Transition Edge Sensor Modeling

Following "Transition-Edge Sensors, K.D. Irwin and G. C. Hilton"

Balance the heat

- Sensor heat capacity C
- P is heat flowing into the sensor
 - For now assume as a constant
- Energy deposition E_{γ} at t_{γ}

$$C\frac{dT(t)}{dt} = P - P_{\text{link}}(T(t), T_{\text{b}}) + E_{\gamma}\delta(t - t_{\gamma})$$

$$P_{\rm link}(T(t), T_{\rm b}) = G(T(t) - T_{\rm b})$$

$$T(t) = \frac{E_{\gamma}}{C} e^{-t/\tau_o} + \left(\frac{P}{G} + T_{\rm b}\right) \qquad \tau_o \equiv C/G$$





$$C\frac{dT(t)}{dt} = P - P_{\text{link}}(T(t), T_{\text{b}}) + E_{\gamma}\delta(t - t_{\gamma})$$
$$P_{\text{link}}(T(t), T_{\text{b}}) = K(T(t)^{n} - T_{\text{b}}^{n})$$



Electro-thermal Feedback (ETF)

 $V_{\rm bias}^2$ P =₽^{V_{bia}} $\geq R(T,I)$ G(T) T_{b} sistance [mOhm] T dR0.102 0.104 0.096 0.098 0.100 Temperature [K]

$$C\frac{dT(t)}{dt} = \frac{V^2}{R(T)} - K(T(t)^n - T_b^n) + E_\gamma \delta(t - t_\gamma)$$

In quiescence:

$$P = \frac{V^2}{R} = K(T^n - T_b^n)$$

- Note, in quiescence T~T_c. Thus P is roughly constant
- Taylor expand with ΔT , note R₀, T₀ are the quiescent values $R = R_0 + \alpha \frac{R_0}{T_0} \Delta T$

$$Crac{d\Delta T}{dt}=rac{V^2}{R_0}-K(T_0^n-T_b^n)-rac{V^2}{R_0^2}rac{dR}{dT}\Delta T-nKT_0^{n-1}\Delta T+E_\gamma\delta(t-t_\gamma)$$

• Define
$$G \equiv \frac{dP}{dT} = nKT^{n-1}$$

$$\Delta \dot{T}(t) = -\left(rac{lpha P}{TC} + rac{G}{C}
ight)\Delta T + rac{E_{\gamma}}{C}\delta(t-t_{\gamma})$$

4

Electro-thermal Feedback (ETF)



$$\Delta \dot{T}(t) = -\left(\frac{\alpha P}{TC} + \frac{G}{C}\right)\Delta T + \frac{E_{\gamma}}{C}\delta(t - t_{\gamma})$$

- Solution is a simple exponential, with
 - $\tau_{\rm eff} = \frac{\tau_0}{1 + \frac{\alpha P_0}{T_0 G}} = \frac{\tau_0}{1 + \frac{\alpha}{n} (1 \left(\frac{T_b}{T_0}\right)^n)}$ Define $\tau_{\rm eff} = \frac{\tau_0}{1 + \frac{\alpha}{n} (1 \left(\frac{T_b}{T_0}\right)^n)}$
 - Define $\phi = 1 \left(rac{T_b}{T_0}
 ight)^n$ $au_{ ext{eff}} = rac{ au_0}{1 + rac{lpha \phi}{n}}$
- In "extreme electrothermal feedback regime"
 - Cold fridge $\rightarrow T_0^n << T_h^n$
 - Excellent detector $\rightarrow a/n >> 1$



- Higher $a \rightarrow$ Faster detector
 - Will see later that higher a also leads to better detector

0.102

T dR

0.104

 $P = \frac{V_{\text{bias}}^2}{R(T)}$

 $\geq R(T,I)$

0.096

0.098

nce [mOhm]

G(T)

T_b

ETF as virtual conductance





0.096

0.098

0.100

Temperature [K]

0.102



- ETF acts like another conductance, in parallel with the natural thermal conductance
- For ETF to be stable

$$\frac{\alpha P}{TG} > -1$$

Naturally satisfied for TES, flipped for NTD

 → TES needs to be "Voltage biased" NTD needs to be "current biased"

TES operation

- Set a **voltage** across the TES (V_{TES})
- Put TES in series with an inductor
- Current in TES \rightarrow magnetic flux in inductor
- $\bullet \quad \rightarrow \text{SQUID measures magnetic flux}$
- \rightarrow SQUID measures current I_{TES}
- Calculate TES resistance with V_{TES}/I_{TES}







Differential equations

$$L\frac{\mathrm{d}I}{\mathrm{d}t} = V - IR_L - IR(T,I)$$

$$C\frac{\mathrm{d}T}{\mathrm{d}t} = -P_{\mathrm{bath}} + P_{\mathrm{J}} + P$$

$$P_{\text{bath}} = K \left(T^n - T^n_{\text{bath}} \right)$$



Quiescent solution and Electro-thermal stability

- Typical scenario:
 - Tb ~ 10 mK
 - T_{TES} (critical temperature Tc) ~ 50 mK
- Joule heating keeps TES at elevated temperature
 P = V²/R
- In Quiescence, TES operates at
 - \circ T₀: Operating temperature, close to Tc
 - \circ R₀: TES operating resistance
 - ~30% of Rn (normal Resistance)
 - P_0 : "Bias power", power to heat TES from Tb to T0
- When particle hit \rightarrow TES heats up \rightarrow R goes up
 - \rightarrow P decreases \rightarrow TES cools back down







More "random" definitions (aka jargons)

Constant temperature dynamic resistance

ce
$$R_{\rm dyn} \equiv \left. \frac{\partial V}{\partial I} \right|_{T_0} = R_0 \left(1 + \beta_I \right)$$

$$\tau \equiv \frac{C}{G}$$

• Natural thermal time constant

• Low frequency loop gain with constant current

$$\mathscr{L}_I \equiv \frac{P_{J_0} \alpha_I}{GT_0}$$

• Current biased thermal time constant

$$\tau_{\rm el} = \frac{L}{R_L + R_0 \left(1 + \beta_I\right)} = \frac{L}{R_L + R_{\rm dyn}}$$
$$\tau_I = \frac{\tau}{1 - \mathscr{L}_I}$$
12

Linearized differential equation

$$L\frac{\mathrm{d}I}{\mathrm{d}t} = V - IR_L - IR(T,I)$$
$$C\frac{\mathrm{d}T}{\mathrm{d}t} = -P_{\mathrm{bath}} + P_{\mathrm{J}} + P$$

• Small signal limit, Taylor expansion, neglecting all higher order terms

$$\frac{\mathrm{d}\delta I}{\mathrm{d}t} = -\frac{R_L + R_0 \left(1 + \beta_I\right)}{L} \delta I - \frac{\mathscr{L}_I G}{I_0 L} \delta T + \frac{\delta V}{L}$$
$$\frac{\mathrm{d}\delta T}{\mathrm{d}t} = \frac{I_0 R_0 \left(2 + \beta_I\right)}{C} \delta I - \frac{\left(1 - \mathscr{L}_I\right)}{\tau} \delta T + \frac{\delta P}{C}.$$

 δV and δP are small perturbations around bias voltage V and input power P

• Put in for later analyses, eg. particle power injection, noise

Matrix differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} \delta I \\ \delta T \end{pmatrix} = - \begin{pmatrix} \frac{1}{\tau_{\mathrm{el}}} & \frac{\mathscr{L}_I G}{I_0 L} \\ -\frac{I_0 R_0 \left(2 + \beta_I\right)}{C} & \frac{1}{\tau_I} \end{pmatrix} \begin{pmatrix} \delta I \\ \delta T \end{pmatrix} + \begin{pmatrix} \frac{\delta V}{L} \\ \frac{\delta P}{C} \end{pmatrix}$$

- Solve for homogeneous equations first \rightarrow setting δV and δP to 0
- Diagonalize the matrix \rightarrow two eigenvalues with two eigenvectors

$$\frac{d}{dt}f_{\pm}(t) = -\lambda_{\pm}f_{\pm}(t)$$

Differential equation solution $\begin{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} \delta I \\ \delta T \end{pmatrix} = -\begin{pmatrix} \frac{1}{\tau_{\mathrm{el}}} & \frac{\mathscr{L}_I G}{I_0 L} \\ -\frac{I_0 R_0 (2 + \beta_I)}{C} & \frac{1}{\tau_I} \end{pmatrix} \begin{pmatrix} \delta I \\ \delta T \end{pmatrix} + \begin{pmatrix} \frac{\delta V}{L} \\ \frac{\delta P}{C} \end{pmatrix} \\ \frac{\mathrm{d}}{\mathrm{d}t} f_{\pm}(t) = -\lambda_{\pm} f_{\pm}(t)$

- Solve for homogeneous equations first \rightarrow setting δV and δP to 0
- Diagonalize the matrix \rightarrow two eigenvalues with two eigenvectors

$$\begin{pmatrix} \delta I \\ \delta T \end{pmatrix} = A_{+}e^{-\lambda_{+}t} \overrightarrow{v_{+}} + A_{-}e^{-\lambda_{-}t} \overrightarrow{v_{-}}$$
$$\frac{1}{\tau_{\pm}} \equiv \lambda_{\pm} = \frac{1}{2\tau_{\rm el}} + \frac{1}{2\tau_{I}} \pm \frac{1}{2}\sqrt{\left(\frac{1}{\tau_{\rm el}} - \frac{1}{\tau_{I}}\right)^{2} - 4\frac{R_{0}}{L}\frac{\mathscr{L}_{I}(2+\beta_{I})}{\tau}}{\tau}}$$
$$\overrightarrow{v_{\pm}} = \begin{pmatrix} \frac{1-\mathscr{L}_{I} - \lambda_{\pm}\tau}{2+\beta_{I}} \frac{G}{I_{0}R_{0}}}{1} \end{pmatrix}$$

Specific solution 1 -- Impulse energy input

• Particle incident and instantaneously thermalization

 $\delta T(0) = \delta T = E/C \qquad \qquad \delta I(0) = 0$

• Solve for A±, then...

$$\delta I(t) = \left(\frac{\tau_I}{\tau_+} - 1\right) \left(\frac{\tau_I}{\tau_-} - 1\right) \frac{1}{(2+\beta_I)} \frac{C\delta T}{I_0 R_0 \tau_I^2} \frac{\left(e^{-t/\tau_+} - e^{-t/\tau_-}\right)}{(1/\tau_+ - 1/\tau_-)}$$
$$\delta T(t) = \left(\left(\frac{1}{\tau_I} - \frac{1}{\tau_+}\right) e^{-t/\tau_-} + \left(\frac{1}{\tau_I} - \frac{1}{\tau_-}\right) e^{-t/\tau_+}\right) \frac{\delta T}{(1/\tau_+ - 1/\tau_-)}$$

• τ +, τ - : "rise time" and "fall time" of a pulse with impulse energy deposition

Specific solution 1 -- Impulse energy input

• When L (inductance) is small, $\tau + << \tau$ -

$$\tau_+ \to \tau_{\rm el},$$

$$\tau_{-} \rightarrow \tau \frac{1 + \beta_{I} + R_{L}/R_{0}}{1 + \beta_{I} + R_{L}/R_{0} + (1 - R_{L}/R_{0})\mathscr{L}_{I}} = \tau_{\text{eff}}$$
$$\mathscr{L}_{I} \equiv \frac{P_{J_{0}}\alpha_{I}}{GT_{0}}$$

- Recall from earlier, when $R_L/R_0 << 1$, "Stiff voltage bias limit"
- $\beta_1 \rightarrow 0$ (not modelled in the first attempt)

$$au_{
m eff} = rac{ au_0}{1 + rac{lpha P_0}{T_0 G}} = rac{ au_0}{1 + rac{lpha}{n} (1 - \left(rac{T_b}{T_0}
ight)^n)}$$



Specific solution 2 -- small sinusoidal power input $\delta P = \operatorname{Re}(\delta P_0 e^{i\omega t})$

• Useful to derive responsivity

$$\frac{d}{dt} \begin{pmatrix} \delta I \\ \delta T \end{pmatrix} = - \begin{pmatrix} \frac{1}{\tau_{\rm el}} & \frac{\mathscr{L}_I G}{I_0 L} \\ \frac{-I_0 R_0 (2 + \beta_I)}{C} & \frac{1}{\tau_I} \end{pmatrix} \begin{pmatrix} \delta I \\ \delta T \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{\delta P_0}{C} \end{pmatrix} e^{i\omega t}$$

• After some math.....

$$\frac{dI}{dP}(\omega) = s_I(\omega) = -\frac{1}{I_0R_0} \frac{1}{(2+\beta_I)} \underbrace{\frac{(1-\tau_+/\tau_I)}{(1+i\omega\tau_+)} \underbrace{(1-\tau_-/\tau_I)}_{(1+i\omega\tau_-)}}_{(1+i\omega\tau_+)} \underbrace{\frac{dT}{dP}(\omega)}_{(1+i\omega\tau_+)} = s_T(\omega) = \frac{1}{G} \frac{\tau_+\tau_-}{\tau^2} \frac{(\tau/\tau_+ + \tau/\tau_- + \mathscr{L}_I - 1 + i\omega\tau_-)}{(1+i\omega\tau_+)(1+i\omega\tau_-)}.$$

- Two low-pass filters
 - For a review of one-pole filter and its response in Laplace domain, see eg. <u>https://www.embeddedrelated.com/showarticle/590.php</u>

Complex impedance

- Impedance of the system in a complex plane, as a function of frequency
 - Given Voltage excitation of ω angular frequency, what's current and phase?

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} \delta I \\ \delta T \end{pmatrix} = - \begin{pmatrix} \frac{1}{\tau_{\mathrm{el}}} & \frac{\mathscr{L}_I G}{I_0 L} \\ -\frac{I_0 R_0 \left(2 + \beta_I\right)}{C} & \frac{1}{\tau_I} \end{pmatrix} \begin{pmatrix} \delta I \\ \delta T \end{pmatrix} + \begin{pmatrix} \frac{\delta V}{L} \\ \frac{\delta P}{C} \end{pmatrix} \qquad \text{Voltage excitation} \\ \text{of } V \omega \\ \text{Fourier} \\ \text{Transform} \qquad \begin{pmatrix} \frac{1}{\tau_{\mathrm{el}}} + i\omega & \frac{\mathscr{L}_I G}{I_0 L} \\ -\frac{I_0 R_0 (2 + \beta_I)}{C} & \frac{1}{\tau_I} + i\omega \end{pmatrix} \begin{pmatrix} I_\omega \\ T_\omega \end{pmatrix} = \begin{pmatrix} \frac{V_\omega}{L} \\ 0 \end{pmatrix}$$

Complex

Complex impedance

$$\begin{pmatrix} \frac{1}{\tau_{el}} + i\omega & \frac{\mathscr{L}_{IG}}{I_{0L}} \\ \frac{-I_{0}R_{0}(2+\beta_{I})}{C} & \frac{1}{\tau_{I}} + i\omega \end{pmatrix} \begin{pmatrix} I_{\omega} \\ T_{\omega} \end{pmatrix} = \begin{pmatrix} \frac{V_{\omega}}{L} \\ 0 \end{pmatrix}$$

$$Z_{\omega} = V_{\omega}/I_{\omega} = R_{L} + i\omega L + Z_{TES}$$

$$Z_{TES} = R_{0}(1+\beta_{I}) + \frac{R_{0}\mathscr{L}_{I}}{1-\mathscr{L}_{I}} \frac{2+\beta_{I}}{1+i\omega\tau_{I}}$$
Or.. complex admittance Y=1/Z
$$Y(\omega) = s_{I}(\omega)I_{0} \frac{\mathscr{L}_{I} - 1}{\mathscr{L}_{I}} (1+i\omega\tau_{I})$$

$$P(\omega) = s_{I}(\omega)I_{0} \frac{\mathscr{L}_{I} - 1}{\mathscr{L}_{I}} (1+i\omega\tau_{I})$$

Complex impedance used to extract TES parameters

• Often takes data at different Tb and different R0



TES Stability

- For a TES to be stable:
 - $\tau \pm$ need to be **real (not oscillating)**
 - $Re(\tau \pm)$ need to be **positive (converging back to quiescence)**

$$\frac{1}{\tau_{\pm}} \equiv \lambda_{\pm} = \frac{1}{2\tau_{\mathrm{el}}} + \frac{1}{2\tau_{I}} \pm \frac{1}{2} \sqrt{\left(\frac{1}{\tau_{\mathrm{el}}} - \frac{1}{\tau_{I}}\right)^{2} - 4\frac{R_{0}}{L}\frac{\mathscr{L}_{I}(2+\beta_{I})}{\tau}}{R_{0}}$$
$$R_{0} > \frac{(\mathscr{L}_{I} - 1)}{(\mathscr{L}_{I} + 1 + \beta_{I})}R_{L}$$

- In large L_1 limit (large amplification), $R_0 > R_L$
- If Re(τ±) is positive, but τ± isn't real → TES is underdamped (oscillate) but will converge → Electrothermal oscillation
- If $\text{Re}(\tau \pm)$ is negative \rightarrow thermal runaway

https://arxiv.org/pdf/2304.14926.pdf

Complex detector modeling





$$\begin{split} & L\frac{dI}{dt} = I_0 * R_0 - I * R_{tes} & \text{Doug Pinckney, undergrad honor thesis} \\ & C_a \frac{dT_a}{dt} = \frac{E}{c\sqrt{2\pi}} e^{\frac{-(t-t_0)^2}{2c^2}} - K_{a,b}(T_a^{n_{a,b}} - T_b^{n_{a,b}}) - K_{a,auq}(T_a^{n_{a,au1}} - T_{au1}^{n_{a,au1}}) - K_{a,g}(T_a^{n_{a,g}} - T_g^{n_{a,g}}) \\ & C_{au1}\frac{dT_{au1}}{dt} = K_{a,au1}(T_{au1}^{n_{a,au1}} - T_a^{n_{a,au1}}) - K_{au1,wb1}(T_{au1}^{n_{au1,wb1}} - T_{wb1}^{n_{au1,wb1}}) \\ & C_g \frac{dT_g}{dt} = K_{a,g}(T_g^{n_{a,g}} - T_a^{n_{a,g}}) - K_{g,si}(T_g^{n_{g,si}} - T_{si}^{n_{g,si}}) \\ & C_{wb1}\frac{dT_{wb1}}{dt} = K_{au1,wb1}(T_{wb1}^{m_{au1,wb1}} - T_{au1}^{n_{au1,wb1}}) - K_{wb1,au2}(T_{wb1}^{m_{wb1,au2}} - T_{au2}^{n_{wb1,au2}}) \\ & C_{wb1}\frac{dT_{wb1}}{dt} = K_{si,au2}(T_{au2}^{n_{si,au2}} - T_{si}^{n_{si,au2}}) - K_{wb1,au2}(T_{au2}^{n_{wb1,au2}} - T_{au2}^{n_{wb1,au2}}) - K_{au2,tes}(T_{au2}^{n_{au2,tes}} - T_{tes}^{n_{au2,tes}}) \\ & C_{si}\frac{dT_{si}}{dt} = K_{g,si}(T_{si}^{n_{g,si}} - T_g^{n_{g,si}}) - K_{si,au2}(T_{si}^{n_{si,au2}} - T_{au2}^{n_{si,au2}}) - K_{si,tes}(T_{si}^{n_{si,tes}} - T_{tes}^{n_{si,tes}}) \\ & - K_{si,m}(T_{si}^{n_{si,m}} - T_m^{n_{si,m}}) \\ & C_{m}\frac{dT_{tes}}{dt} = I^2 R_{tes} - K_{au2,tes}(T_{tes}^{n_{au2,tes}} - T_{au2}^{n_{au2,tes}}) - K_{si,tes}(T_{tes}^{n_{si,tes}} - T_{si}^{n_{si,tes}}) - K_{tes,m}(T_{tes}^{n_{tes,m}} - T_m^{n_{tes,m}}) \\ & C_m\frac{dT_m}{dt} = K_{tes,m}(T_m^{n_{tes,m}} - T_{tes}^{n_{tes,m}}) - K_{si,m}(T_m^{n_{si,m}} - T_{si}^{n_{si,m}}) - K_{m,wb2}(T_{wb2}^{n_{m,wb2}} - T_{wb2}^{n_{m,wb2}}) \\ & C_{wb2}\frac{dT_{wb2}}{dt} = K_{m,wb2}(T_{wb2}^{n_{m,wb2}} - T_m^{n_{m,wb2}}) - K_{wb2,b}(T_{wb2}^{n_{wb2,b}} - T_b^{n_{wb2,b}}) \\ \end{array}$$

Differential equation \rightarrow Matrices (time, frequency)



One note about noise and resolution

- With responsivity s₁, can derive theoretical resolution:
 - Model various noise
 - Johnson noise, thermal fluctuation noise, etc.
 - Convert to "Noise Equivalent Power"

$$\bigcirc \qquad \Delta E_{\rm rms} = \left(\sqrt{\int_0^\infty \frac{4\,df}{{\rm NEP}(f)^2}}\right)^{-1}$$

• Irwin's derivation shows

$$\Delta E_{\rm rms} = \sqrt{4k_{\rm B}T^2 C \frac{1}{\alpha} \sqrt{\frac{n}{2}}}$$

• Will come back to this next class

Noise Modeling

- Characterize noises in an idealized TES system in four categories:
 - Internal Johnson Noise
 - External Johnson Noise
 - Thermal Fluctuation Noise
 - "Downstream" Amplifier noise
- General steps:
 - Calculate noise sources from each component
 - Derive "Noise Equivalent Power" (NEP) with responsivity of each component
 - Integrate NEP to get predicted resolution
 - This assumes "optimal filter" is used
 - Weighs each frequency component with signal-to-noise



https://en.wikipedia.org/wiki/Johnso n%E2%80%93Nyquist_noise

Johnson noise

• Generated by thermal motions of charge carriers

$$\sqrt{\overline{v_n^2}} = \sqrt{4k_BTR\Delta f}$$

$$\sqrt{\overline{i_n^2}} = \sqrt{rac{4k_BT\Delta f}{R}}$$

• For a circuit with components at different temperatures

$$S_V = \bar{V}_{j,passive}^2(f) = \sum_{passive} S_V = 4k_b \sum_{passive} T_i R$$
$$S_I = \bar{I}_{j,passive}^2(f) = \frac{4k_b}{|Z_{loop}|^2} \sum_{passive} T_i R_i$$



29

Johnson noise modeling

- Usually quantify noise in $\sqrt{i_n^2}/\Delta f$
- Can direct model noise when TES is normal/superconducting
 - I.e. No electro-thermal feedback

PA SC, 0V





Passive vs "active" Johnson Noise

- Johnson noise in non-TES part of the circuit wiggles current in the circuit
- Johnson noise in TES does that, plus heating up TES
 - Lead to changing of resistance of TES

$$V_B + \delta V = I_0 R_{sh} + L \frac{dI}{dt} + I_0 R_0 \qquad \left(\delta T\right)^{-M} \qquad \left(\frac{\delta P}{C}\right)^{-M} \qquad \left(\frac{\delta P}{C}\right)^{-M} \qquad \left(\frac{\delta P}{C}\right)^{-M} \qquad \left(\frac{\delta P}{C}\right)^{-M} = I_0 V_0 \qquad \delta P = -I_0 \delta V_0 \qquad \delta P =$$



Thermal Fluctuation Noise

- TES, with heat capacity of C, is in thermal contact with Tb
- Heat conducting through G can fluctuate
 - $\circ \rightarrow$ Thermal Fluctuation Noise
 - $S_{P_{\rm TFN}} = 4k_{\rm B}T_0^2 G \times F(T_0, T_{\rm bath}) \; , \label{eq:SPTFN}$



- F(T₀, T_{bath}) is to correct for the fact that TES and bath are not in thermal equilibrium for short time scale
 - $\circ~$ In ideal case (Tb \rightarrow 0, G $\rightarrow~$ $nl_{0}{}^{2}R_{0}^{}/T_{0}^{}),~F\rightarrow$ $^{1\!\!/_{2}}$

$$S_{I_{\rm TFN}}(\omega) = 4k_{\rm B}T_0^2 G \times F(T_0, T_{\rm bath})|s_I(\omega)|^2$$

"Downstream" electronics noise

- TES usually read out with SQUID and a bunch of amplifiers.
- Can quantify the electronics noise in the input-coil referenced current noise
 - Convert all noises to the amount of current fluctuation through TES

$$S_{P_{\mathrm{amp}}}(\omega) = rac{S_{I_{\mathrm{amp}}}(\omega)}{|s_I(\omega)|^2}$$

• The total noise is the sum of all components



Noise modeling -- TES in transition



NEP and resolution

• Noise Equivalent Power $NEP(\omega) = \sqrt{S_P(\omega)}$.

$$\Delta E_{\rm rms} = \left(\sqrt{\int_0^\infty \frac{4df}{\rm NEP}(f)^2} \right)^-$$

- Derived with "optimal filter"
- Idealized TES, strong electro-thermal feedback, zero load resistance, with delta energy input

$$\Delta E_{\rm rms} = \sqrt{4k_{\rm B}T^2C\frac{1}{\alpha}\sqrt{\frac{n}{2}}}$$

• For macroscopic crystal detector, need to include phonon propagation time $\Delta E_{rms} = \sqrt{2k_bT_c^2G\left(au_{pulse} + rac{2 au_{TES}}{n}
ight)}$



QET technology

Quasiparticle-trap-assisted
 Electrothermal-feedback Transition-edge sensors

$$\Delta E_{\rm rms} = \sqrt{4k_{\rm B}T^2C\frac{1}{\alpha}\sqrt{\frac{n}{2}}}$$

- Minimizing C is important...
- SuperCDMS way -- Athermal phonon detector
 - Not include crystal heat capacity
 - Cool crystal strongly down to bath temperature
 - Use TES electron-phonon coupling as weak G
- Min TES volume while max phonon collection:
 - Aluminum fins
 - Superconductor, no heat capacity accounted







QET modelling

- Cost:
 - \circ Al needs minimum 2 Δ energy to excite $\Delta(T=0)=1.764\,k_{
 m B}T_{
 m c}$
 - ~334 ueV, from BCS theory
 - Need to pick up phonons fast before they down-convert too much
 - Quasiparticle transportation has finite probability of funneling energy to TES
- Overall, captured as an energy efficiency





$$\Delta E_{rms} = rac{1}{\epsilon} \sqrt{2k_b T_c^2 G\left(au_{pulse} + rac{2 au_{TES}}{n}
ight)}$$

Some optimizations can be performed



37