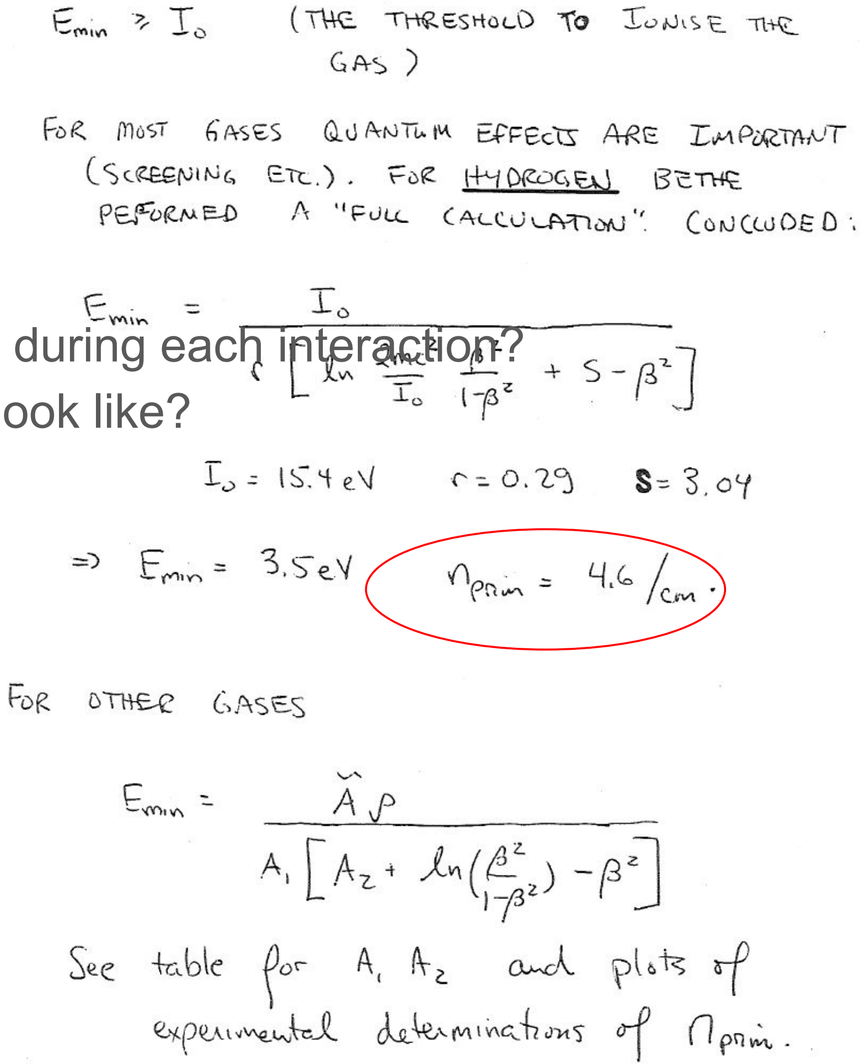
Ionization Yield

Recall…

- Question:
- How many ionizations are happening during each interaction?
- What does the statistical distribution look like?



$E_{min} \geq I_0$ (THE THRESHOLD TO IONISE THE GAS)

FOR MOST GASES QUANTUM EFFECTS ARE IMPORTANT (SCREENING ETC.). FOR HYDROGEN BETHE PEFORMED A "FULL CALCULATION". CONCLUDED:

$$E_{min} = \frac{I_0}{r\left[\ln \frac{2mc^2}{I_0} \frac{\beta^2}{1-\beta^2} + S - \beta^2\right]}$$

$I_0 = 15.4\,eV$ $r = 0.29$ $S = 3.04$

$\Rightarrow E_{min} = 3.5\,eV$ $n_{prim} = 4.6/cm$.

FOR OTHER GASES

$$E_{min} = \frac{\breve{A}\rho}{A_1\left[A_2 + \ln\left(\frac{\beta^2}{1-\beta^2}\right) - \beta^2\right]}$$

See table for A_1 A_2 and plots of experimental determinations of n_{prim}.

Mean number of electron-ion pairs in gas

- For energy deposition E in a media with mean ion-electron pair creation W, the mean number of ion-electron pair is
  <N> = E / W
- W~30 eV for gas
- Note, W is higher than the excitation or ionization potential

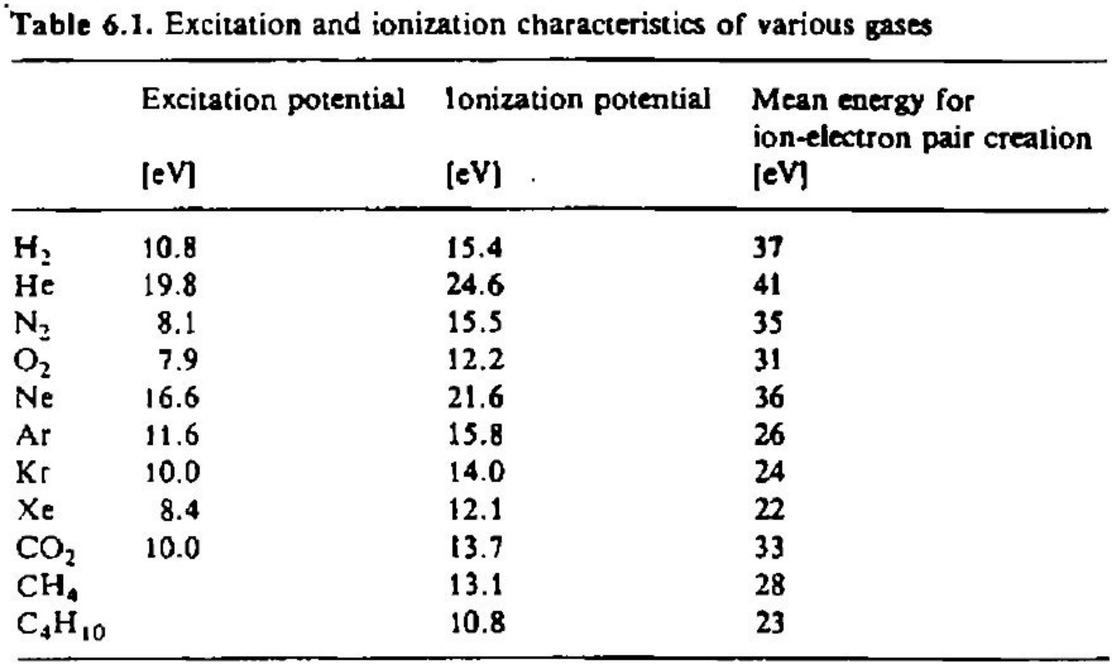


Table 6.1. Excitation and ionization characteristics of various gases

	Excitation potential [eV]	Ionization potential [eV]	Mean energy for ion-electron pair creation [eV]
H_2	10.8	15.4	37
He	19.8	24.6	41
N_2	8.1	15.5	35
O_2	7.9	12.2	31
Ne	16.6	21.6	36
Ar	11.6	15.8	26
Kr	10.0	14.0	24
Xe	8.4	12.1	22
CO_2	10.0	13.7	33
CH_4		13.1	28
C_4H_{10}		10.8	23

Ionization in semi-conductor

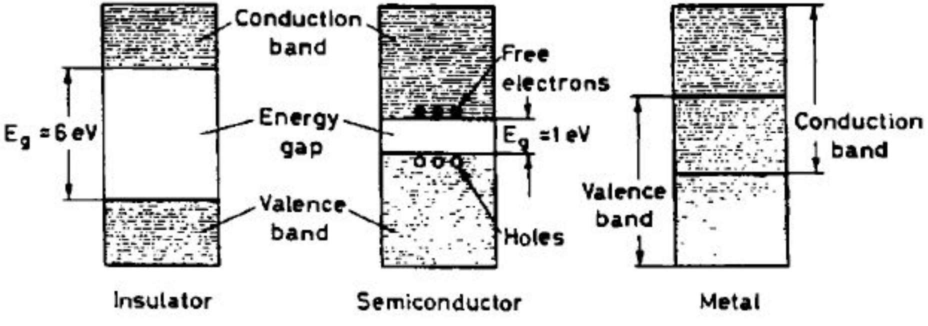
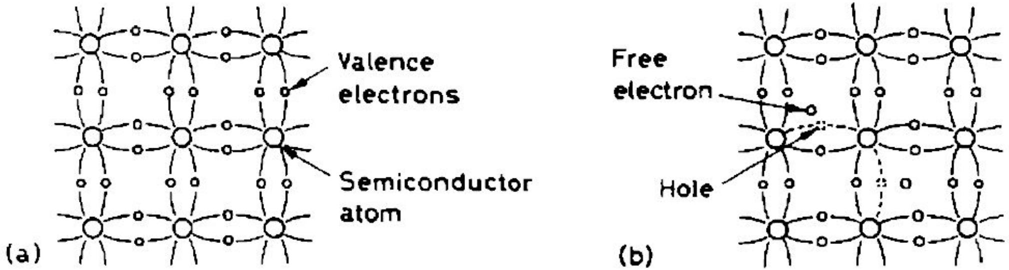


Fig. 10.1. Energy band structure of conductors, insulators and semiconductors



- In semiconductor, ionization is the process of exciting electron-hole pairs
  - Instead of completely free electrons
- For heavy charged particles, Bethe-Bloch still applies
  - Initial ionization can excite electrons to completely free state (or not), then cascade to multiple electron-hole pairs
- Other interactions dumping energy into the **electron system** go through the same electron-hole cascade process
  - Processes depositing energy into the **nuclear system** is a bit different

Ionization yield in semiconductor

- W for semiconductor (often denoted as ϵ), is (again) higher than band gap
- Can be explain by kinematic phase space
  - Electron and hole split energy
  - Secondary ionizations doesn't always have perfect energy split
- ~x10 lower than gas
  - More charge carriers generated
  - Often a good thing, though could cause a dense cloud if too many are generated

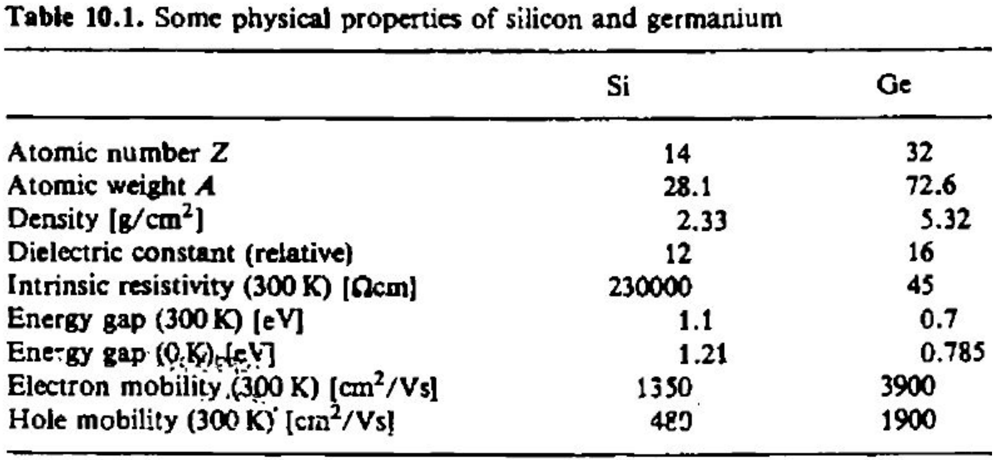


Table 10.1. Some physical properties of silicon and germanium

	Si	Ge
Atomic number Z	14	32
Atomic weight A	28.1	72.6
Density [g/cm^2]	2.33	5.32
Dielectric constant (relative)	12	16
Intrinsic resistivity (300 K) [Ωcm]	230000	45
Energy gap (300 K) [eV]	1.1	0.7
Energy gap (0 K) [eV]	1.21	0.785
Electron mobility (300 K) [cm^2/Vs]	1350	3900
Hole mobility (300 K) [cm^2/Vs]	480	1900

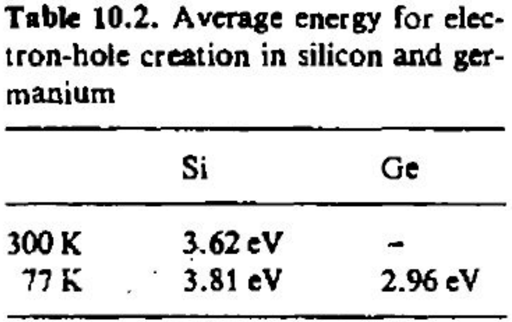


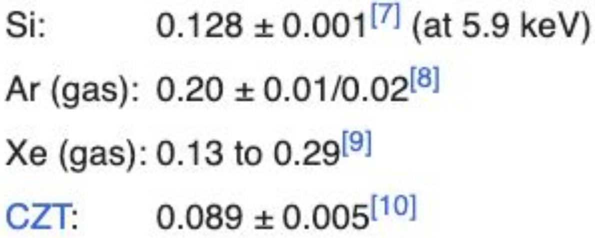
Table 10.2. Average energy for electron-hole creation in silicon and germanium

	Si	Ge
300 K	3.62 eV	–
77 K	3.81 eV	2.96 eV

A side note on statistics: Poisson distribution

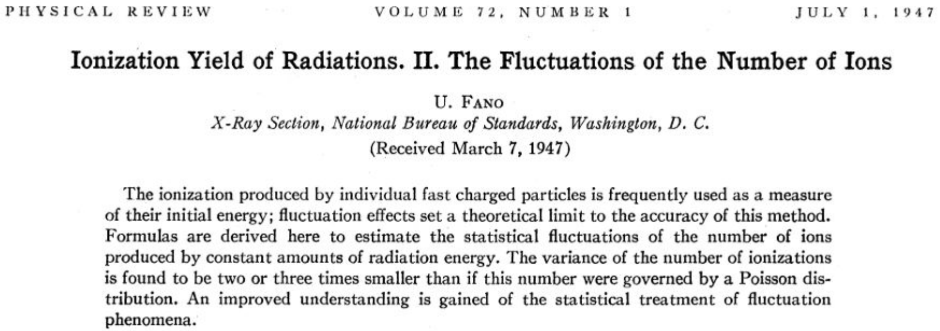
- “If I observe N events between x_1 and x_2 (a bin of a histogram), the uncertainty of N is sqrt(N)”
- ← N follows Poisson distribution
    - At large N, it can be approximated as a Gaussian with mean of N, and width of sqrt(N)
- ← There is a large sample of **independent** particles. In a certain period of time, each particle has a tiny probability of going through a certain process
    - a Binomial distribution where the number of trials, n, gets very large and p, the probability of success, is small
    - Can be approximated by a Poisson distribution
    - https://math.oxford.emory.edu/site/math117/connectingPoissonAndBinomial/

Fluctuation of ionization yield



Si:	0.128 ± 0.001[7] (at 5.9 keV)
Ar (gas):	0.20 ± 0.01/0.02[8]
Xe (gas):	0.13 to 0.29[9]
CZT:	0.089 ± 0.005[10]

- $\langle N \rangle = E / \epsilon$
- Doesn’t follow Poisson distribution…
- Lots of charges comes from secondary ionizations
    - Breaks the “independent” condition in Poisson distribution
- “Fano Factor”
    - $\sigma^2(N) = F * \langle N \rangle$, F ofen <1
    - Energy conservation constraining randomness of ionization



PHYSICAL REVIEW VOLUME 72, NUMBER 1 JULY 1, 1947

Ionization Yield of Radiations. II. The Fluctuations of the Number of Ions

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X-Ray Section, National Bureau of Standards, Washington, D. C.

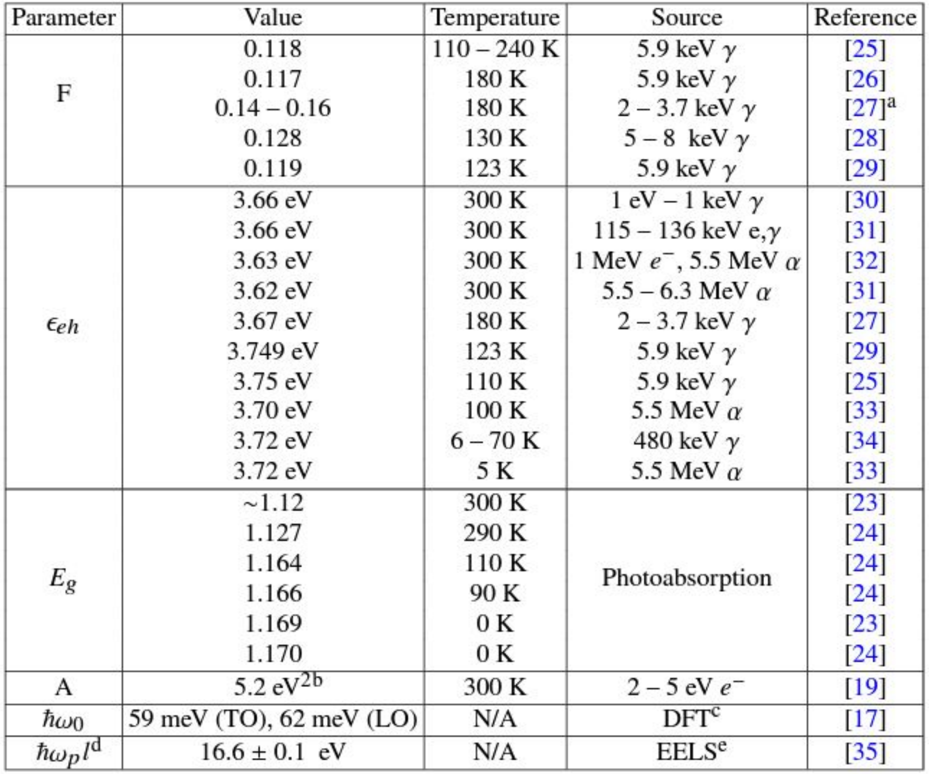
(Received March 7, 1947)

The ionization produced by individual fast charged particles is frequently used as a measure of their initial energy; fluctuation effects set a theoretical limit to the accuracy of this method. Formulas are derived here to estimate the statistical fluctuations of the number of ions produced by constant amounts of radiation energy. The variance of the number of ionizations is found to be two or three times smaller than if this number were governed by a Poisson distribution. An improved understanding is gained of the statistical treatment of fluctuation phenomena.

https://arxiv.org/pdf/2004.10709.pdf

Near ionization threshold

- These averaged behaviors break near ionization threshold
- Example scenarios:
  - Photon incident on silicon (band gap 1.1 eV, ϵ = 3.8 eV)
    1. 2 eV photon on silicon
    2. 10 eV photon on silicon
    3. 0.9 eV photon on silicon…
- Both ϵ and F seem to be temperature and energy dependent…



Parameter	Value	Temperature	Source	Reference
F	0.118	110 – 240 K	5.9 keV γ	[25]
	0.117	180 K	5.9 keV γ	[26]
	0.14 – 0.16	180 K	2 – 3.7 keV γ	[27][a]
	0.128	130 K	5 – 8 keV γ	[28]
	0.119	123 K	5.9 keV γ	[29]
ϵ_{eh}	3.66 eV	300 K	1 eV – 1 keV γ	[30]
	3.66 eV	300 K	115 – 136 keV e,γ	[31]
	3.63 eV	300 K	1 MeV e^-, 5.5 MeV α	[32]
	3.62 eV	300 K	5.5 – 6.3 MeV α	[31]
	3.67 eV	180 K	2 – 3.7 keV γ	[27]
	3.749 eV	123 K	5.9 keV γ	[29]
	3.75 eV	110 K	5.9 keV γ	[25]
	3.70 eV	100 K	5.5 MeV α	[33]
	3.72 eV	6 – 70 K	480 keV γ	[34]
	3.72 eV	5 K	5.5 MeV α	[33]
E_g	~1.12	300 K	Photoabsorption	[23]
	1.127	290 K		[24]
	1.164	110 K		[24]
	1.166	90 K		[24]
	1.169	0 K		[23]
	1.170	0 K		[24]
A	5.2 eV^{2}[b]	300 K	2 – 5 eV e^-	[19]
$\hbar\omega_0$	59 meV (TO), 62 meV (LO)	N/A	DFT[c]	[17]
$\hbar\omega_{pl}$[d]	16.6 ± 0.1 eV	N/A	EELS[e]	[35]

https://arxiv.org/pdf/2004.10709.pdf

Modeling (near threshold) ionization yield

- Often model ionization yield with some integral equation
    - Eg.

$$n(E_r) = 1 + \int_{E=0}^{E_r - E_g} dE\, P(E, E_r) \langle N(E) \rangle$$

- Based on micro physics with assumptions
    - Eg. how does energy split between electrons and holes
- Specify initial and boundary conditions
- Solve integral equation numerically, or via Monte Carlo methods
- Similar approach used to model nuclear recoil ionization yield

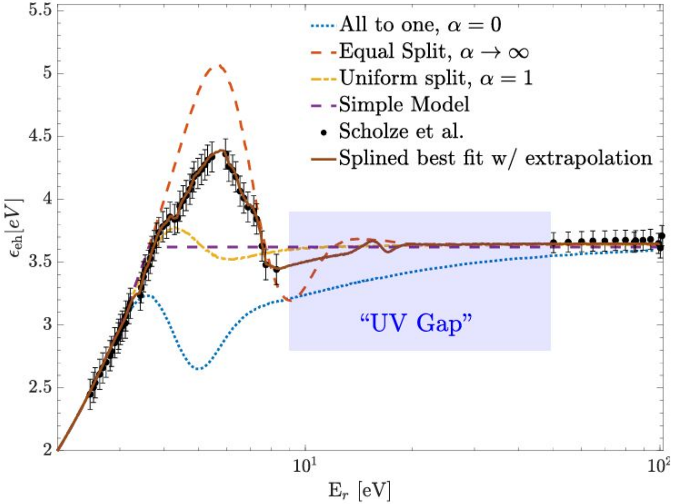
$$\epsilon_{eh}(E_r) = \begin{cases} \infty & E_r < E_g \\ E_r & E_g \le E_r < 2E_g \\ \epsilon_{imp}(E_r) & E_r \ge 2E_g \\ \epsilon_{eh,\infty} & E_r \to \infty \end{cases}$$

$$F(E_r) = \begin{cases} 0 & E_r < 2E_g \\ F_{imp}(E_r) & E_r \ge 2E_g \\ F_\infty & E_r \to \infty \end{cases}$$

https://arxiv.org/pdf/2004.10709.pdf

Near threshold ionization yield

- Result of a recent model



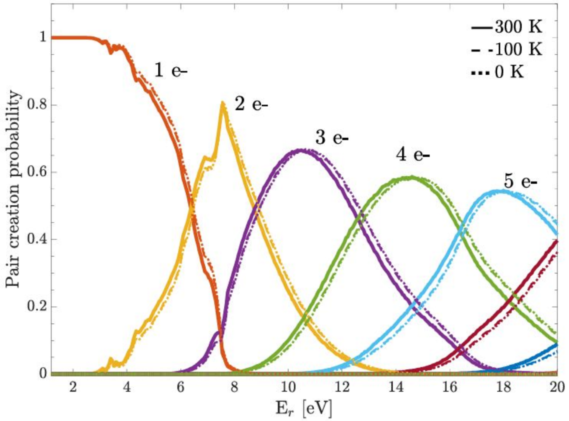
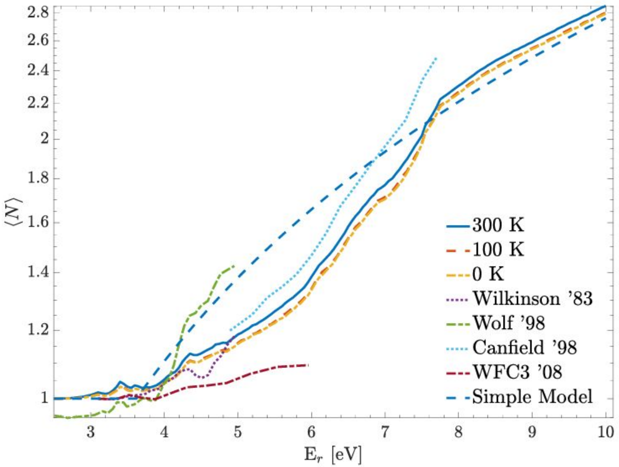
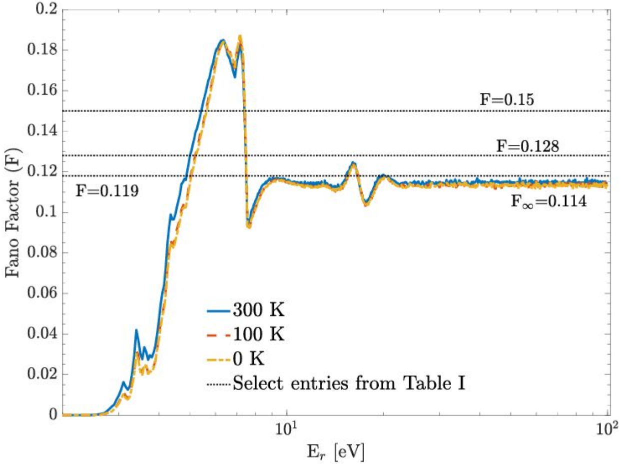


FIG. 6. Pair-creation probability distributions for best-fit model at 0 K, 100 K and 300 K (former curves effectively overlap). These lines are to be interpreted as the probability to ionize the labeled number of charge pairs for a given deposited energy. These are *not* PDFs in that only the sum of curves across a given point in energy is normalized to 1.



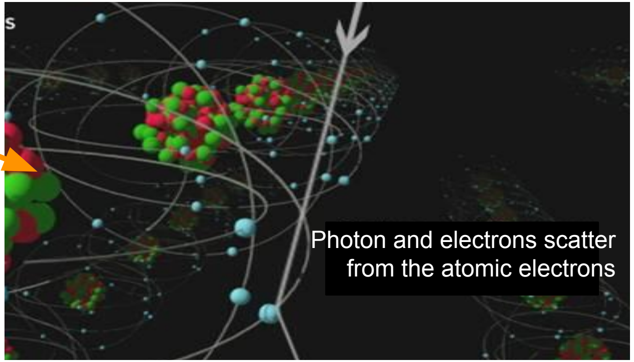


Nuclear Recoils

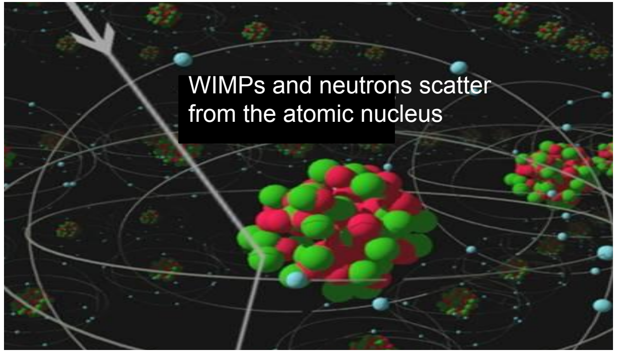
ELECTRON RECOILS (ER)

Gamma: Most prevalent background

Beta: on the surface or in the bulk



Photon and electrons scatter from the atomic electrons



WIMPs and neutrons scatter from the atomic nucleus

NUCLEAR RECOILS (NR)

Neutron: NOT distinguishable from WIMP

Alphas: almost always a surface event

Recoiling parent nucleus: yet another surface event

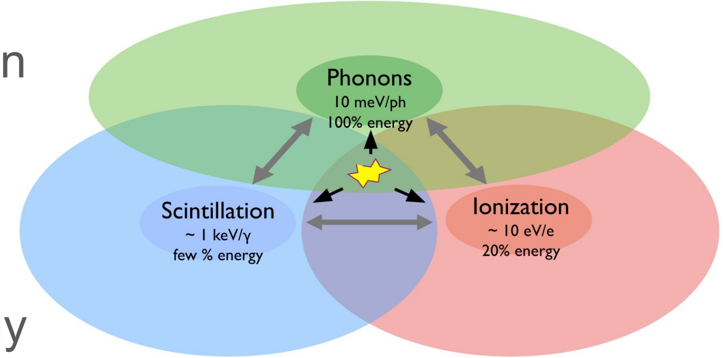
https://arxiv.org/pdf/2311.02257.pdf

Energy loss of a recoiling nucleus in a medium

- Energy transfer from a slow-moving nucleus to electrons is inefficient
  - Due to large mass disparity
- Only a small fraction of energy is channeled into the electron system
  - → Ionization, excitation, scintillation, etc.
- Majority of energy is transferred to atomic motion
  - → Heat
- Ionization/scintillation yield (Quenching factor)

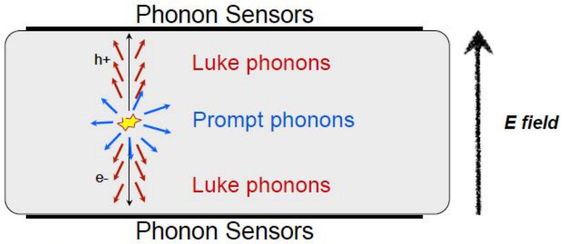
$Q = \frac{E_I}{E_R}$, E_I: ionization energy; E_R: recoil energy

- Can model with software like SRIM
- Or with analytical models
- (Neither is great for near threshold)



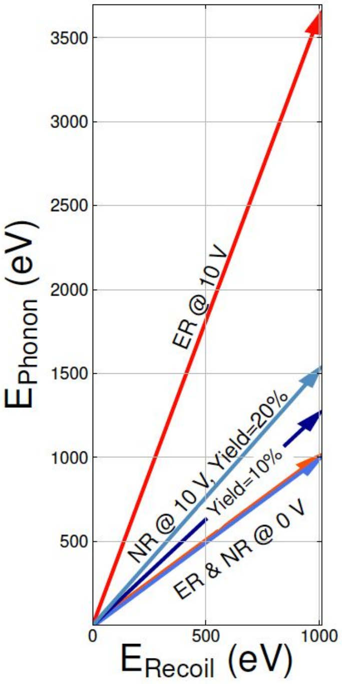
Getting Y wrong →
Misinterpreting WIMP signal!

Why is this important?



$$
\begin{aligned}
E_{\mathrm{Phonon}} &= E_{\mathrm{Recoil}} + E_{\mathrm{NTL}} \\
&= E_{\mathrm{Recoil}} + n_{\mathrm{eh}} e^{-} V \\
&= E_{\mathrm{Recoil}} + E_{\mathrm{Recoil}}/\epsilon \cdot Y(E_{\mathrm{Recoil}}) e^{-} V
\end{aligned}
$$

- ϵ: average e-h creation energy
  - $\sim$4 eV for Si, $\sim$3 eV for Ge
- $Y(E_{\mathrm{Recoil}})$: ionization yield/quenching factor
  - For electron recoil, $Y = 1$
  - For nuclear recoil, it's energy dependent



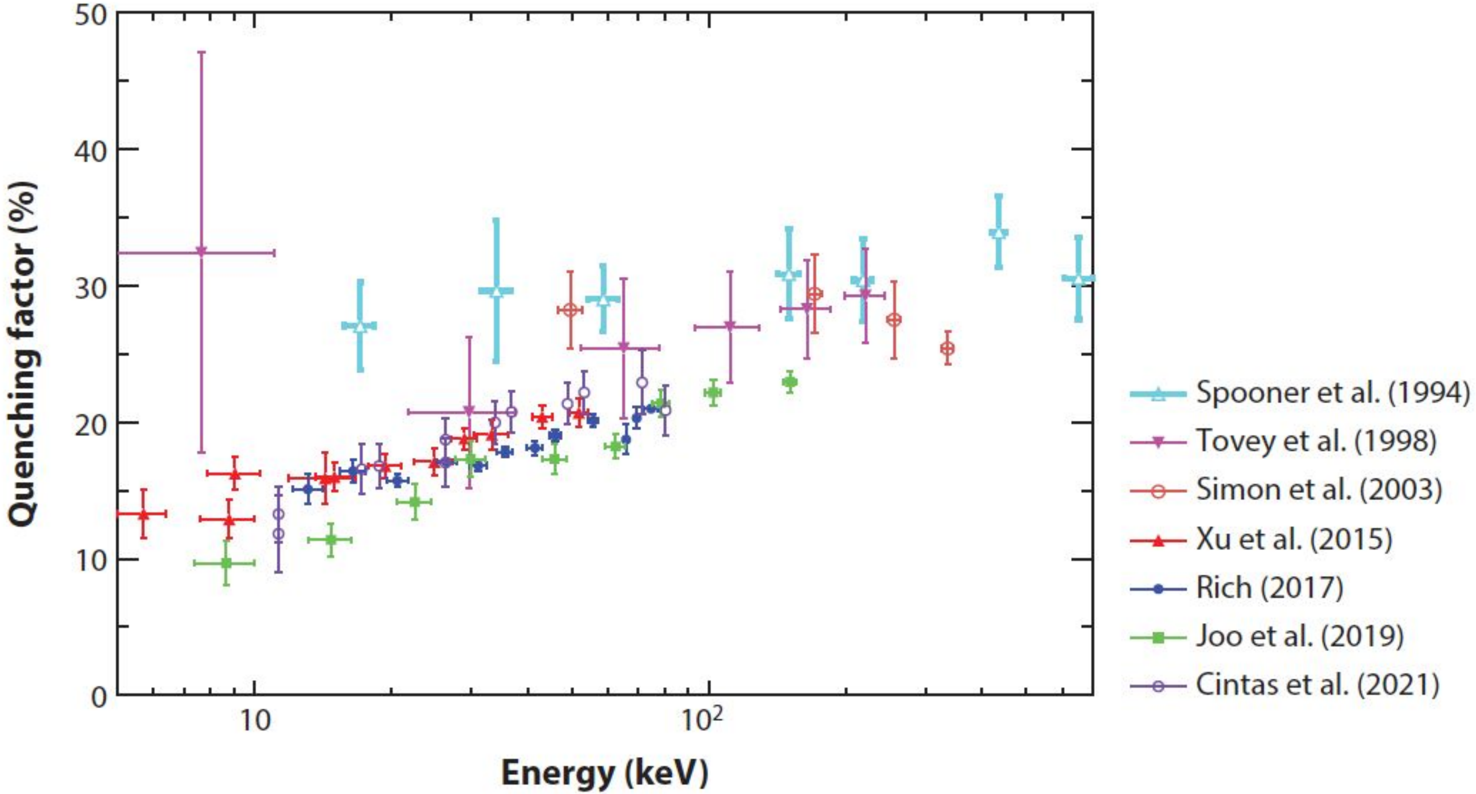
Scintillation yield -- Birks' law

- Scintillation yield per path length

$$\frac{\mathrm{d}L}{\mathrm{d}x} = \frac{S}{1 + kB\frac{\mathrm{d}E}{\mathrm{d}x}} \frac{\mathrm{d}E}{\mathrm{d}x},$$

- S is the scintillation efficiency
  dE/dx is the linear energy transfer by the ionizing particle to the medium
  kB is the Birks quenching coefficient
- Derived for organic scintillators
- Applicable to some inorganic scintillators as well

Near threshold scintillating yield (Quenching factor)



Slide from Y. Sarkis @ Magnificent CEvNS 2020
Phys.Rev.D101,102001(2020)

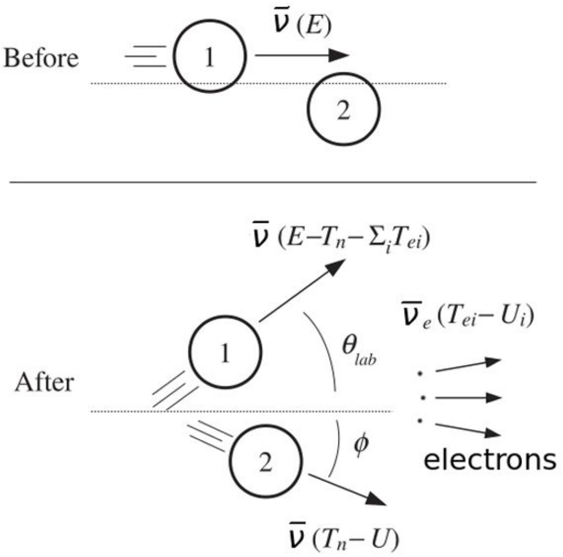
Ionization yield -- Lindhard model

Trying to relax a few of these assumptions

$$\underbrace{\int d\sigma_{n,e}}_{\textit{total cross section}} \left[\underbrace{\bar{\nu}\left(E - T_n - \sum_i T_{ei}\right)}_{\text{A}} + \underbrace{\bar{\nu}\left(T_n - U\right)}_{\text{B}} \overset{-}{\cancel{+}} \underbrace{\bar{\nu}(E)}_{\text{C}} + \underbrace{\sum_i \bar{\nu}_e\left(T_{ei} - U_{ei}\right)}_{\text{D}} \right] = 0 \qquad (2)$$

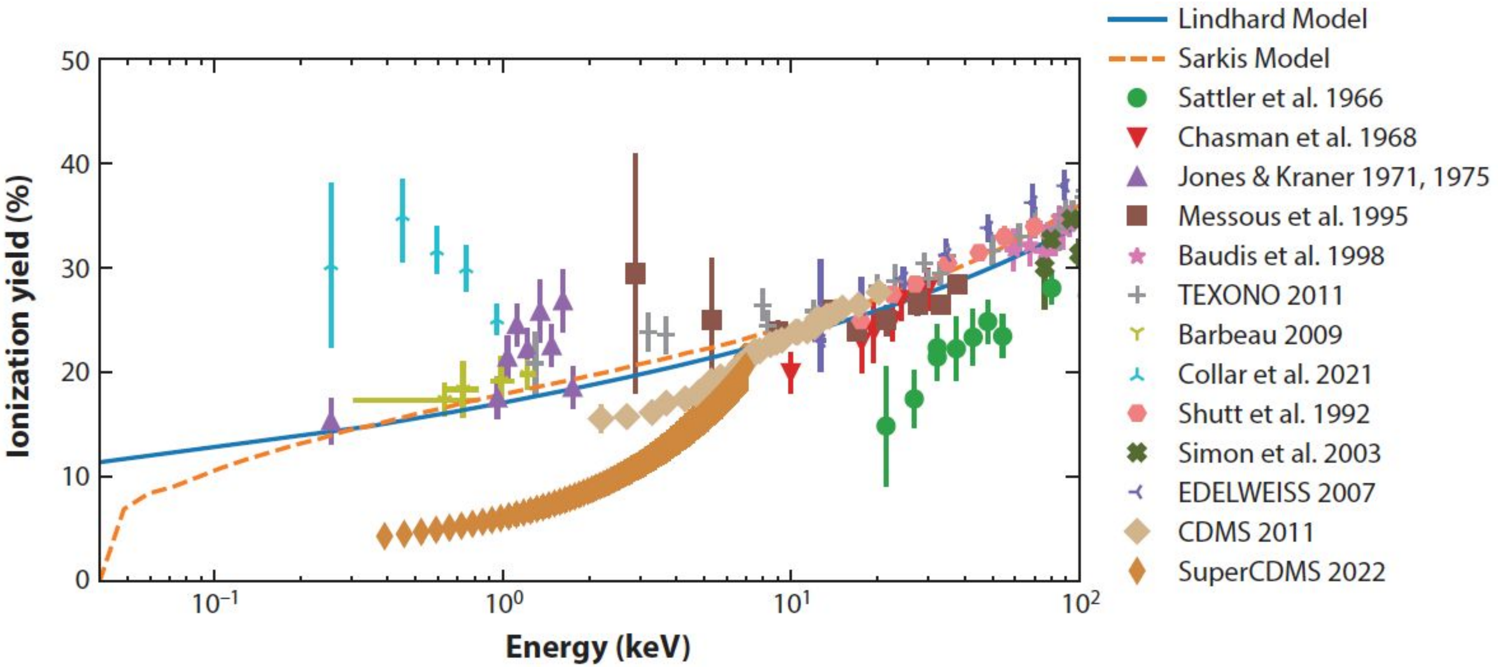
Lindhard's (five) approximations

- I. Neglect contribution to atomic motion coming from electrons.
- II. **Neglect the binding energy, U = 0.**
- III. The energy transferred to ionized electrons is small compared to that transferred to recoiling ions.
- IV. Effects of electronic and atomic collisions can be treated separately.
- V. T_n is also small compared to the energy E.

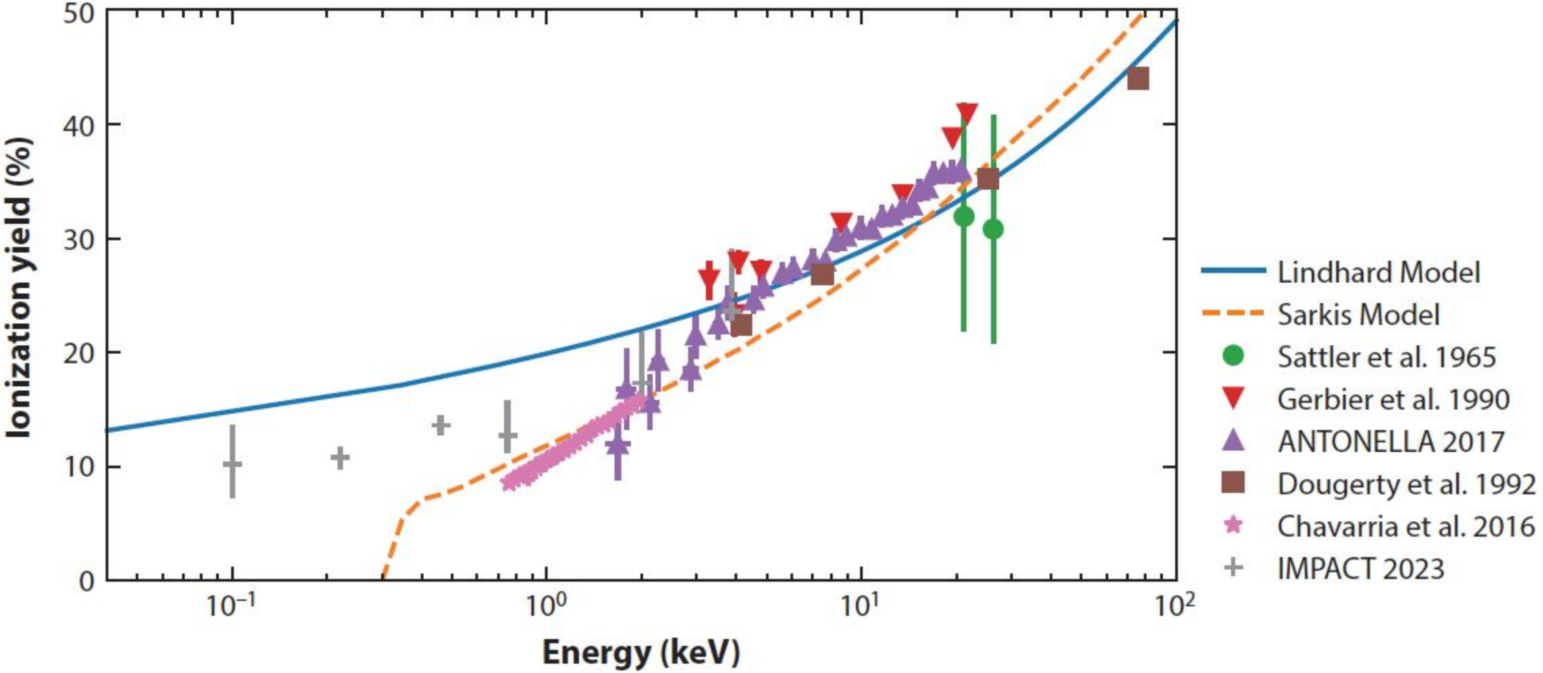


Germanium NR ionization yield

- State of the “art”



Silicon NR ionization yield



Typical measurement scheme

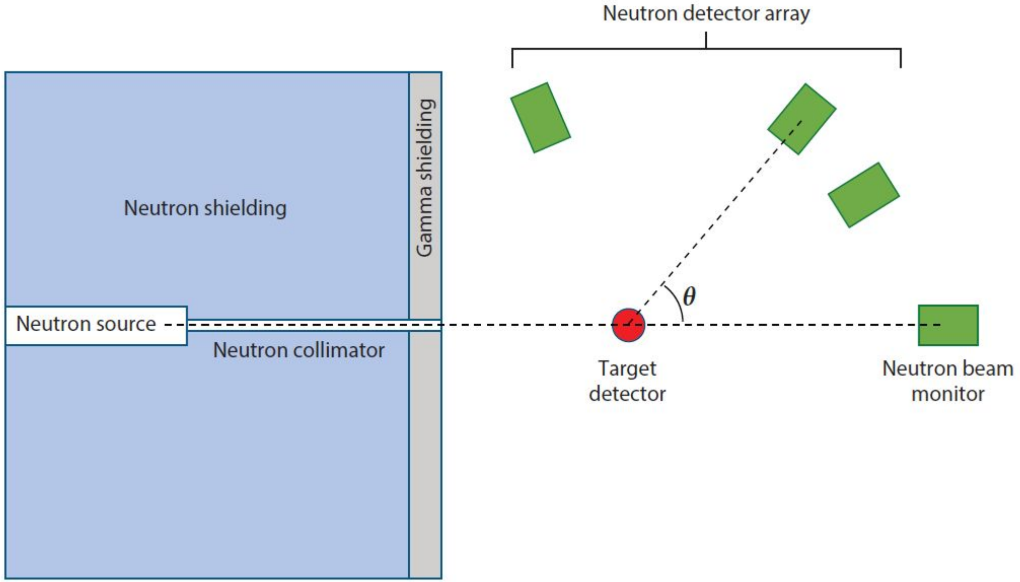
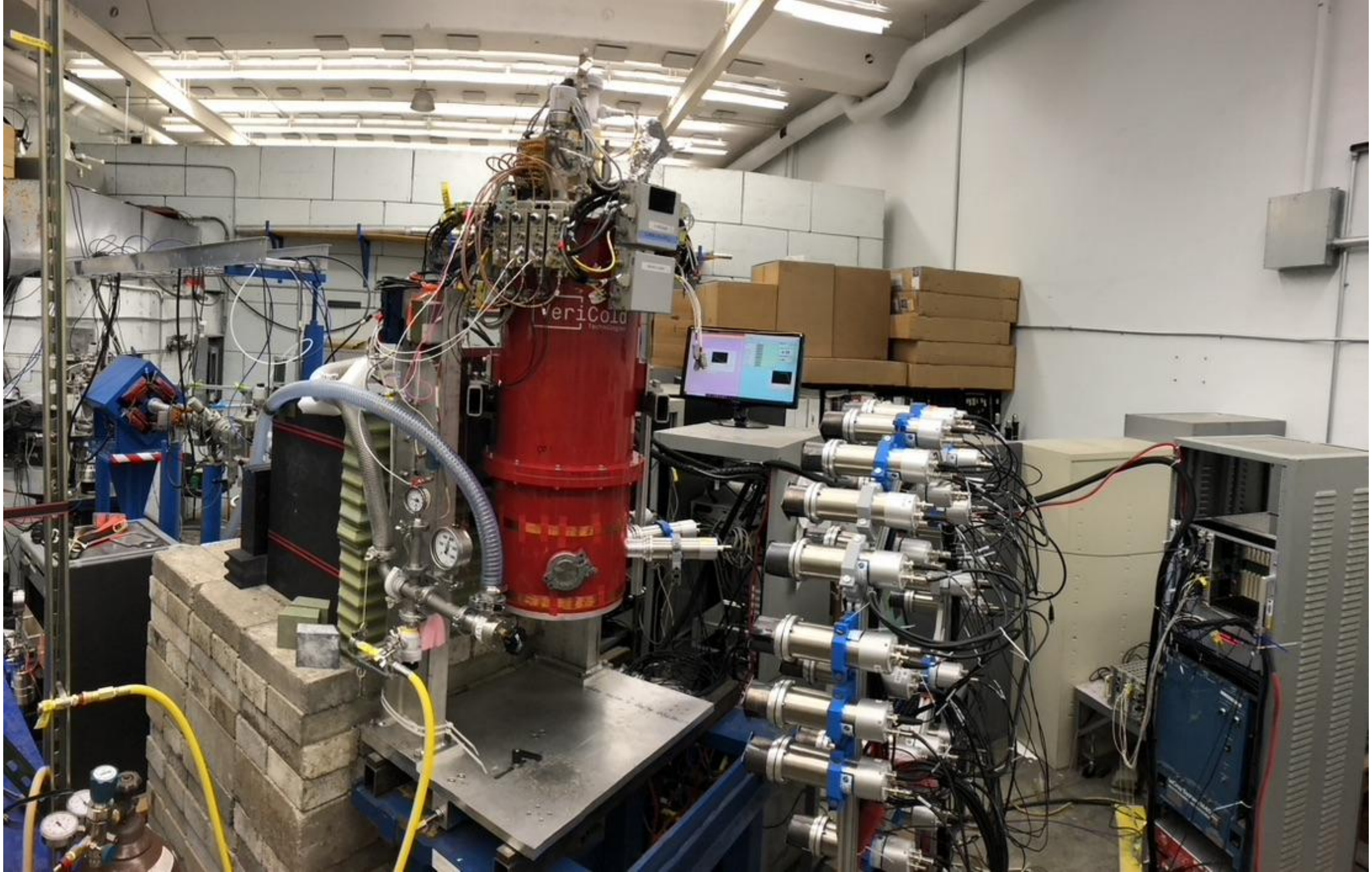


Figure 1

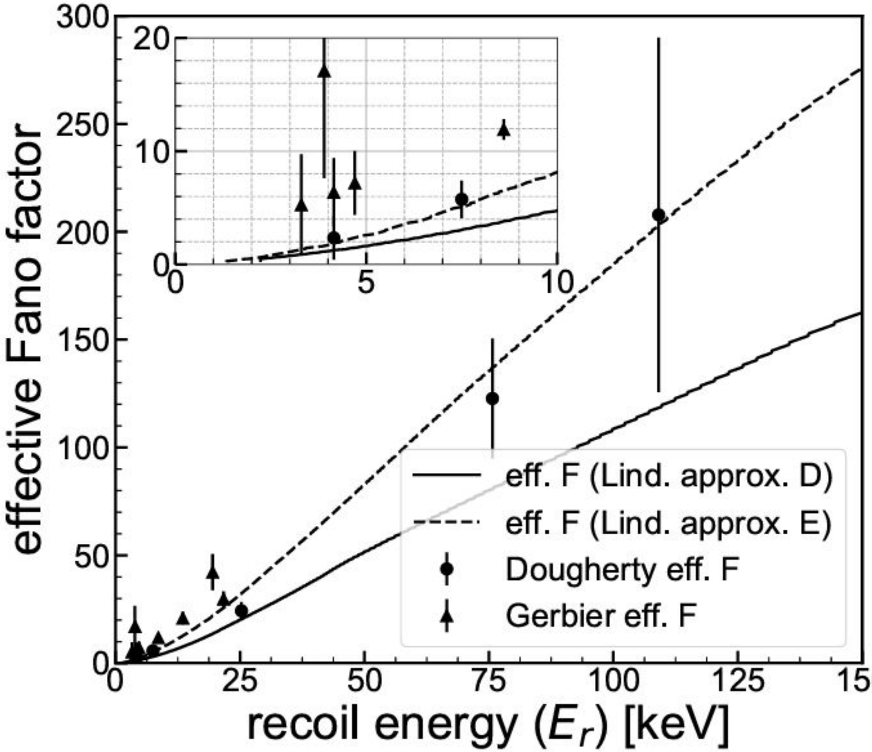
Illustration of a typical experimental setup to measure nearly monoenergetic nuclear recoils using elastic neutron scatters. Neutrons of known energy produce nuclear recoils with a well-defined energy distribution when scattering off a target at a specific angle. The neutron scatter angle is usually informed by placing detectors with gamma-neutron discrimination capabilities at fixed locations, where direct neutrons from the source should be suppressed with shielding. The neutron passage inside the shielding (collimator) may be tapered to improve beam purity, and a neutron detector may be placed right behind the target detector as a beam monitor.


VeriCold
Technologies

https://arxiv.org/pdf/2206.13639.pdf

Nuclear Recoil “Fano Factor”?

- Multiple random processes
  - Nucleus vs electron energy partitioning
  - Electron cascade
- Resulting in an inflated effective Fano Factor
- In principle do not even expect Poisson/Gaussian distributions
- Might need more data to tell…



Migdal effect

- Lower probability, but unquenched ionization
- Lots of experiments use this to lower detection threshold

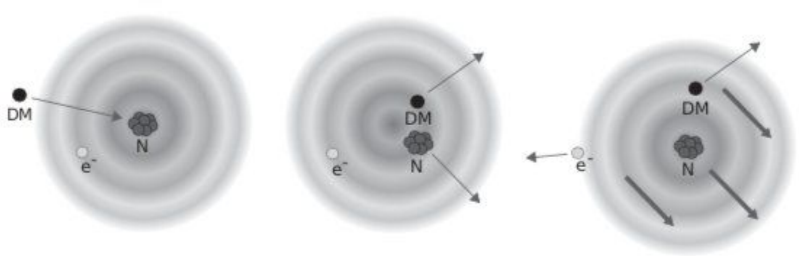
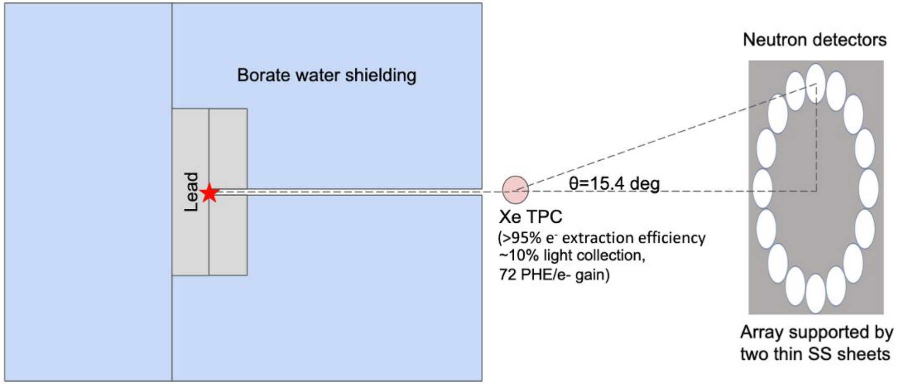


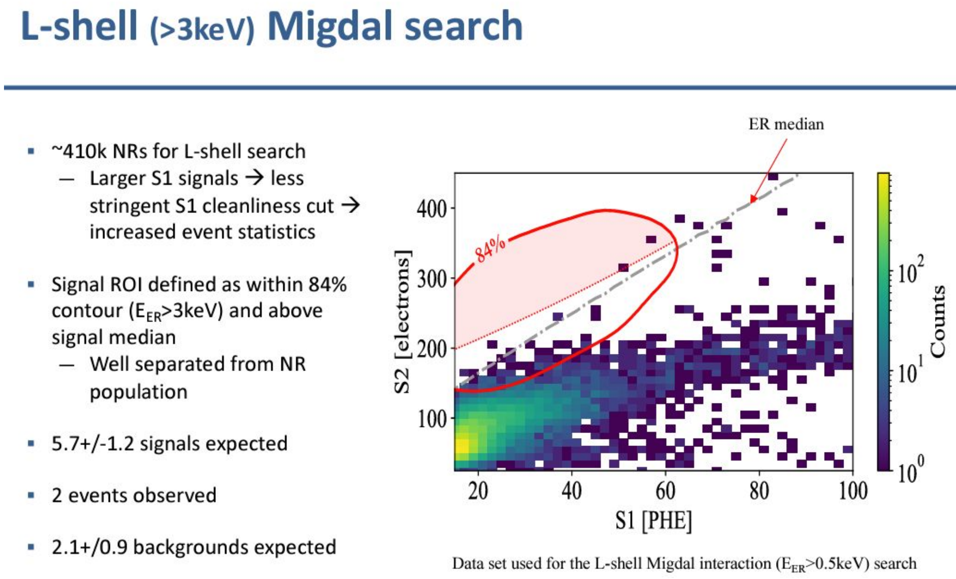
FIG. 1. Illustration of electron emission from nuclear recoils. If a DM particle scatters off a nucleus (left), we can assume that immediately after the collision the nucleus moves relative to the surrounding electron cloud (middle). The electrons eventually catch up with the nucleus, but individual electrons may be left behind and are emitted, leading to ionization of the recoiling atom (right).

Phys. Rev. Lett. **121**, 101801

arXiv:2307.12952

(Maybe?) Migdal non-observation?





L-shell (>3keV) Migdal search

- ~410k NRs for L-shell search
  - Larger S1 signals → less stringent S1 cleanliness cut → increased event statistics
- Signal ROI defined as within 84% contour (E_{ER}>3keV) and above signal median
  - Well separated from NR population
- 5.7+/-1.2 signals expected
- 2 events observed
- 2.1+/0.9 backgrounds expected

Data set used for the L-shell Migdal interaction (E_{ER}>0.5keV) search

Need more data……