on either side of the lamp), the $\nu = \nu_0$ appears as a linearly polarized wave with $E_w$ parallel to $B$, and the $\nu = \nu_0 \pm \mu_0 B$ appear as linearly polarized waves with $E_w$ perpendicular to $B$.

**Apparatus**

The experimental setup is shown in fig. 5 (the actual power source used may be different from the one in the picture). The electromagnet is situated on a stand that can rotate to allow one to observe the lamp either axially (and so parallel to the field lines), or transversely (perpendicular to the field). The lamp fits nicely into the holder on top of the electromagnet, just between the poles — they have been adjusted to be as close together as possible and still accommodate the lamp and the Hall probe, with the pole pieces clamped in place to prevent them from
attracting each other when the magnetic field is applied. The diagram of the optical device used to observe the effect is shown in fig. 6.

Figure 5: The apparatus used in the Zeeman effect experiment: the electromagnet with the power source and the optical instrument are shown.

Figure 6: A schematic diagram of the optical setup used in the Zeeman effect experiment, including the Lummer-Gehrcke plate.

Experiment

In this experiment one wishes to measure the Zeeman splitting of the red line of Cadmium (regular $^{112}$Cd), resulting from an electron’s transition between two
states, $5^1D_2$ to $5^1P_1$, in a weak magnetic field. From the electron configuration for the two states (described by the expressions above) we may infer that we will witness the “normal” Zeeman effect, when a magnetic field is applied (because the spins are zero ³). The electromagnet and the power supply provided will produce uniform fields up to 0.8 Tesla. You may use a Hall probe to measure $B$ at the location of the Hg-Cd discharge lamp, which is inserted between the magnet pole pieces, and build a calibration curve. The magnet assembly can be rotated to allow you to view the Cd red lines both parallel and perpendicular to $B$.

A red filter is used to remove extra emission lines (other than the red one): it is positioned in front of the Lummer-Gehrke plate. When the magnetic field is turned on, each of the red interference lines splits into two or three lines, depending on the direction of observation and polarization filter orientation. A measurement of this splitting, $\Delta S$, can be made with respect to the spacing $\Delta A$ of successive interference lines with the field off. It must be clarified that by the spacing $\Delta S$ we mean \textit{half} the spacing between the two splitting lines (as that is the true displacement due to the field of any one line from the original unsplit line). We can obtain a relationship between the shift of the frequency of this spectral line and the magnetic field that causes it. From Einstein’s relation

$$\Delta E = h\nu = 2\pi\hbar\delta\nu$$

If we use the expression for the energy of the transition we derived above (3) and equate the two, we obtain

$$\delta\nu = \frac{\mu_0 m_l B}{2\pi\hbar} = \frac{m_l B e}{4\pi m}$$

According to our selection rule (see [1] for an explanation of selection rules — these are usually obtained experimentally), $m_l$ can only take on the values $-1, 0, 1$. Hence we obtain

$$\delta\nu = 0, \pm \frac{B e}{4\pi m}$$

Now we can find the frequency difference $\Delta\nu$ through the wavelength difference,

³Electron configurations are normally designated as $n^{2s+1}L_l$, where $n$ is the principal quantum number, $s$ is the spin quantum number, and $l$ is the azimuthal quantum number, and $L$ is a placeholder for the letters corresponding to different orbitals, as mentioned earlier.
\[ \Delta \lambda = \frac{\lambda^2 \sqrt{\eta^2 - 1}}{2d(\eta^2 - 1 - \eta \lambda \frac{d\eta}{d\lambda})} \]  

(5)

where \( \lambda = 643.8 \text{ nm} \) for the red line in cadmium, \( d = 4.04 \text{ mm} \) is the thickness of the Lummer plate, and \( \eta = 1.4567 \) is the refractive index of quartz glass of the Lummer plate. From Cauchy’s equation for refractive index

\[ \eta(\lambda) = B + \frac{C}{\lambda^2} + \frac{D}{\lambda^4} + \ldots \]

we see that, to first non-constant order

\[ \frac{d\eta}{d\lambda} = -\frac{2C}{\lambda^3} \]

and so the last term in the denominator of equation (4) is

\[ \eta \lambda \frac{d\eta}{d\lambda} = -\frac{2\eta C}{\lambda^2} \]

Usually the numbers \( C \) are around \( 0.005 \mu m^2 \) - after everything is factored in, the term is still small enough compared to 1 to be neglected. Thus we obtain

\[ \Delta \lambda = \frac{\lambda^2}{2d} \frac{1}{\sqrt{\eta^2 - 1}} \]  

(6)

Now we can measure the splitting \( \Delta S \) relative to the distance between adjacent spectral lines \( \Delta A \). Since both these quantities will be small, we may use an approximation:

\[ \frac{\delta \lambda}{\Delta \lambda} \simeq \frac{\Delta S}{\Delta A} \]

where by \( \Delta S \) we denote the displacements of the split lines, as before. Using the expression for \( \Delta \lambda \)

\[ \delta \lambda = \frac{\Delta S \lambda^2}{\Delta A} \frac{1}{2d \sqrt{\eta^2 - 1}} \]
According to the well-known relation $c = \lambda \nu$, where $c$ is the speed of light, we obtain

$$|\delta \nu| = \frac{c}{\lambda^2} \delta \lambda$$

and thus, combining (4) and (6) and the above

$$\frac{B}{4\pi m} = |\delta \nu| = \frac{c}{\lambda^2} \delta \lambda = \frac{\Delta S}{\Delta A} \frac{c}{2d} \frac{1}{\sqrt{\eta^2 - 1}}$$

or, finally

$$\frac{e}{m} B = \frac{2\pi c}{d} \frac{\Delta S}{\Delta A} \frac{1}{\sqrt{\eta^2 - 1}} \quad (7)$$

Equation (7) can be used to create a linear fit between the magnetic field $B$ and the spectral line separation ratio, $\Delta S/\Delta A$, from which the charge-to-mass ratio for the electron can be inferred.

The discrete nature of the Cd red lines (the split ones) in a magnetic field is consistent with a model of the Cd atom in which the total orbital angular momentum vector can only exist in discrete orientations with respect to the direction of the applied magnetic field. This so-called space quantization is a major success of the Schrödinger model of the atom and of quantum mechanics as we know it today. A further test of theory can be made by measuring the polarization of the three Cd lines in the magnetic field. Your measurements should be made both parallel and perpendicular to $B$. Use the polaroid and the quarter wave plate.

**Procedure**

First and foremost, be careful with this apparatus. When in doubt, ask your demonstrator to check it. Some rules in regards to using the Hall effect gaussmeter:

1. The Hall effect gaussmeter probe is fragile. Treat it with care.

2. In the container of the gaussmeter, there is a zero chamber. Before you conduct any measurements of the magnetic field, insert the probe completely into the zero chamber, then calibrate it by adjusting the circular dial until the gauge reads zero.
3. Avoid contact between the Hall probe and the electromagnet: it can result in melting of the probe.

The electromagnet (and its power source) is an integral part of the setup. A few rules of exploitation:

1. The poles of the electromagnet will experience an attractive force when the current is turned on. You must tighten down the pole pieces before turning on the magnet power supply, to prevent them from shifting and possibly breaking the light bulb.

2. Always turn the voltage down to zero before turning off the power supply. Likewise, when turning it on, make sure that the starting voltage/current is set to zero to prevent overvoltage. If at any point you see the green light next to the “overvoltage” label, turn the voltage down immediately.

3. As a rule of thumb, avoid changing the voltage and/or current of the power supply too quickly.

4. Keep note of the temperature of the electromagnet: if it gets too hot, turn off the power supply and wait for it to cool down.

Using the power source

When you wish to set the power source to a particular current value, keep in mind that the power source is a complicated circuit with multiple resistors and capacitors, and as such has trouble maintaining the exact current that it is initially set to over even a short period of time, decreasing by about half an ampere in a few minutes (you might want to connect a multimeter in series with the electromagnet to monitor the current more closely 4). To get around this, note that the two sets of knobs regulating the current and voltage on the power source can both act as “principal” or as “secondary” controls. That is, turning one of them first sets a ceiling for the values of the other: for example, if we turn the current knob to about 8 A (assume both are originally at zero), we see that there is no current or voltage running until we turn the voltage knob away from zero. However, as we

4DO NOT use the front ports on the power source to monitor the current — you will be connecting the ammeter in parallel! Voltage, however, can be measured this way.
increase the voltage, we see that the current will only go up to 8 A, no matter how much we increase the voltage past this point. In this example, the current is the secondary control (it sets the maximum current value), while the voltage knob is the principal control (it actually controls the current/voltage running through the electromagnet). This provides a way to get around the decrease of current issue: first set the current limit where you wish, and then increase the voltage a bit past that value: that way the current will remain where you set the limit long enough for you to make a measurement.

Degaussing the electromagnet

If you measure the magnetic field between the poles while the current is turned off, you may notice that the field is not exactly zero: this is due to residual magnetism “stored” in the electromagnet from the previous experiment. Hence before conducting any measurements, make sure that the electromagnet is free of any residual magnetism (the procedure is otherwise known as “degaussing” and applies to any magnetic circuit you might use in the future). The way to achieve it is the following: turn on the power supply, and increase the current to about 9 A. Then slowly turn it down to zero, turn off the power supply, and reverse polarity (switch the banana plugs on the back of the electromagnet). Turn the power supply back on and increase the current from zero to −8 A (the signs are arbitrary, as long as the polarity is the opposite of what it was before). Now turn it back to zero, turn off the power supply and reverse polarity again. Continue descending in this manner, reversing polarity each time, until you hit about 1 A; then decrease your increments (it is convenient to use the “Fine” knob at this point). This effectively eliminates any “stray” magnetism left over in the device: if you now measure the magnetic field with the current turned off, you should be fairly close to zero (a field value within ±0.3 mT of zero is good enough).

Building a calibration curve and maintaining hysteresis

As is apparent from the previous section, it is required that we know the value of the magnetic field between the poles of the electromagnet while measuring a given Zeeman splitting; however, when the lamp is in the holder, it is inconvenient to use the Hall probe. Instead, we will establish what is called a calibration curve,
before conducting our measurements. A calibration curve is a curve that associates (in this case) a given current value, as output by the multimeter, with a magnetic field value, as measured by the probe. By changing the current and measuring the field it produces, we obtain the relationship of the field with the current, which we then plot and conduct a fitting on: the fit function is then capable, when given a set of current values (something that is easy to monitor while measuring the Zeeman splitting), of producing a set of corresponding magnetic field values (so that we do not have to measure them with the probe). The following procedure was taken from [4], with minor adjustments:

In order to calibrate the magnet and verify the reproducibility of a desired field, obtain the following data. Cycle the magnet to maximum current $I$ (which you choose to be slightly above the range of currents you intend to probe — a value around 16 A is most advisable) and then to zero. Beginning with $I = 0$ obtain data for $B$ versus $I$ up to the maximum $I$ you selected before, using steps of about 1 A. Then obtain data for $B$ versus $I$ as you decrease the current from the maximum value back to zero. Finally, obtain two more points by running the current up to about 4 A and 6 A. Upon completion of data taking for these final two points, run the current up to its maximum value and then down to zero, which leaves the magnet ready to begin your Zeeman effect experiments.

Present the data as a plot of $B$ versus $I$, distinguishing between points taken while increasing the current and while decreasing the current. Also, distinctly show the final two points, which is indicative of the reproducibility of a magnetic field using the suggested cycling procedure.

Fit the points of your $B$ versus $I$ curve corresponding to increasing $I$s with a smooth (linear or parabolic) curve. Use this curve to obtain magnetic field values for the Zeeman effect experiments. You will no longer need the gaussmeter.

**Obtaining separation measurements**

You will use the cross-hair in the telescope in conjunction with the gauge to measure the separations between the split and unsplit spectral lines. Since we will only be using the ratio of the original spectral line separation to that of the split lines, no calibration of the gauge is required. This is a general principle worth noting.
When conducting measurements of the separation of the split lines, as well as of the original unsplit ones, it is advisable to consistently choose the same sets of lines. This is due to the fact that, as you can see through the optical instruments, the separation between the lines varies vertically. Choose one set for which the lines are sufficiently widely spaced, yet are still bright and sharp, then choose the other two above and below the first one, then average the ratios obtained for the three and use the average as your ratio value for the corresponding magnetic field value. Moreover, since you are measuring the same sets of lines, and the separations between the original unsplit lines are unaffected by the presence of a magnetic field, it is possible to use the separation between these lines as a way to determine the error in the reading of the optical dial: just look at the deviations in these separation values.

**Quarter wave plate**

One of the devices useful in investigating the polarization state of spectral lines is the so-called *quarter-wave plate*. It is an optical device that changes the polarization of the light passing through it. Normally quarter wave plates are made out of *birefringent materials* — these have a different index of refraction depending on direction of propagation of (as well as the polarization of) the light incident on them. Birefringent crystals have an optic axis (sometimes several): it is defined to be the direction on the crystal along which light can pass through the crystal without being split into two perpendicular components (it does not suffer *birefringence*). By comparison, light passing through in any other direction gets doubly refracted, producing two different light beams travelling at different speeds — these two new beams are called ordinary (O) beam (parallel to the optic axis) and extraordinary (E) (perpendicular to it). The indices of refraction for each ray, \( n_O \) and \( n_E \), are different because of the birefringent properties of the crystal. Hence there will be a non-zero optical path difference between the two beams. If we let \( d \) be the distance travelled by the beams, then we have for the optical path difference

5By “a set of lines” we mean two original, unsplit spectral lines, and the six lines they split into. You would want to use the original lines to measure unsplit lines separation, and then to choose one of them and measure the separation between the farthest two of the three lines it splits into.

6Optical path difference is defined as the product of the actual path travelled by the beam with the index of refraction of the medium through which this distance was travelled, \( r = nd \).
path difference

\[ \Delta = r_E - r_O = d(n_E - n_O) \]

Hence the two beams will be out of phase upon exit, by the amount:

\[ \delta = \frac{2\pi}{\lambda} d(n_E - n_O) \]

By preparing the plate so that the phase difference is exactly 90°, or \( \lambda/4 \) (hence the name “quarter wave”), the two resulting beams will be polarized at right angles to each other — thus their electric field vectors will always oscillate perpendicular to one another. But this is exactly the model for a circularly polarized beam of light (see [4] for further explanations of circular light polarization). Moreover, because of the principle of reversibility of light, if a circularly polarized beam of light is incident on a quarter-wave plate, then the exiting beam will be linearly polarized: a quarter-wave plate can both convert linearly polarized light to circularly polarized light and vice versa. Hence, used in conjunction with a polarization filter, it can cause the extinction of circularly polarized beams (such as the ones visible in this experiment when looking at the lamp in the direction parallel to the magnetic field) by first converting them to linearly polarized beams and then orienting the filter at a right angle to their polarization vector. This will prove useful when investigating polarization of different light beams.

Questions

1. Build the calibration curve, according to the directions given in “Procedure” section. Use Python for the fit (and all subsequent data analysis in this lab!).

2. Take measurements of the current and the separations of the split lines, as well as the separation of the original spectral lines, and plot the ratio of the two versus the magnetic field, values of which you can obtain from the current values via the calibration curve. Do this for both orientations of the electromagnet (you might want to take off the polarization filter and the quarter wave plate when viewing the lamp from the side, i.e. parallel to the magnetic field). Obtain the charge-to-mass ratio, \( e/m \), as the slope of the graph.
3. Investigate the polarization of the different components of incoming light, both parallel and perpendicular to the magnetic field, by using the polarization filter and the quarter wave plate. Describe the extinction arrangement (i.e. orientation of polarization filter and the quarter wave plate with respect to incoming beam for which one (or several) of the beam components vanish) for both cases and explain why that is the case, based on your knowledge of the polarization of the beam components.

4. Would you expect your results in (3) to be different if you used the polarization filter and the quarter wave plate first in front of the Lummer-Gehrcke plate and then behind it?

References


*The lab manual was revised in 2014 by P. Albanelli and S. Fomichev.*