INTERACTIONS OF CHARGED PARTICLES WITH MATTER

- FUNDAMENTALLY RESPONSIBLE FOR "WORKING" OF ALL PARTICLE DETECTORS
  - TRACKERS - MINIMAL INTERACTION
  - CALORIMETERS - MAXIMAL INTERACTION

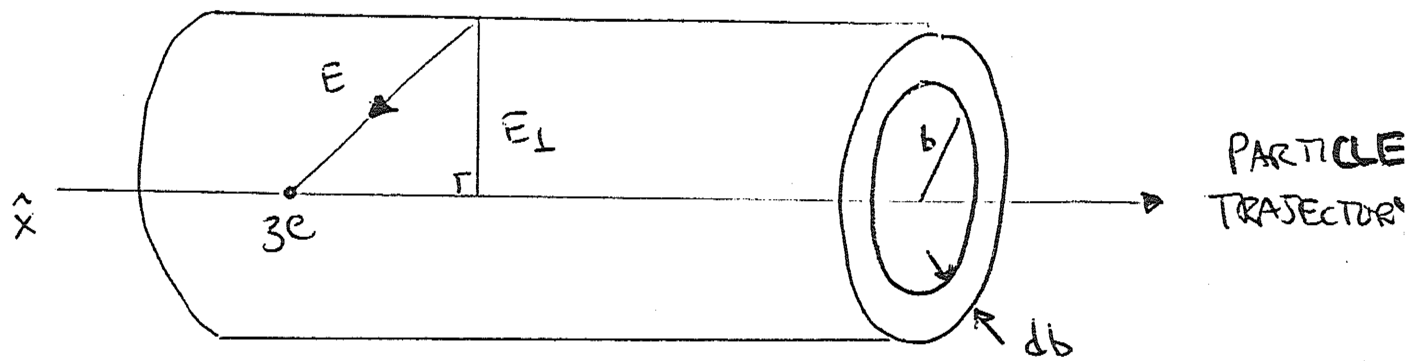
(EVEN NEUTRAL PARTICLES MUST BE CONVERTED TO CHARGED PARTICLES BEFORE BEING DETECTED)

CONSIDER:

1) IONIZATION
   - AVERAGE ENERGY LOSS
   - LANDAU TAIL

2) MULTIPLE SCATTERING

INFLUENCE OF PARTICLE (CHARGE z) ON NEARBY ELECTRONS



MOMENTUM TRANSFERRED $\quad \vec{p} = p_{\parallel} \hat{u}_{\parallel} + p_{\perp} \hat{u}_{\perp}$

$$p_{\parallel} = 0$$

$$p_{\perp} = \int_{-\infty}^{\infty} e E_{\perp} \, dt$$

$$= \frac{e}{v} \int E_{\perp} \, dx$$

$$= \frac{e}{v} \frac{1}{2\pi b} \int_{\text{surface}} E_{\perp} (2\pi b) dx$$

$$= \frac{e}{v} \frac{1}{2\pi b} \left(\frac{ze}{\epsilon_0} \right)$$

(2)

Energy transferred $\quad \mathbf{E} = p^2/2m$

$$E = \left(\frac{ze^2}{\epsilon_0}\right)^2 \frac{1}{2mc^2} \frac{1}{4\pi^2} \frac{1}{b^2} \frac{1}{\beta^2}$$

N.B. $\quad \dfrac{dE}{E} = -\dfrac{2db}{b}$

$$\boxed{\text{Now define: } \phi(E,E')\,dE'dx \text{ to be the probability that a particle loses energy between } E' \to E'+dE \text{ in path length } dx}$$

$$\Rightarrow \phi(E,E')\,dE'dx = \underbrace{-n_0}_{\#\text{ electrons/unit volume}}(2\pi b)\,db\,dx$$

$$= n_0 \pi b^2\,dx\left(-\frac{2db}{b}\right)$$

$$= n_0 \pi \frac{z^2 r_e^2 (mc^2)}{\beta^2} \frac{dE'}{E'^2} dx$$

$$\equiv \frac{\overset{\frown}{A}(\rho\,dx)}{\beta^2} \frac{dE'}{E'^2}$$

$$\breve{A} = 0.154 \frac{z^2 Z}{A} \frac{MeV\ cm^2}{gm}$$

Z = Charge of Media

A = Atomic # of Media

PRIMARY IONIZATION

- MEAN NUMBER OF "ENCOUNTERS" BETWEEN E'_{min} and E'_{max} (CAUSE "SINGLE IONIZATION") PER UNIT LENGTH

$$\frac{n_{prim}}{\Delta x} = \left\langle \frac{dN}{dx} \right\rangle = \int_{E'_{min}}^{E'_{max}} \phi(E', E)\, dE'$$

$$= \frac{\breve{A}\rho}{\beta^2} \int_{E_{min}}^{E_{max}} \frac{dE'}{E'^2}$$

$$= \frac{\breve{A}\rho}{\beta^2} \left(\frac{1}{E_{min}} - \frac{1}{E_{max}} \right)$$

FOR MOST GASES $\frac{\breve{A}\rho}{E_{max}} \ll 1$

SO AS $\beta \to 1$ THIS CAN BE IGNORED

$$\Rightarrow \frac{n_{prim}}{\Delta x} \simeq \frac{\breve{A}\rho}{\beta^2} \frac{1}{E_{min}}$$

$E_{min} \geq I_0$ (THE THRESHOLD TO IONISE THE GAS)

FOR MOST GASES QUANTUM EFFECTS ARE IMPORTANT (SCREENING ETC.). FOR HYDROGEN BETHE PEFORMED A "FULL CALCULATION". CONCLUDED:

$$E_{min} = \frac{I_0}{r\left[\ln \frac{2mc^2}{I_0} \frac{\beta^2}{1-\beta^2} + S - \beta^2\right]}$$

$I_0 = 15.4\,\text{eV}$ $r = 0.29$ $S = 3.04$

$\Rightarrow$ $E_{min} = 3.5\,\text{eV}$ $n_{prim} = 4.6\,/\text{cm}$.

FOR OTHER GASES

$$E_{min} = \frac{\tilde{A}\rho}{A_1\left[A_2 + \ln\left(\frac{\beta^2}{1-\beta^2}\right) - \beta^2\right]}$$

See table for A_1 A_2 and plots of experimental determinations of n_{prim}.

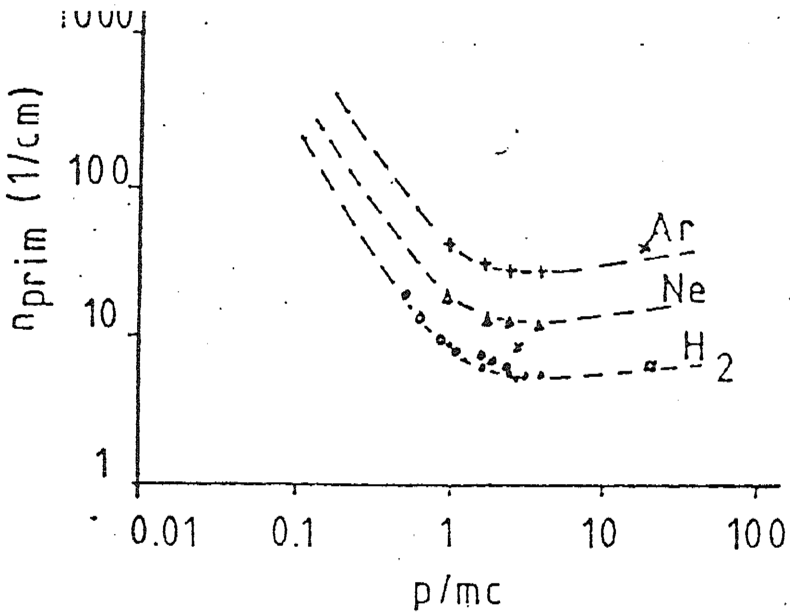


Fig. 1 Primary ionisation. Points: measured, --- Bethe's formula adjusted

Table I

gas	density (mg/cm^3) (0^oC, 1atm)	I_o (eV)	A_1	A_2	n_{prim} (cm^{-1})
H_2	0.0899	15.4	Bethe's	formula	4.6
H_e	0.1785	24.6	0.244	11.64	3.5
N_e	0.9004	21.7	0.844	10.89	11.4
A_r	1.7837	15.8	1.828	11.45	25.8
X_e	5.8510	12.1	3.554	11.31	49.6
N_2	1.2506	15.5	1.941	11.43	27.1
O_2	1.4290	12.2	2.079	11.28	28.9

ENERGY LOSS

$$< \frac{dE}{dx} > = \int_{E_{min}}^{E_{max}} E' \, \phi(E', E) \, dE'$$

$$= \frac{\breve{A} \rho}{\beta^2} \int_{E_{min}}^{E_{max}} \frac{dE'}{E'}$$

$$= \frac{\breve{A} \rho}{\beta^2} \ln \left(\frac{E_{max}}{E_{min}} \right)$$

SHORT OF DOING FULL Q.E.D. CALCULATION NEED SOME PHYSICAL ARGUMENTS FOR THE ORDER OF MAGNITUDE OF E_{min} AND E_{max}.

E_{min} CORRESPONDS TO b_{max}

DURATION OF INTERACTION SHOULD SHORT COMPARED TO ORBITAL PERIOD OF THE ATOM.

$$\frac{b}{v} \lesssim \frac{1}{\nu} \qquad (h\nu = I_0)$$

$$\Rightarrow \quad b_{max} = \frac{v}{\nu} f_{max}$$

AS PARTICLE BECOMES RELATIVISTIC $\vec{E}$ EXTENDS BY A FACTOR $\gamma \left(= \frac{1}{\sqrt{1-\beta^2}}\right)$.

$$\Rightarrow \quad b_{max} = \frac{v\gamma}{\nu} f_{max}$$

b_{min}: CAN'T BE SHORTER THAN THE deBroglie WAVELENGTH OF THE ELECTRON IN THE CENTRE OF MASS SYSTEM OF THE PROJECTILE (AS V_{cm}(electron) $\simeq V_{incident}$ COLLISION CANNOT CAUSE IONIZATION).

$$\lambda_{deBroglie} = \frac{h}{mc\beta\gamma}$$

$$\Rightarrow \quad b_{min} = \frac{h}{mc\beta\gamma} f_{min}$$

$$\frac{b_{max}}{b_{min}} = \frac{v\gamma}{\nu} \quad \frac{mc\beta\gamma}{h} \quad f'$$

$$= \frac{mc^2}{h\nu} \beta^2\gamma^2 f'$$

7) BETHE'S CALCULATION SAYS $f' = 2$ ⊕ TERM INDEPENDENT OF β.

PUTTING THIS ALL BACK TOGETHER

$$\frac{dE}{dx} = 4\pi r_e^2 \left(3^2 \frac{ZN}{A}\right) mc^2 \frac{1}{\beta^2}\left[\ln \frac{2mc^2}{I}\left(\frac{\beta^2}{1-\beta^2}\right) - \beta^2 - \frac{d}{2}\right]$$

$$\boxed{\frac{dE}{dx} \simeq 0.31 \frac{Z}{A}\left\{\frac{\text{MeV cm}^2}{g}\right\} \frac{3^2}{\beta^2}\left[\ln\left(\frac{2mc^2\gamma^2\beta^2}{I}\right) - \beta^2 - \frac{d}{2}\right]} \quad \text{I}$$

INTERPRETATION:

a) $\frac{dE}{dx} \sim \frac{1}{\beta^2}$ AT LOW β

Collision time $\tau \simeq \frac{1}{\beta}$

$\Rightarrow$ momentum transfer $\sim 1/\beta$

$\Rightarrow$ Energy $\sim 1/\beta^2$.

b) As $\beta \to 1$

$\ln(\beta^2\gamma^2) \simeq \ln(p)$ dominates

Known as relativistic rise.

STARTS AT $\simeq \beta\gamma \simeq 3.5 - 4$.

c) Recall that we argued $b_{max} \sim \gamma b_0$ since at very high energies $E_\perp$ "spreads out" $\sim \gamma$.

This cannot continue indefinitely.

Material begins to polarise – depending on its density (hence symbol d)

$$\Rightarrow \frac{d}{2} = \ln \beta^2 \gamma^2 - \ln \frac{I^2}{\hbar^2 \omega_p^2} - 1$$

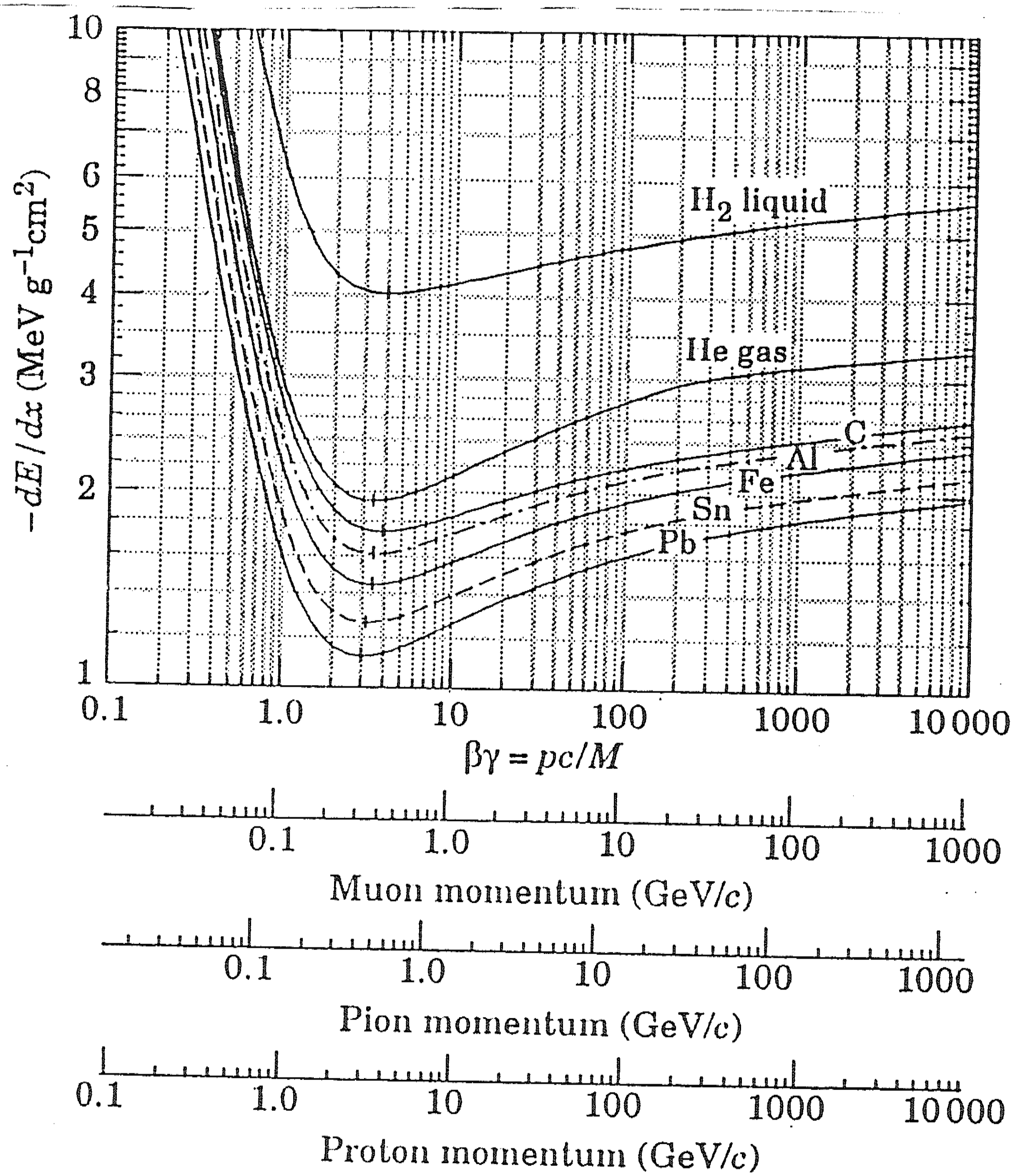
ω_p is plasma frequency of the material through which the particle is passing

$$\omega_p = \left(\frac{n_0 e^2}{\epsilon_0 m} \right)^{1/2}$$

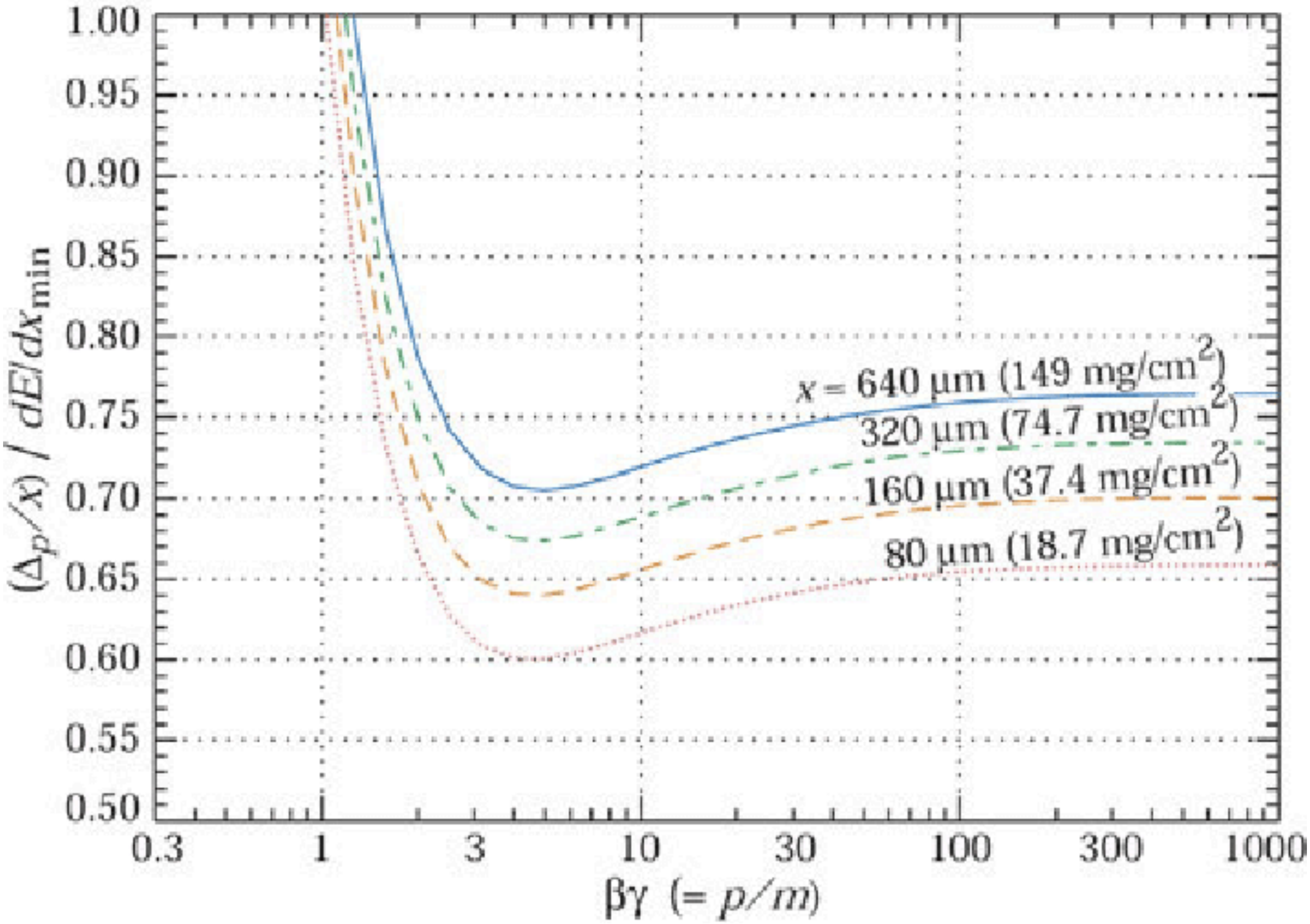
The density effect cancels the relativistic rise for:

$$\gamma_{plateau} \simeq \frac{I}{\hbar \omega_p}$$

$\gamma_{plateau} \sim 1000$ in gases ~ 10 in silicon.



10
8
6
5
4
3
2
1
$-dE/dx$ (MeV g^{-1}cm^2)
H_2 liquid
He gas
C
Al
Fe
Sn
Pb
0.1
1.0
10
100
1000
10 000
$\beta\gamma = pc/M$
0.1
1.0
10
100
1000
Muon momentum (GeV/c)
0.1
1.0
10
100
1000
Pion momentum (GeV/c)
0.1
1.0
10
100
1000
10 000
Proton momentum (GeV/c)



1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
(Δ_p/x) / dE/dx_{min}
0.3
1
3
10
30
100
300
1000
$\beta\gamma$ (= p/m)
x = 640 µm (149 mg/cm^2)
320 µm (74.7 mg/cm^2)
160 µm (37.4 mg/cm^2)
80 µm (18.7 mg/cm^2)

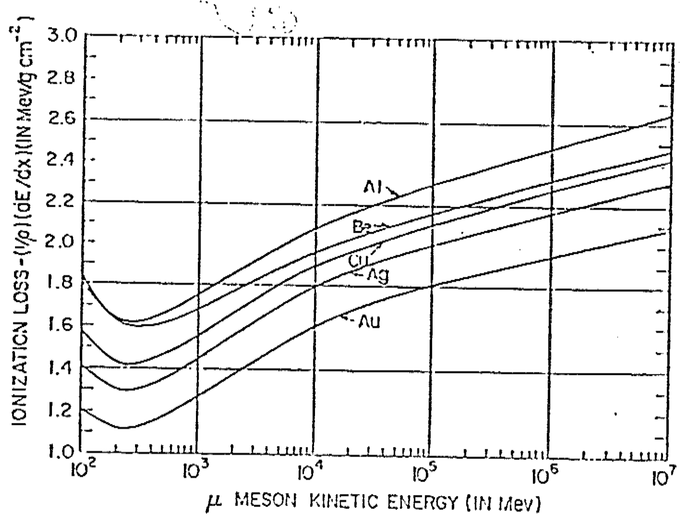


FIG. 4. The average ionization loss of μ mesons in Be, Al, Cu, Ag, and Au, as a function of the μ-meson kinetic energy [Eq. (1.1.19)].

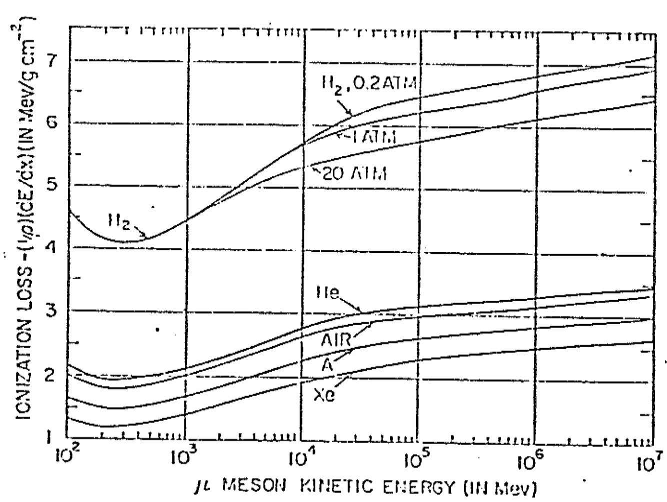


FIG. 5. The average ionization loss of μ mesons in H_2, He, air, Ar, and Xe, as a function of the μ-meson kinetic energy [Eq. (1.1.19)]. The curves for He, air, Ar, and Xe pertain to normal pressure.

IF YOU HAVE A MIXTURE OF MATERIALS (e.g. A SAMPLING CACORIMETER - SEE LATER) THEN YOU CAN SUM THE ENERGY LOSS FROM THE DIFFERENT COMPONENTS :

$$\frac{dE}{dx} = \sum_j w_j \left.\frac{dE}{dx}\right|_j$$

WHERE ω_j IS THE FRACTION (BY MASS) OF THE j^{th} COMPONENT

$\left.\frac{dE}{dx}\right|_j$ IS I FOR j^{th} COMPONENT.

FLUCTUATIONS ABOUT MOST PROBABLE ENERGY LOSS (LANDAU DISTRIBUTION)

FOR VERY SMALL IMPACT PARAMETER COLLISIONS THERE IS A (LOW PROBABILITY) CHANCE OF PRODUCING A VERY HIGH ENERGY SECONDARY ELECTRON ~ δ-RAY.

WHILE IT HAS HIGH-ENERGY COMPARED TO "NORMAL" IONISATION IT HAS LOW ENERGY COMPARED TO RELATIVISTIC PARTICLE IN THE FIRST PLACE.

$\Rightarrow$ δ ELECTRON IS VERY HEAVILY IONISING $\left(\frac{1}{\beta^2}\right)$

LANDAU WORKED OUT FIRST APPROXIMATION TO THIS DESCRIBE SUCH A DISTRIBUTION:

$$\boxed{\phi(\lambda) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}(\lambda + e^{-\lambda})}}$$

$$\lambda = (E - E_{mp})/\xi$$

NORMALISED DEVIATION FROM MOST PROBABLE ENERGY LOSS.

$$\xi = \frac{1}{2}(4\pi r_e^2)\, mc^2 (z^2 Z)\left(\frac{N_A \rho}{A}\right)\frac{\Delta x}{\beta^2}$$

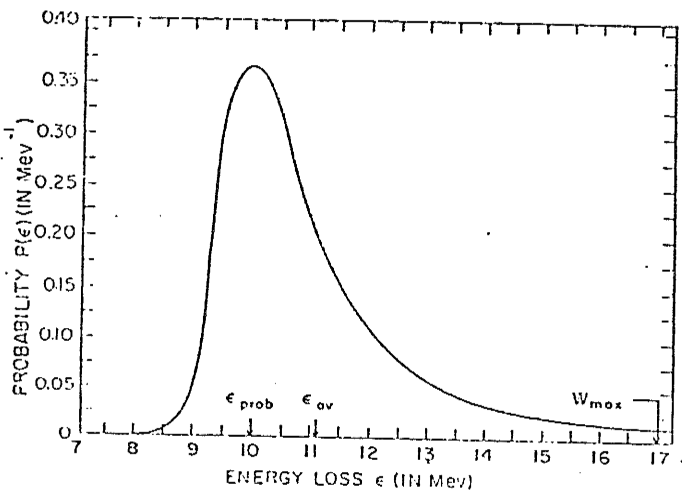


FIG. 3. The Landau distribution of energy losses ϵ for 3-Bev protons traversing a thickness 6.97 gm/cm² of Be, for which ϵ_{prob} = 10 Mev, ϵ_{Av} = 11.10 Mev, and W_{max} = 17 Mev.

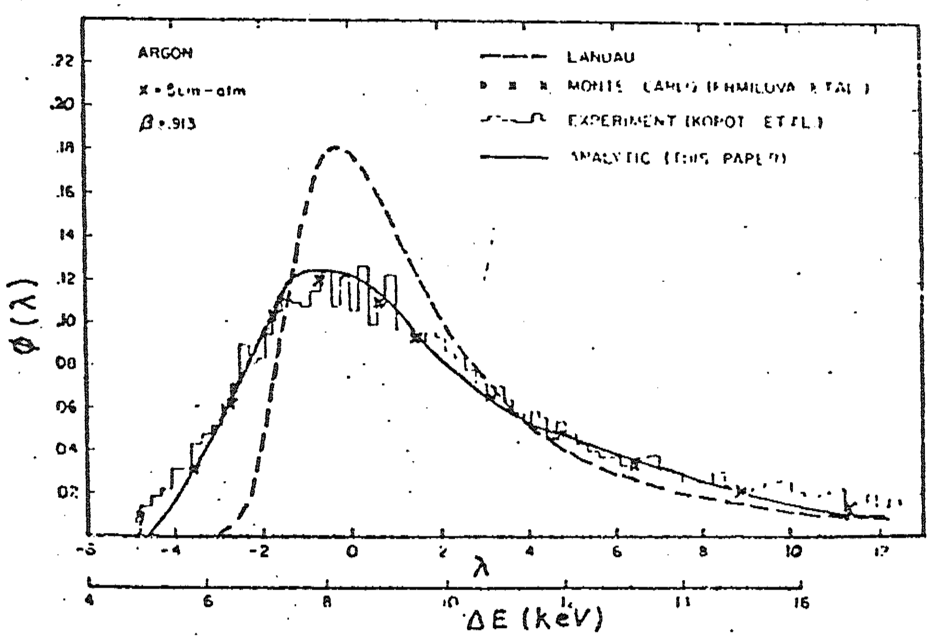


Fig. 17 Energy loss distribution. Comparison of calculations and measurements.

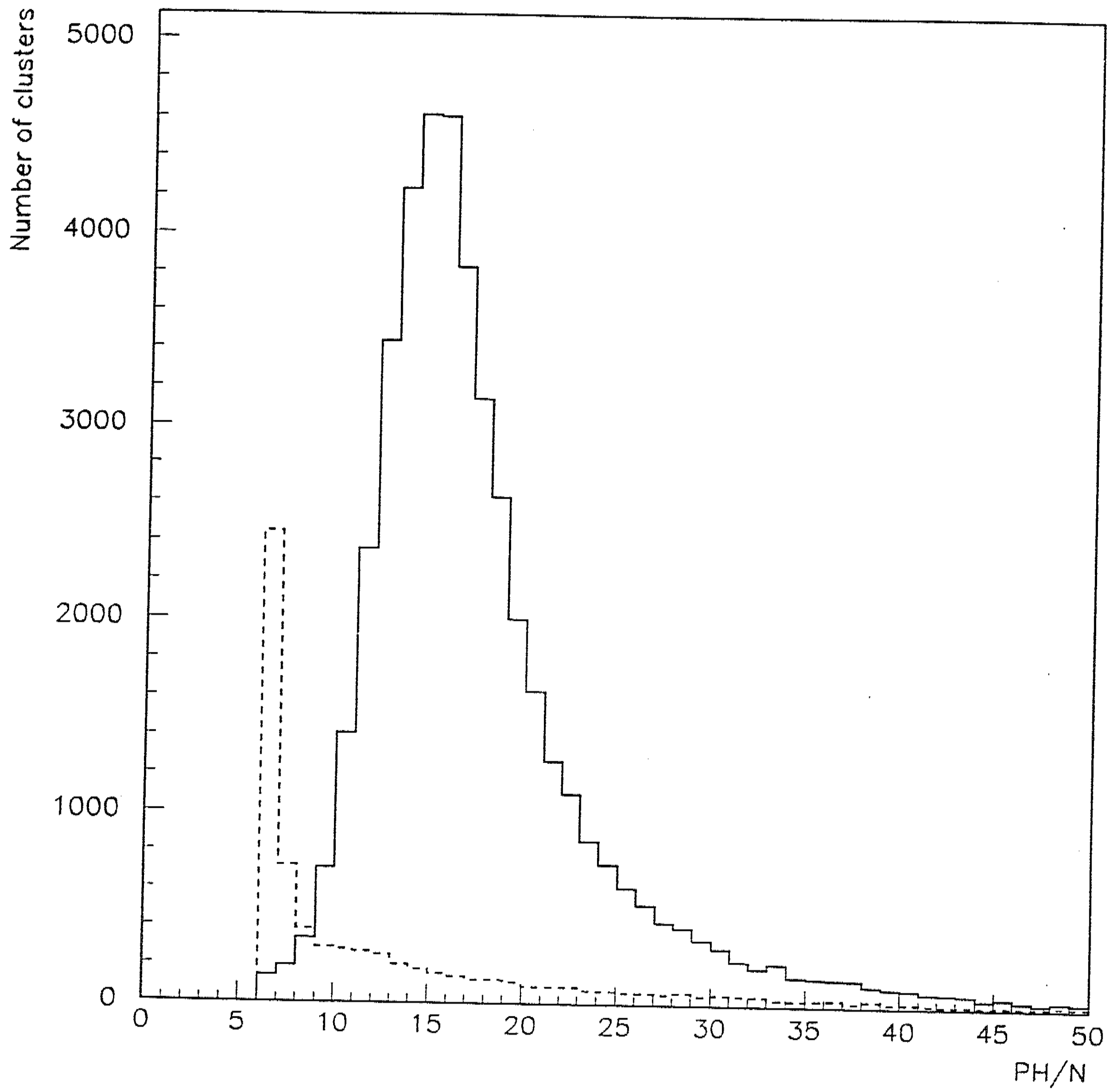


Figure 20: PH/N for clusters with (full-line) and without (dashed-line) a track associated to them for the Inner layer modules. The PH/N has been normalized to the minimum track length in traversing the silicon.

DELTA ELECTRONS HAVE AT LEAST TWO IMPLICATIONS FOR PARTICLE DETECTOR PERFORMANCE.

1) SOME TRACKERS USE dE/dx FOR PARTICLE ID (SEE LATER). LANDAU TAIL CAN MASK 20-40% EFFECTS OF DIFFERING PARTICLE MASSES IF NOT PROPERLY ACCOUNTED FOR.

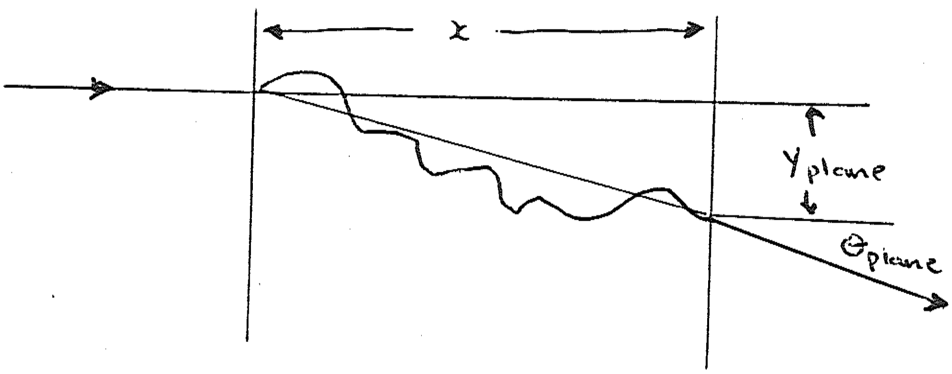
2) δ- RAYS CAN BE "ORTHOGONAL" TO ORIGINAL PARTICLE TRAJECTORY. IN VERY PRECISE TRACKERS (SILICON) THIS CAN DETERIORATE POSITION RESOLUTION.

FAR FROM AN EXACT SCIENCE. BEWARE OF LIMITATIONS OF PARAMETRISATION (II).

MULTIPLE SCATTERING

AS PARTICLES TRAVERSE A MEDIUM THEY ARE DEFLECTED BY MANY SMALL ANGLES – COLLISIONS MAINLY WITH NUCLEI IN THE MATERIAL.

THIS IS COULOMB SCATTERING.



IF WE DEFINE $\theta_0 = \theta_{plane}^{rms} = \frac{1}{\sqrt{2}}\,\theta_{space}^{rms}$

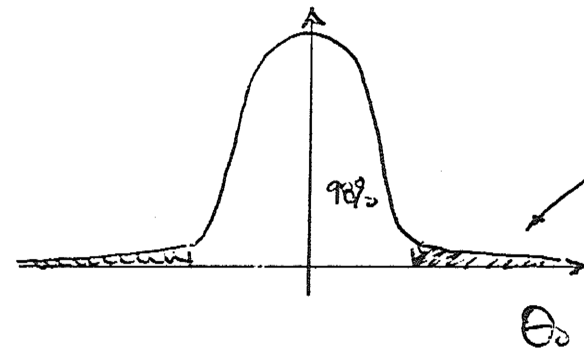
THEN WE CAN APPROXIMATE 98% OF CASES BY A SIMPLE GAUSSIAN WITH:

III $$\theta_0 = \frac{13.6\ \text{MeV}}{\beta c p}\ \sqrt{\frac{x}{X_0}}\left\{1 + 0.038 \ln\left(\frac{x}{X_0}\right)\right\}$$

X_0 IS THE <u>RADIATION LENGTH</u> OF MEDIUM

13

NB: DISTRIBUTION OF Θ_0 LOOKS LIKE:



LONG TAILS DUE TO HARD SCATTERS

cf LYNCH, DAHL NIM B58 (1991)

FOR $Z=1$ $\quad \beta \to 1$ FORMULA III VARIES

FROM MEASUREMENTS BY LESS THAN 10%

FOR: $10^{-3} < \frac{x}{X_0} < 10^{2}$

OFTEN:

$$\boxed{\Theta_0 \simeq \frac{13.6}{\beta c p}\sqrt{\frac{x}{X_0}}}$$

IS ENOUGH.

IN PRACTICAL APPLICATIONS (PARTICLE TRAVERSING MORE THAN 1 MATERIAL) IT IS BETTER TO COMPUTE COMBINED x & X_0 AND APPLY FORMULA II ONCE. SCATTERING ANGLES ARE NOT INDEPENDENT $\Rightarrow$ CANNOT JUST SUM IN QUADRATURE.

RADIATION LENGTH

- DEFINED AS THE THICKNESS OF MATERIAL IN UNITS OF $\left(\frac{g}{cm^2}\right)^{-1}$ OVER WHICH A HIGH ENERGY ELECTRON LOSES ALL BUT e^{-1} OF ITS ENERGY

$$\frac{1}{X_0} = 4\pi\alpha r_e^2 \frac{N_A}{A} \left\{ Z^2\left[L_{RAD} - f(Z)\right] + Z L'_{RAD} \right\}$$

L_{RAD}	L'_{RAD}	Z	Element
5.31	6.14	1	H
4.79	5.62	2	He
4.74	5.81	3	Li
4.71	5.92	4	Be
$\ln(184\,Z^{-1/3})$	$\ln(1194\,Z^{-2/3})$	>4	Others

$$f(Z) = a^2\left[\frac{1}{1+a^2} + 0.202 \mp 0.037\,a^2 + \ldots\right]$$

$$a = \alpha Z$$

Dahl PROVIDES A MORE SUCCINCT FORM:

$$\boxed{X_0 = \frac{716.4\left(\frac{g}{cm^2}\right)\ A}{Z(Z+1)\ \ln\left(287/\sqrt{Z}\right)}}$$

Z^2 is dominant term.

ENERGY LOSS MECHANISM HERE IS BREMSSTRAHLUNG <u>NOT</u> IONISATION (SEE LATER WITH CALORIMETERS)