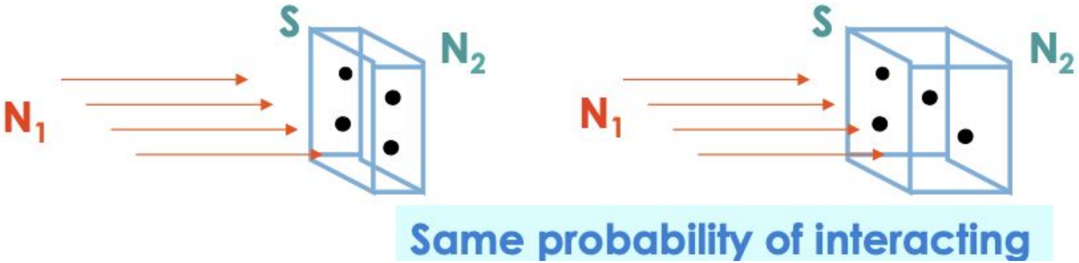
Particle Interactions

Particle Interactions

- Consider five different kinds of interactions
  - **Energy loss of heavy charged particles**
  - Electron and positron energy loss (light particles!)
  - Interaction of Photons
  - ~~Electromagnetic showers~~
  - ~~Strong interactions of hadrons: hadron showers~~

Interaction Probability

- Interaction probability depends on density and thickness of target
  - P proportional to ρ d ($1/cm^2$)
    - ρ is the number density of the target particles
  - Take a target with N_2 particles and surface S: $N_2/S = \rho d$
  - The cross-section (σ) givens the interaction probability for each incident particle
  - Interaction rate given by $R=N_1 \sigma N_2/S=(N_1 \sigma \rho d)$



Mean Free Path

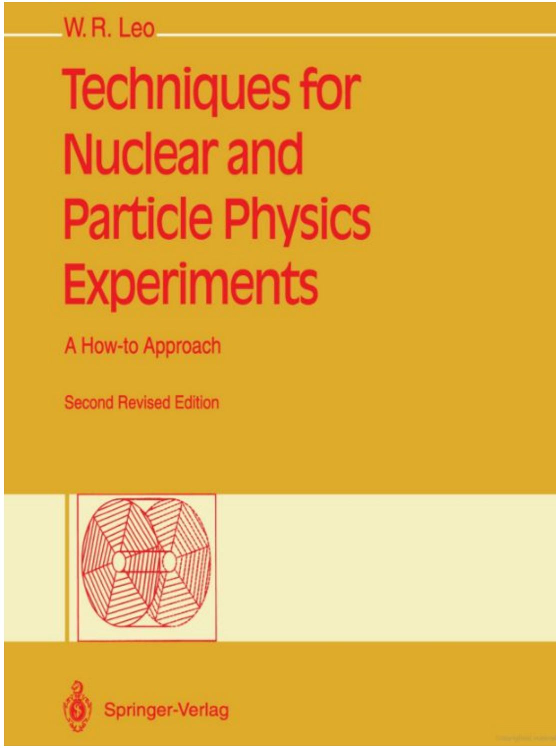
- Mean distance between collisions of particle traversing matter given by:
  - Determined by the interaction rate of particle traveling a distance dx and then undergoing an interaction
    - $R/d = N_1\ \sigma\ \rho$
  - **Probability** of an interaction between x and x+dx
    - $w\,dx = (R/d)\,dx / N_1 = \sigma\ \rho\ dx$
  - Probability of not interacting
    - $P(x+dx) = P(x)\,(1 - w\,dx) \rightarrow d\,P(x) / dx = -wP(x)$
    - $P(x) = e^{-wx}$
- Mean free path $\lambda = 1/w = 1/(\sigma\ \rho)$ and $P(x) = \exp(-x/\lambda)$

Heavy Charged Particles -- Ionisation

- Includes all charged particles besides electrons and positrons
  - At relatively low energies (keV to GeV)
  - Energy loss dominated by electromagnetic interactions with atomic electrons (orbits 10^{-10} m vs. 10^{-14} m for nuclear size)
  - Transfer part of the energy of the particle to an atomic electron
    - Results in ionisation of atom (typical binding energies < keV)
- The cross-section is pretty small (10^{-17} cm^2) but the huge density (6×10^{23} /g) makes this is the most important contribution
  - 10 MeV protons lose all their energy in less than 1 mm of Cu
- Sometimes the electrons liberated in initial collision are able to ionise other atoms
  - Called δ-electrons and leave localised blobs of ionisation

Here we dive into some derivations…

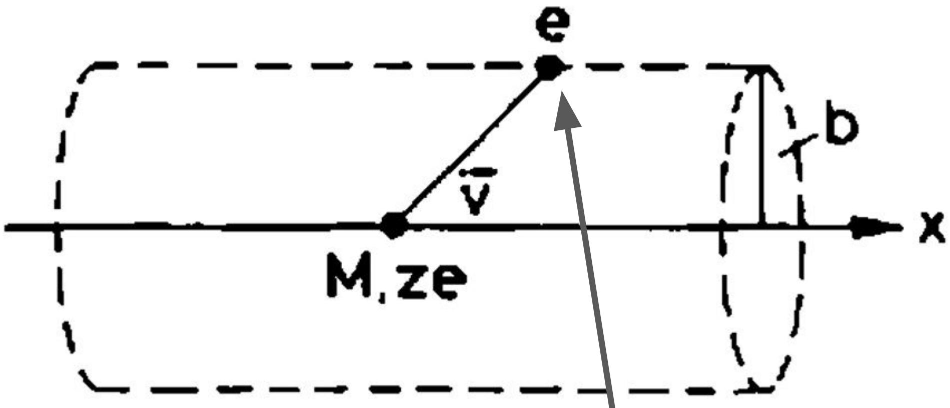
Following Chapter 2 of Leo, then William’s notes



Baseline assumptions:

- Incoming charged particles are heavy
  - Heavier than electrons so inelastic collisions with electrons don’t change its trajectory much
- Not too heavy to be comparable to nuclei
  - Otherwise effects of nuclei collisions need to be considered
- Energy transferred in each collision is small
- Number of collisions per path length is huge
- Averaged energy loss per unit length: *stopping power* or simply dE/dx,

Classical derivations



Assume interaction happens fast and the electron doesn't move

- Calculate momentum impulse on an electron

$$I = \int F\,dt = e\int E_\perp\,dt = e\int E_\perp \frac{dt}{dx}\,dx = e\int E_\perp \frac{dx}{v}$$

- Gauss's Law

$$\int E_\perp 2\pi b\,dx = 4\pi z e\,, \qquad \int E_\perp\,dx = \frac{2ze}{b}\,,$$

- So

$$I = \frac{2ze^2}{bv} \qquad \Delta E(b) = \frac{I^2}{2m_e} = \frac{2z^2e^4}{m_e v^2 b^2}$$

Classical derivations (II)

- Electron density N_e, look at a shell (b → b+db, dx), energy lost to all electrons in this shell is

$$-dE(b) = \Delta E(b)\, N_e\, dV = \frac{4\pi z^2 e^4}{m_e v^2} N_e \frac{db}{b} dx$$

- Can't integrate from b=0 to b→∞: invalid for large and small b
- Need to estimate b_{max} and b_{min} that are reasonable.

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{b_{max}}{b_{min}}$$

Classical derivations (III)

- For b_{min}, recall max energy gained in a collision is $2\gamma^2 m_e v^2$
  - Thus
$$b_{min} = \frac{ze^2}{\gamma m_e v^2}$$
- For b_{max}, energy transfer needs to be fast. If the electron bound state frequency is ν, the interaction time $t/\gamma = b/(\gamma v)$ needs to be smaller than 1/ (averaged ν)
$$b_{max} = \frac{\gamma v}{\bar{\nu}}$$
- $$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{\gamma^2 m v^3}{z e^2 \bar{\nu}}.$$ Fine for α, not good for p

Now define: $\phi(E,E')\,dE'dx$ to be the probability that a particle loses energy between $E' \to E'+dE$ in path length dx

$$\Rightarrow \phi(E,E')\,dE'dx = -n_0(2\pi b)\,db\,dx$$

↳ # electrons / unit volume

$$= n_0 \pi b^2\,dx\left(\frac{-2db}{b}\right)$$

$$= n_0 \pi \frac{z^2 r_e^2 (mc^2)}{\beta^2} \frac{dE'}{E'^2}\,dx$$

$$\equiv \frac{\breve{A}\,(\rho dx)}{\beta^2} \frac{dE'}{E'^2}$$

3)

$$\breve{A} = 0.154 \frac{z^2 Z}{A} \frac{\text{MeV cm}^2}{\text{gm}}$$

Z = Charge of Media

A = Atomic # of Media

PRIMARY IONIZATION

- MEAN NUMBER OF "ENCOUNTERS" BETWEEN E'_{min} and E'_{max} (CAUSE "SINGLE IONIZATION") PER UNIT LENGTH

$$\frac{n_{prim}}{\Delta x} = \left\langle \frac{dN}{dx} \right\rangle = \int_{E'_{min}}^{E'_{max}} \phi(E', E)\, dE'$$

$$= \frac{\breve{A}\rho}{\beta^2} \int_{E_{min}}^{E_{max}} \frac{dE'}{E'^2}$$

$$= \frac{\breve{A}\rho}{\beta^2} \left(\frac{1}{E_{min}} - \frac{1}{E_{max}} \right)$$

FOR MOST GASES $\frac{\breve{A}\rho}{E_{max}} \ll 1$ SO AS $\beta \to 1$ THIS CAN BE IGNORED

$$\Rightarrow \frac{n_{prim}}{\Delta x} \simeq \frac{\breve{A}\rho}{\beta^2} \frac{1}{E_{min}}$$

$E_{min} \geq I_0$ (THE THRESHOLD TO IONISE THE GAS)

FOR MOST GASES QUANTUM EFFECTS ARE IMPORTANT (SCREENING ETC.). FOR HYDROGEN BETHE PEFORMED A "FULL CALCULATION". CONCLUDED:

$$E_{min} = \frac{I_0}{r\left[\ln \frac{2mc^2}{I_0}\frac{\beta^2}{1-\beta^2} + S - \beta^2\right]}$$

$I_0 = 15.4\,eV \qquad r = 0.29 \qquad S = 3.04$

$\Rightarrow E_{min} = 3.5\,eV \qquad n_{prim} = 4.6/cm.$

FOR OTHER GASES

$$E_{min} = \frac{\tilde{A}\rho}{A_1\left[A_2 + \ln\left(\frac{\beta^2}{1-\beta^2}\right) - \beta^2\right]}$$

See table for A_1 A_2 and plots of experimental determinations of n_{prim}.

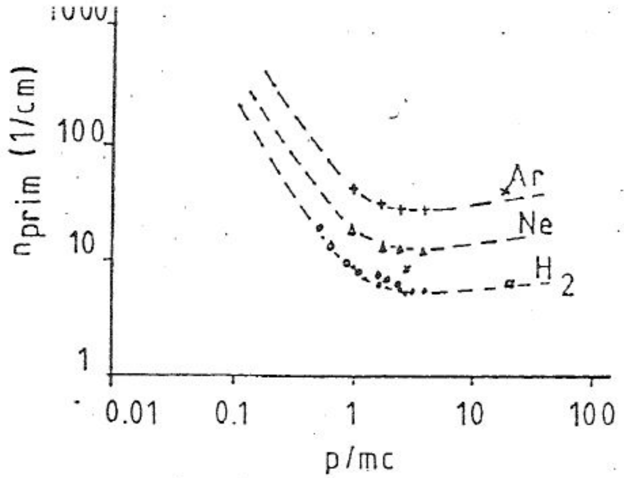


Fig. 1 Primary ionisation. Points: measured, --- Bethe's formula adjusted

Table I

gas	density (mg/cm^3) (0^oC, 1atm)	I_o (eV)	A_1	A_2	n_{prim} (cm^{-1})
H_2	0.0899	15.4	Bethe's	formula	4.6
H_e	0.1785	24.6	0.244	11.64	3.5
N_e	0.9004	21.7	0.844	10.89	11.4
A_r	1.7837	15.8	1.828	11.45	25.8
X_e	5.8510	12.1	3.554	11.31	49.6
N_2	1.2506	15.5	1.941	11.43	27.1
O_2	1.4290	12.2	2.079	11.28	28.9

Bethe-Bloch formula

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2 m_e \gamma^2 v^2 W_{max}}{I^2} \right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right], \quad (2.27)$$

with

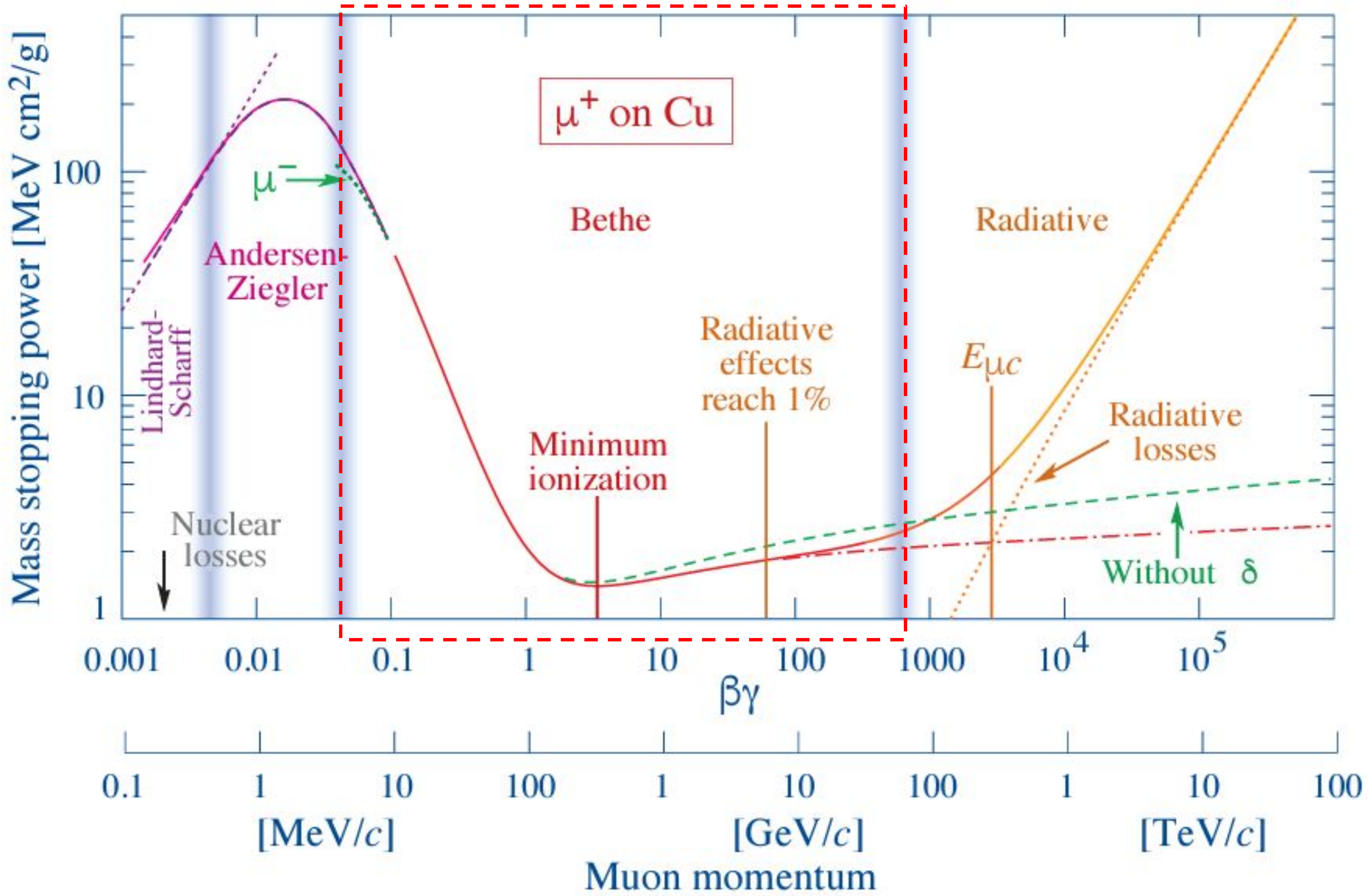
$$2\pi N_a r_e^2 m_e c^2 = 0.1535 \text{ MeVcm}^2/\text{g}$$

r_e: classical electron radius $= 2.817 \times 10^{-13}$ cm
m_e: electron mass
N_a: Avogadro's number $= 6.022 \times 10^{23}$ mol^{-1}
I: mean excitation potential
Z: atomic number of absorbing material
A: atomic weight of absorbing material

ρ: density of absorbing material
z: charge of incident particle in units of e
$\beta =$ v/c of the incident particle
$\gamma =$ $1/\sqrt{1-\beta^2}$
δ: density correction
C: shell correction
W_{max}: maximum energy transfer in a single collision.

$$W_{max} = \frac{2 m_e c^2 \eta^2}{1 + 2s\sqrt{1+\eta^2} + s^2},$$

where $s = m_e/M$ and $\eta = \beta\gamma$.



μ+ on Cu
Mass stopping power [MeV cm²/g]
100
10
1
μ−
Bethe
Radiative
Andersen-Ziegler
Lindhard-Scharff
Radiative effects reach 1%
Eμc
Radiative losses
Minimum ionization
Nuclear losses
Without δ
0.001
0.01
0.1
1
10
100
1000
10⁴
10⁵
βγ
0.1
1
10
100
1
10
100
1
10
100
[MeV/c]
[GeV/c]
[TeV/c]
Muon momentum

Interpretation

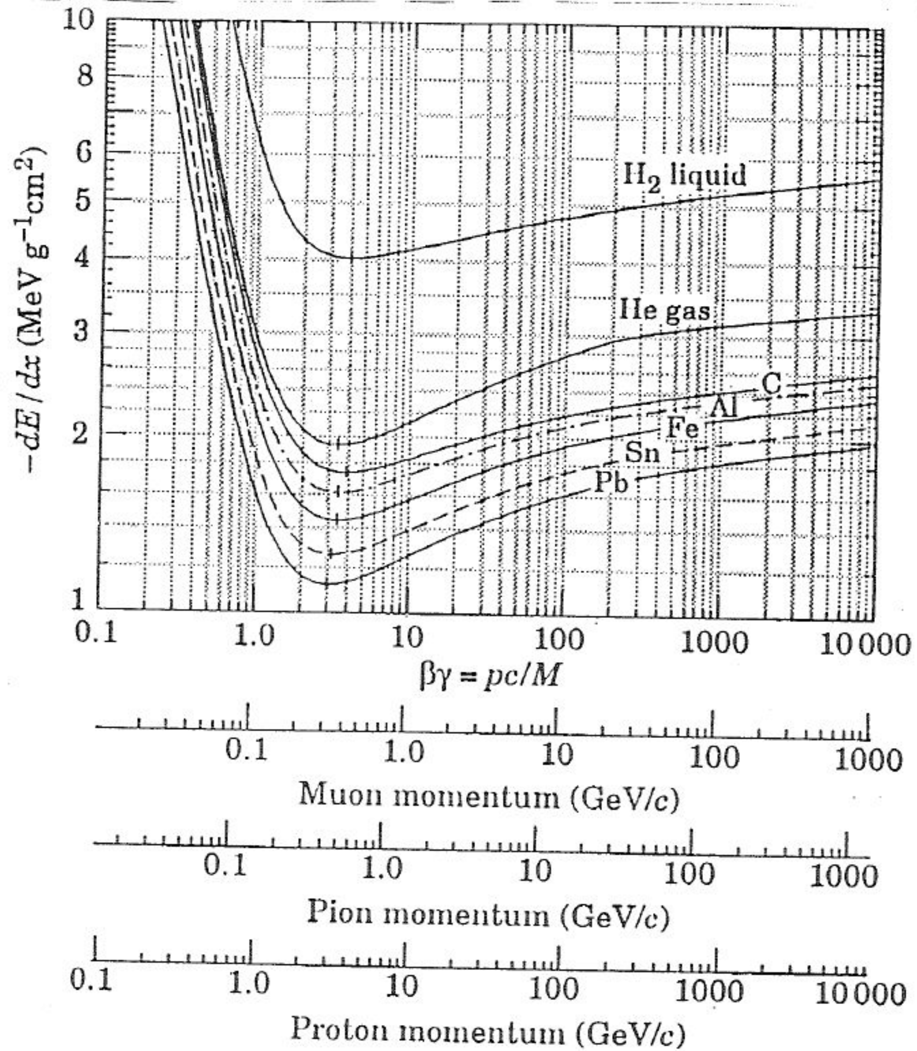
$$\frac{dE}{dx} \simeq 0.31 \frac{Z}{A} \left\{ \frac{\text{MeV cm}^2}{g} \right\} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2mc^2\gamma^2\beta^2}{I} \right) - \beta^2 - \frac{d}{2} \right] \quad (I)$$

- At low β, $dE/dx \sim 1/\beta^2$
  - Collision time ~ 1/β, momentum transfer ~ 1/β, energy ~ $1/\beta^2$
- As $\beta \rightarrow 1$, $ln(\beta^2\gamma^2) \sim ln(p)$ dominates
  - "Relativistic rise"
  - Starts at βɣ~3.5-4
- Material begins to polarise as ɣ goes even higher
  - Density correction
    ω_p is the plasma frequency

$$\frac{d}{2} = \ln \beta^2\gamma^2 - \ln \frac{I^2}{\hbar^2\omega_p^2} - 1$$

$$\omega_p = \left(\frac{n_0 e^2}{\epsilon_0 m} \right)^{1/2}$$

- The density effect cancels the relativistic rise around $\gamma_{plateau} \simeq \frac{I}{\hbar\omega_p}$, ~1000 in gases, ~10 in silicon

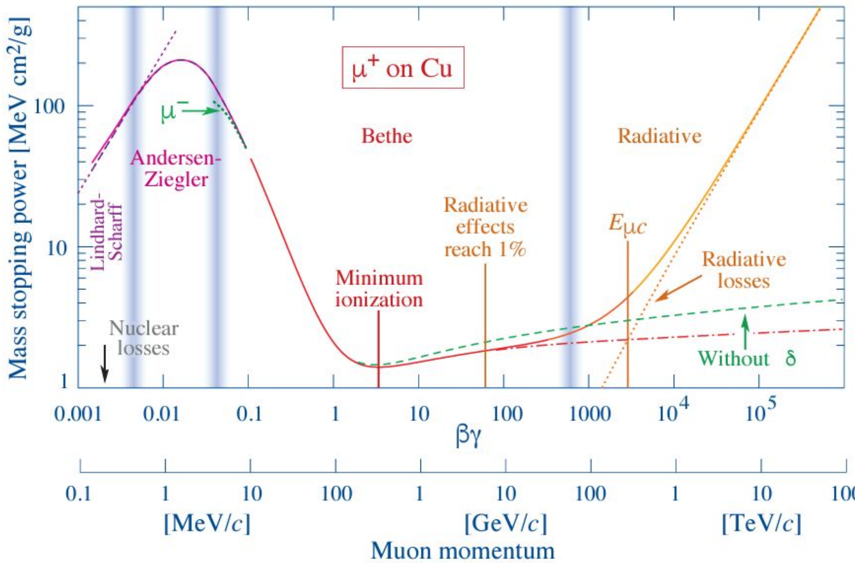


$-dE/dx$ (MeV g^{-1}cm^2)
H_2 liquid
He gas
C
Al
Fe
Sn
Pb
0.1
1.0
10
100
1000
10 000
$\beta\gamma = pc/M$
Muon momentum (GeV/c)
Pion momentum (GeV/c)
Proton momentum (GeV/c)

Minimum Ionizing Particles (MIPs)

- Most relativistic particles (e.g., cosmic-ray muons) have mean energy loss rates close to the minimum

$$-\frac{dE}{dx}\Big|_{min} \sim 2\mathrm{MeV\ cm^2/g}$$



IF YOU HAVE A MIXTURE OF MATERIALS (e.g. A SAMPLING CALORIMETER - SEE LATER) THEN YOU CAN SUM THE ENERGY LOSS FROM THE DIFFERENT COMPONENTS :

$$\frac{dE}{dx} = \sum_j w_j \left.\frac{dE}{dx}\right|_j$$

WHERE ω_j IS THE FRACTION (BY MASS) OF THE j^{th} COMPONENT

$\left.\frac{dE}{dx}\right|_j$ IS (I) FOR j^{th} COMPONENT.

FLUCTUATIONS ABOUT MOST PROBABLE ENERGY LOSS (LANDAU DISTRIBUTION)

FOR VERY SMALL IMPACT PARAMETER COLLISIONS THERE IS A (LOW PROBABILITY) CHANCE OF PRODUCING A VERY HIGH ENERGY SECONDARY ELECTRON ~ δ-RAY.

WHILE IT HAS HIGH-ENERGY COMPARED TO "NORMAL" IONISATION IT HAS LOW ENERGY COMPARED TO RELATIVISTIC PARTICLE IN THE FIRST PLACE.

$\Rightarrow$ δ ELECTRON IS VERY HEAVILY IONISING $\left(\frac{1}{\beta^2}\right)$

LANDAU WORKED OUT FIRST APPROXIMATION TO THIS DESCRIBE SUCH A DISTRIBUTION:

$$\phi(\lambda) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\lambda + e^{-\lambda})}$$

$$\lambda = (E - E_{mp})/\xi$$

NORMALISED DEVIATION FROM MOST PROBABLE ENERGY LOSS.

$$\xi = \frac{1}{2}(4\pi r_e^2)\, mc^2 (z^2 Z)\left(\frac{N_A \rho}{A}\right)\frac{\Delta X}{\beta^2}$$

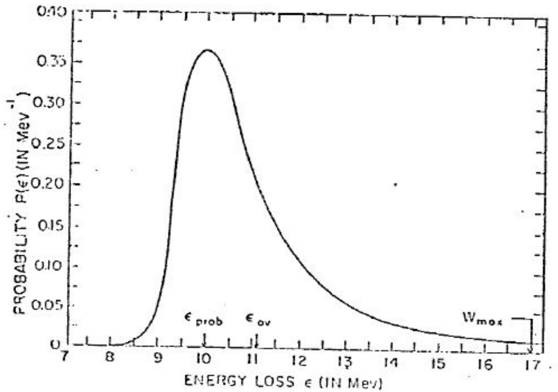


FIG. 3. The Landau distribution of energy losses ϵ for 3-Bev protons traversing a thickness 6.97 gm/cm² of Be, for which ϵ_{prob} = 10 Mev, ϵ_{Av} = 11.10 Mev, and W_{max} = 17 Mev.

Note: Most probable energy isn't averaged energy

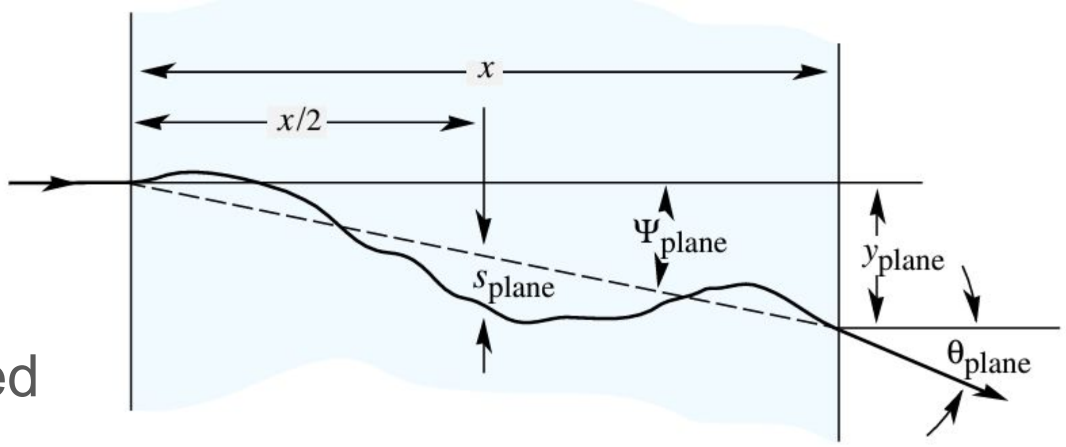
DELTA ELECTRONS HAVE AT LEAST TWO IMPLICATIONS FOR PARTICLE DETECTOR PERFORMANCE.

1) SOME TRACKERS USE dE/dx FOR PARTICLE ID (SEE LATER). LANDAU TAIL CAN MASK 20-40% EFFECTS OF DIFFERING PARTICLE MASSES IF NOT PROPERLY ACCOUNTED FOR.

2) δ-RAYS CAN BE "ORTHOGONAL" TO ORIGINAL PARTICLE TRAJECTORY. IN VERY PRECISE TRACKERS (SILICON) THIS CAN DETERIORATE POSITION RESOLUTION.

FAR FROM AN EXACT SCIENCE. BEWARE OF LIMITATIONS OF PARAMETRISATION (II).

Multiple Scattering

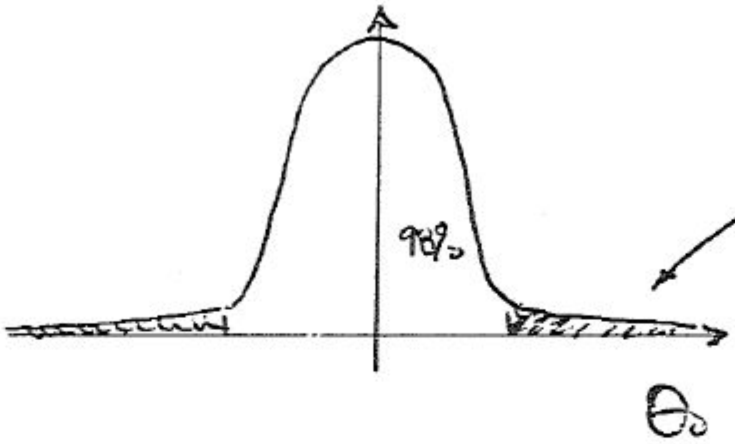


- As particles traverse a medium they are deflected by many small angles
  - Collisions mainly with nuclei in the material
  - Columb scattering
- For ~98% of cases can approximate by Gaussian with

$$\theta_0 = \theta^{\text{rms}}_{\text{plane}} = \frac{1}{\sqrt{2}}\theta^{\text{rms}}_{\text{space}} = \frac{13.6\ \text{MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}}\left[1 + 0.038 \ln(\frac{x\, z^2}{X_0 \beta^2})\right]$$

- X_0 is the radiation length of the medium

NB: DISTRIBUTION OF Θ_0 LOOKS LIKE:



LONG TAILS DUE TO HARD SCATTERS

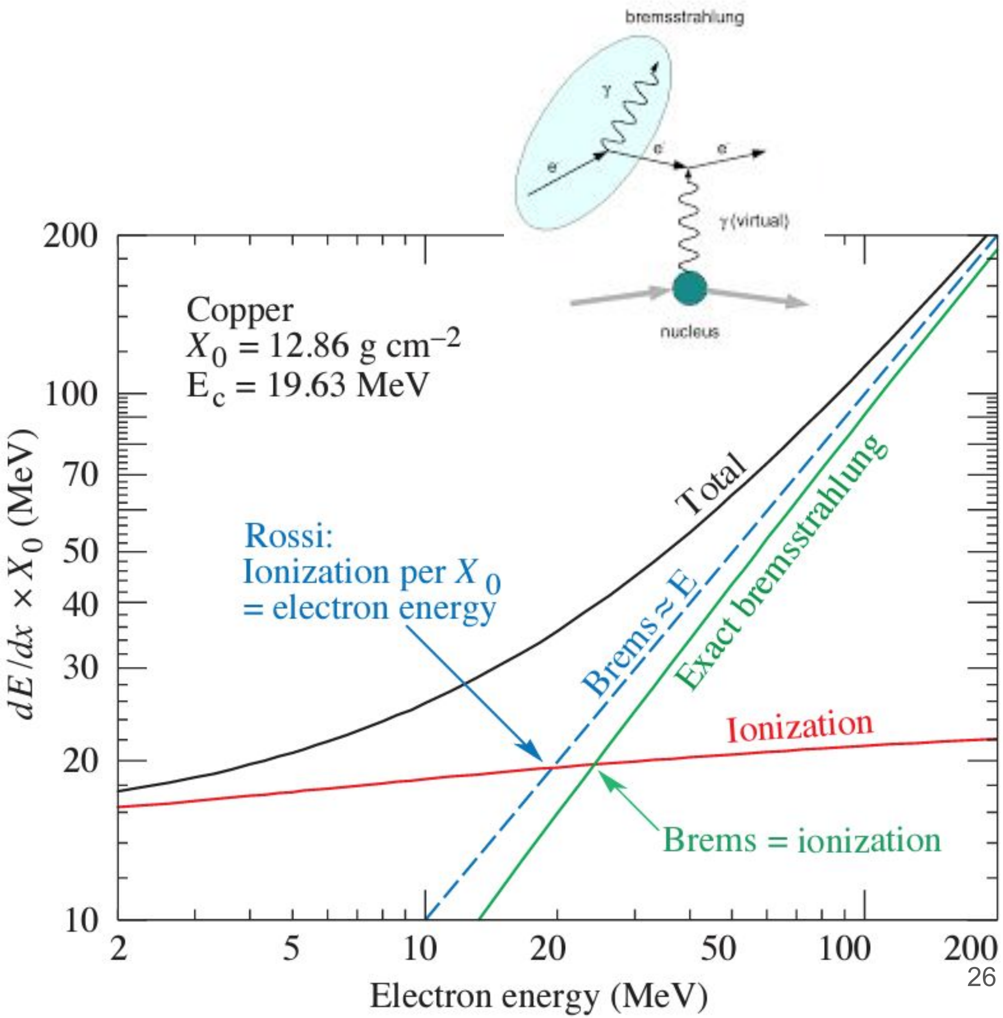
cf LYNCH, DAHL NIM B58 (1991)

$$\theta_0 = \theta^{\text{rms}}_{\text{plane}} = \frac{1}{\sqrt{2}}\theta^{\text{rms}}_{\text{space}} \quad = \frac{13.6\ \text{MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}}\left[1 + 0.038 \ln\left(\frac{x z^2}{X_0 \beta^2}\right)\right]$$

- For z=1, β→1, above formula varies from measurements by less than 10%
- For 1E-3 < x/x_0 < 1E2, often enough: $\theta_0 \sim \frac{13.6}{\beta c p}\sqrt{\frac{x}{x_0}}$
- In practical applications (particle traversing more than 1 material), it is better to compute combined x & x_0 and apply above formula once.
  - Scattering angles are not independent → Cannot just sum in quadraqure

Bremsstrahlung

- For electrons, Brem dominates in high energy, ionisation in low energy
- Where the two breaks even is called **“critical energy”**



Radiation Length

- Define as the thickness of material in units of $(g/cm^2)^{-1}$
  - Over which a **high energy electron** loses all but e^{-1} of its energy

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A} \left\{ Z^2 \left[L_{\rm rad} - f(Z) \right] + Z\, L'_{\rm rad} \right\}$$

$$f(Z) = a^2 \left[(1+a^2)^{-1} + 0.20206 - 0.0369\, a^2 + 0.0083\, a^4 - 0.002\, a^6 \right] \qquad a = \alpha Z$$

Table 34.2: Tsai's $L_{\rm rad}$ and $L'_{\rm rad}$, for use in calculating the radiation length in an element using Eq. (34.25).

Element	Z	$L_{\rm rad}$	$L'_{\rm rad}$
H	1	5.31	6.144
He	2	4.79	5.621
Li	3	4.74	5.805
Be	4	4.71	5.924
Others	> 4	$\ln(184.15\, Z^{-1/3})$	$\ln(1194\, Z^{-2/3})$

Photon interactions

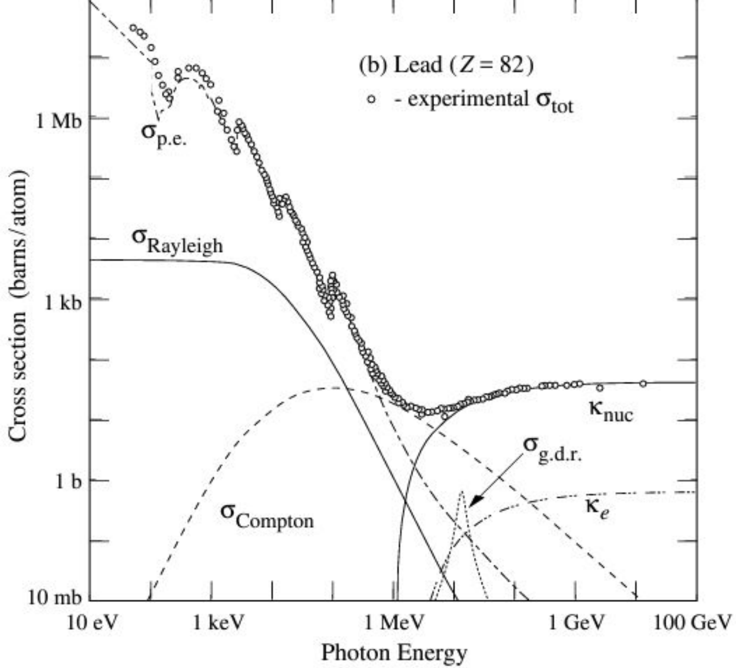


Figure 34.15: Photon total cross sections as a function of energy in carbon and lead, showing the contributions of different processes [50]:

$\sigma_{\text{p.e.}}$ = Atomic photoelectric effect (electron ejection, photon absorption)

σ_{Rayleigh} = Rayleigh (coherent) scattering–atom neither ionized nor excited

σ_{Compton} = Incoherent scattering (Compton scattering off an electron)

κ_{nuc} = Pair production, nuclear field

κ_e = Pair production, electron field

$\sigma_{\text{g.d.r.}}$ = Photonuclear interactions, most notably the Giant Dipole Resonance [51]. In these interactions, the target nucleus is usually broken up.

Original figures through the courtesy of John H. Hubbell (NIST).