PHY2405 Accelerator Problems

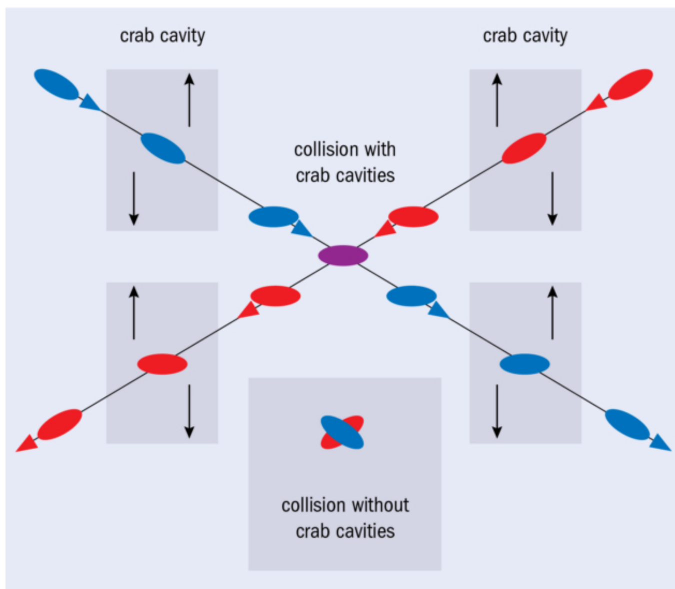
January 15, 2024

1) Crossing Angle: In class we talked about the luminosity that results from Gaussian bunches that collide head-to-head. Suppose instead that they collide with a small crossing angle α between their directions of motion. Assume that the distribution function of each bunch is still Gaussian along its direction of motion with a standard deviation of σ_z. Show that the expression of luminosity becomes:

$$\mathcal{L} = \frac{fN^2}{4\pi\sigma^2}\frac{1}{\sqrt{1+\alpha^2(\sigma_z^2/4\sigma^2)}}$$

Sometimes a finite crossing angle between colliding beams is necessary because parasitic collisions. Collisions between the intended bunch and the 'next bunch' from the next beam happen so close to the interaction region that they can leave collision remnants in the detector confusing the measurement of the target bunch collisions. Apart from the added complexity need to 'tilt' the beams before bringing them in to collision, here you are computing the luminosity penalty.

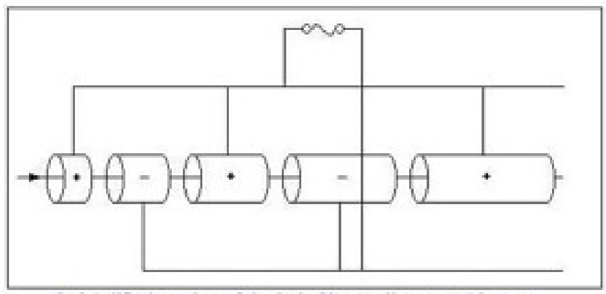
In fact there are colliders in operation where they introduce even more beam-gymnastics to rotate the beams just before they collide to recover this lumi loss. These are called "crab crossings" and add quite a bit of complexity to the final focus around the experiments.



To be clear, what I am asking you to calculate in this problem is overlap integral that corresponds to the picture in the bottom middle labelled': "collision without crab cavities".

2) Fixed Target Lumi: High interaction rates (luminosity) are more readily achieved in fixed target operation (ie. with a beam incident on a solid target) than in a collider mode (where two counter-rotating beams encounter each other at the centre of an experiment). Suppose a beam of 10^{11} protons per second is incident on a liquid hydrogen target that is one metre long. Calculate the effective luminosity for this fixed target situation. You will need to find the density of liquid hydrogen (PDG might be a good place to start).

3) Linear Accelerator: A linear accelerator is often constructed with copper tubes along a common axis with an alternating voltage of frequency f applied between neighbouring tubes. Inside each of the tubes the electric field vanishes (it acts as a Faraday shield from fields that are present outside the tubes). Between the tube ends, an alternating electric field accelerates particles traveling along the beam axis. For a fixed voltage and frequency, the tube length must increase with distance to maintain the correct phase relationship between the accelerating particles and the field. Determine the first three tube lengths for a frequency f, accelerating voltage V, injection energy E_i, particle mass m and charge e. You can neglect the travel time of the particles across the gaps between the tubes. In practice these are kept quite small so that the particles experience the maximum field when they are present in the gap.



A simplified version of the Luis Alvarez linear accelerator.

To be clear, a bunch of protons (or electrons – so do you computations for an arbitrary mass: m) arrives at the left side of the structures in the picture above. It is timed just right so that it will be in the middle of the gap between the first and second drift tubes to experience the maximum accelerating voltage "V". Half and RF cycle (of frequency f) later the beam will end up at the centre of the gap between the 2nd and 3rd tubes and will, again be 'pushed' with maximum voltage V', thus accelerated further.

4) Next Collider: Consider building a 50 TeV on 50 TeV pp collider (CERN has started geological surveys for a tunnel that is something like the length you will calculate below).

(a) Given a maximum dipole strength of 10 T, what is the minimum circumference for such a machine? The actual circumference would have to accommodate focusing magnets, accelerating cavities, injection/extraction locations and some detectors. From a current example (perhaps the LHC?) determine what the typical filling factor is (ie. length of dipoles vs. total physical circumference) and estimate how big a 50-on-50 TeV collider would, realistically, need to be.

(b) Assuming an initial peak luminosity target of 10^{35} (once the machine is built there will undoubtedly be intensity upgrades discovered, implemented etc., but this is a good starting point for the physics) per interaction region, a total pp interaction cross-section of 100 mb and a 30% live time, how many collisions will be observed by a detector in one year.

(c) For an energy gain of 6 MeV per turn, how many orbits are required to ramp the accelerator to its ultimate energy? How long will this take?

(d) If the machine can accommodate 10^{14} protons in each beam, find the total beam current in amps, the total beam energy in MJ and the synchrotron radiation power lost as the beams bend around the machine in the dipoles. This synchrotron power will be absorbed (mostly) in the liquid He refrigeration system for the dipole magnets.

(e) If this beam were extracted and targeted on liquid hydrogen target, what would the resulting centre of mass energy be? Ie. turn the usual luminosity vs. centre of mass energy (Collider vs. Fixed target) argument around: Would a fixed target machine like this have high enough beam energy to make W bosons, top quarks or Higgs bosons, at presumably much higher rates than any collider?