UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER EXAMINATIONS 2003

PHY 255F – Oscillations and Waves

Duration — 3 hours

Aids allowed: calculators
student-supplied aid sheet (two sides of 8.5 x 11 inch sheet, hand-written)

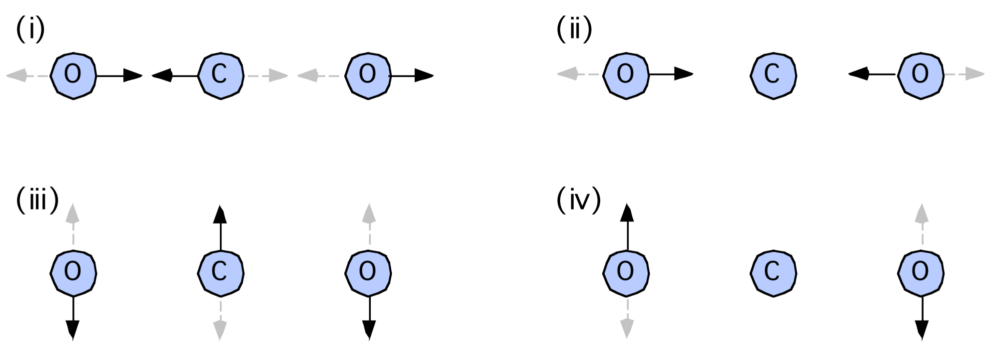
INSTRUCTIONS: ANSWER ALL FIVE QUESTIONS IN THE MAIN PART.
MARKS ARE SHOWN IN LEFT MARGIN. TOTAL MARKS = [100]

[20] 1. *Principles and terms of oscillations and waves*
Explain succinctly (*i.e.,* three sentences or less) the meaning *and* significance of each of the following, in the context of waves and oscillations. Your answer should make clear not only what the term or concept is, but put it in context and make clear why it is important.

[4] i) normal modes of oscillation
[4] ii) transverse oscillations, and longitudinal oscillations
[4] iii) phase speed, and group speed
[4] iv) impedance, and impedance-matching
[4] v) Fourier series, and Fourier transforms

[20] 2. *Multiple Choice Section* — In each of the following questions, select the answer A–F which best responds to the question. Please mark your answers in your exam answer booklet, in the order presented. You may give a brief explanation also, if you think no answer is quite appropriate. Part marks may be given for certain wrong answers which have merit.

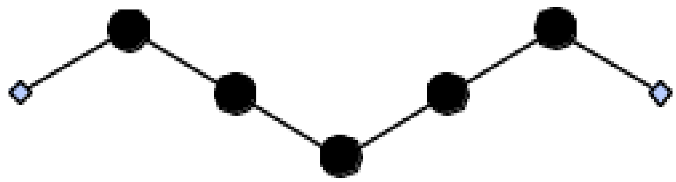
[4] a) Which of the figures below is *not* a normal mode for the vibration of the molecule CO_2?



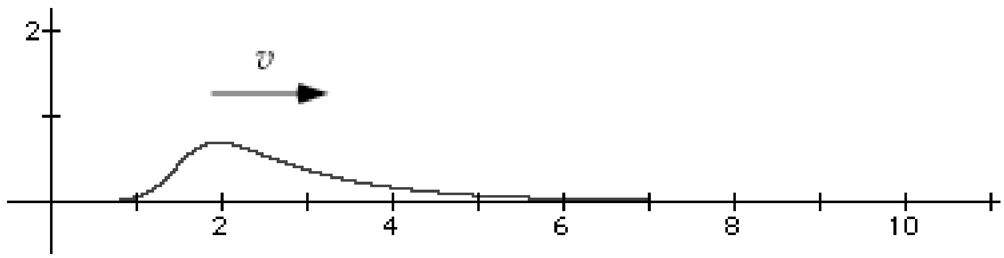
A. (iv) and (iii)
B. (iii)
C. (ii)
D. (iv)
E. none of the above is a correct answer

[4] b) Five identical weights (●) are attached to a string of length $L = 30$ cm. The masses are equally spaced from each other and from the anchor points (◇) at the ends of the string. The tension in the string is $T = 100$ N, and the masses of the weights are each $m = 50$ g. At one point the instantaneous displacements look like the figure below — what is the frequency of this oscillation?

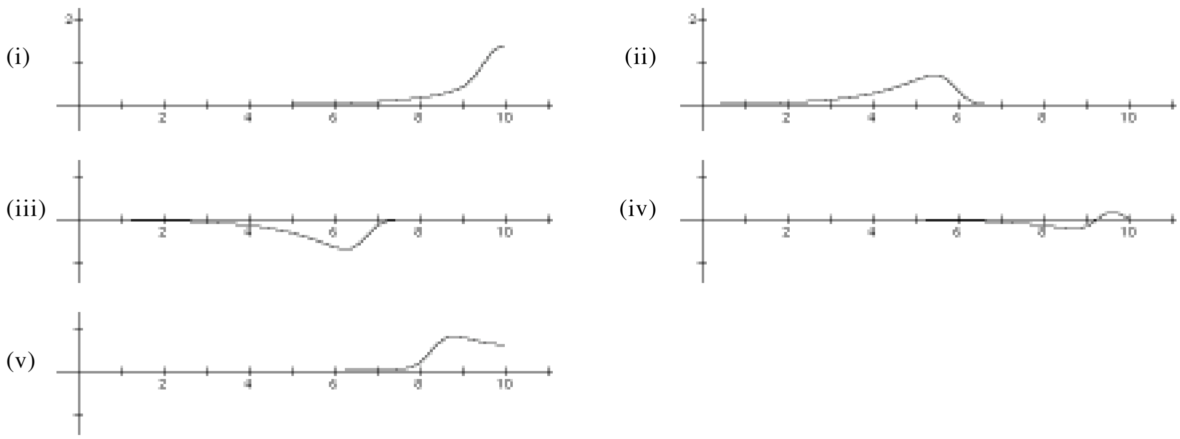
A. 20 Hz
B. 22 Hz
C. 24 Hz
D. 408 Hz
E. none of the above



[4] c) The figure below shows a travelling wave on a string, propagating towards an end of the string which is not fixed, but which is instead free to slide back and forth (transversely) along a wire without friction; it provides no force in the transverse direction. This sliding attachment adds no mass — the end of the string is the same mass density as the rest of the string. The string end is at 10 units for the scale shown below.



Which of the following graphs correctly represents this pulse at a later time? Take it that the pulse may have reflected, but only at x = 10.

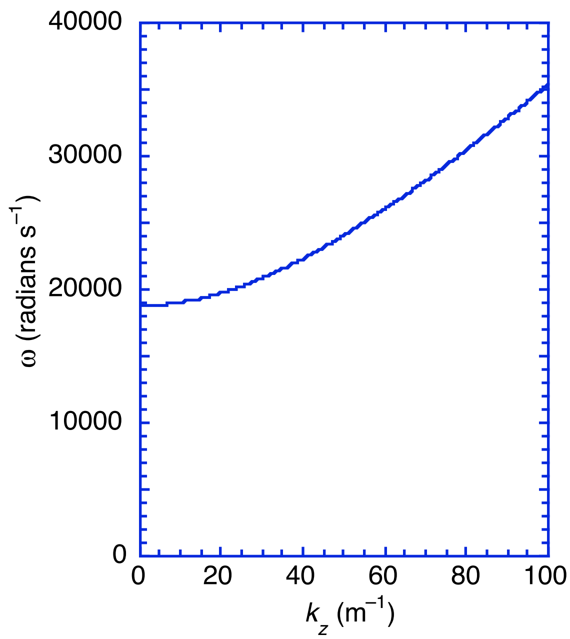


A. (i) and (ii)
B. (i) and (iv)
C. (iii)
D. (iii) and (iv)
E. (i), (ii) and (v)
F. no answer above is correct

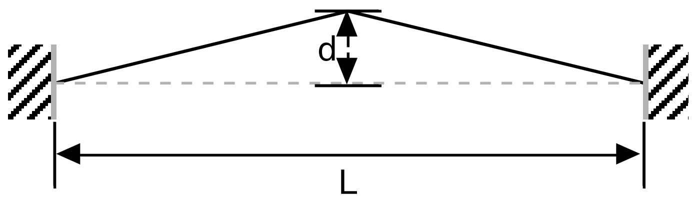
[Question #2 continues ...]

[4] d) The graph at right shows the dispersion relation for a particular sound-wave travelling in an acoustic waveguide — here a duct which has a rectangular cross-section. What does it show about the phase speed v_ϕ and group speed v_g?

A. $v_\phi = v_g$
B. $v_\phi \approx 900\ \mathrm{m\,s^{-1}}$, $v_g \approx 100\ \mathrm{m\,s^{-1}}$
C. $v_\phi \approx 0.001\ \mathrm{m\,s^{-1}}$, $v_g \approx 0.01\ \mathrm{m\,s^{-1}}$
D. $v_\phi \approx 100\ \mathrm{m\,s^{-1}}$, $v_g \approx 900\ \mathrm{m\,s^{-1}}$
E. $v_\phi \approx 200\ \mathrm{m\,s^{-1}}$, $v_g \approx 300\ \mathrm{m\,s^{-1}}$
F. none of the above



[4] e) A guitar string is plucked by pulling the centre of the string aside by a distance d and then releasing it. Thus, the string's initial position at time t = 0 is illustrated by the figure at right. Which of the following equations gives the first few terms of the Fourier series representing the subsequent motion?



A. $d\cos(\frac{L}{2}x) + d\cos(Lx) + d\cos(\frac{3L}{2}x) + \ldots$

B. $\sin(\omega_0 t)\left(a_1\cos(\frac{L}{2}x) + a_2\cos(Lx) + a_3\cos(\frac{3L}{2}x) + \ldots\right),\ a_1 \neq a_2 \neq a_3$

C. $\cos(\omega_0 t)\left(a_1\cos(\frac{L}{2}x) + a_2\cos(Lx) + a_3\cos(\frac{3L}{2}x) + \ldots\right),\ a_1 \neq a_2 \neq a_3$

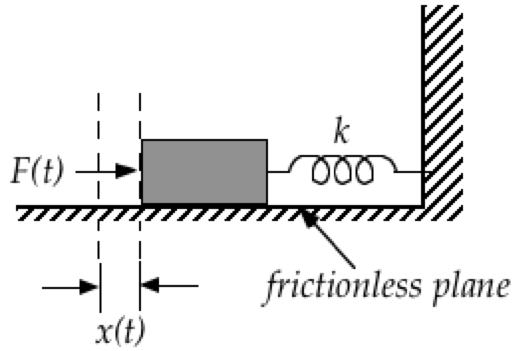
D. $\left(a_1\cos(\frac{L}{2}x)\cos(\omega_o t) + a_2\cos(Lx)\cos(2\omega_o t) + a_3\cos(\frac{3L}{2}x)\cos(3\omega_o t) + \ldots\right),$
$a_1 \neq a_2 \neq a_3$

E. $\left(a_1\sin(\frac{L}{2}x)\cos(\omega_o t) + a_2\sin(Lx)\cos(2\omega_o t) + a_3\sin(\frac{3L}{2}x)\cos(3\omega_o t) + \ldots\right),$
$a_1 \neq a_2 \neq a_3$

F. none of the above

[20] 3. *Driven damped harmonic oscillators*

One design problem in the production of machine guns is that the periodic recoil of the bolt mechanism due to the gun firing must not excite a resonance in the system that would cause the gun to break. (The bolt is the part of a gun that puts a bullet in the chamber; its recoil automatically loads the next bullet). A simple model for a particular machine gun is shown at right. The recoil-force on the bolt (mass $m = 0.4$ kg) due to successive firing of bullets is represented by a general periodic external force (in Newtons) of the form



$F(t) = 25 \sin(60t)$. The spring opposing the recoil of the bolt provides a linear restoring force, with a spring constant $k = 2000$ N m^{-1}. Initial conditions are $x(0) = 0$ m and $\left.(dx/dt)\right|_{t=0} = 0$ m s^{-1}. Using the model system as a guide:

[4] i) Draw a force diagram, and write the *full* equation of motion for the mass. Show that the units of each term are the same (*i.e.,* verify that the equation is dimensionally consistent). Keep the forcing frequency in a general form ω for now.

[4] ii) This system is characterized by a very high Q ($Q >> 1$). What terms can be neglected from the equations of motion? Why? Apply this approximation to the governing differential equation and obtain a simpler form. Find the natural frequency ω_o of the system in this approximation.

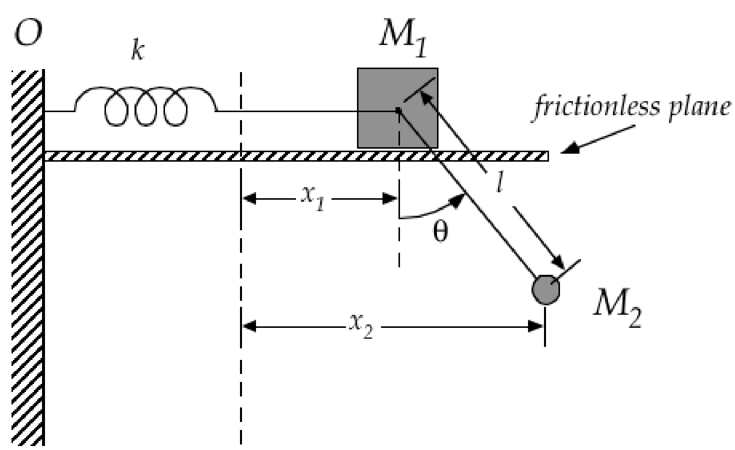
[4] iii) Find $x(t)$ for the given initial conditions, still with arbitrary frequency ω. Show that your expression has units of length.

[4] iv) Sketch the displacement amplitude response as a function of the frequency ω.

[4] v) For the specified frequency in this system, what is the amplitude response? On your plot, identify the range of frequencies at which the gun is *least likely* to break.

[20] 4. *Coupled oscillations*

The sketch at right shows a mass M_1 on a frictionless plane, connected to support O by a spring of stiffness k. Mass M_2 is supported by a string of length l from M_1.



[7] i) Starting from $F = ma$, and assuming that $x_2 - x_1$ is always small, derive the equations of motion of M_1 and M_2:

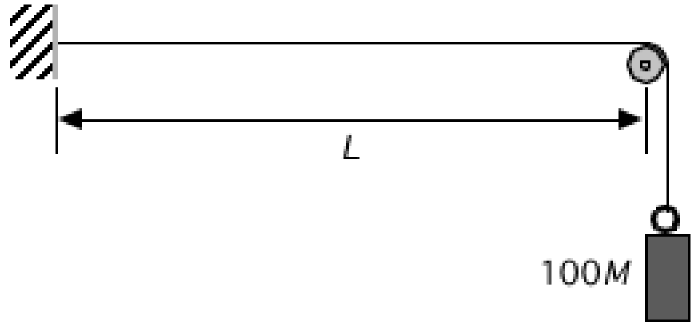
$$M_1 \ddot{x}_1 = -k\,x_1 + M_2 \frac{g}{l}(x_2 - x_1)$$

$$M_2 \ddot{x}_2 = -M_2 \frac{g}{l}(x_2 - x_1)$$

[7] ii) For $M_1 = M_2 = M$, use the equations to obtain the normal frequencies of the system.

[6] iii) What are the normal-mode motions for $M_1 = M_2 = M$ and $g/l >> k/M$?

[20] 5. *Transverse travelling waves*



A string is stretched between two points a distance L apart. The string has total mass M. The tension is produced by a mass $100M$ hanging from one end of the string after it passes over a pulley wheel.

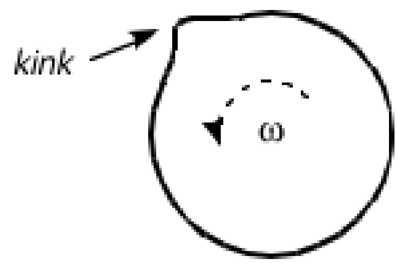
[5] i) It is observed that a pulse requires 0.1 s to travel from one end of the string to the other. What must be the length L? What are the frequencies of the normal modes?

[5] ii) Derive the formula for the *total energy* of vibration for the string oscillating in its *n*th normal mode with amplitude A. [HINT: save time — think of the *easiest* way to find the energy]

[5] iii) Calculate the total energy of vibration of the same string if it is vibrating in the following superposition of normal modes:

$$y(x,t) = A_1 \sin\left(\frac{\pi x}{L}\right)\cos(\omega_1 t) + A_3 \sin\left(\frac{3\pi x}{L}\right)\cos\left(\omega_3 t - \frac{\pi}{4}\right)$$

Show that this energy is equal to the sum of energies of the two normal modes taken separately.



A closed loop of string of mass M is rotated rapidly at a constant angular speed ω, as shown in the figure at right. The radius of the loop is R. A tension T is set up in the string, circumferentially, because of its rotation.

[5] iv) By considering the instantaneous centripetal acceleration of a small segment of the string, show that the tension must be equal to

$$T = \frac{M\omega^2 R}{2\pi}$$

The string is suddenly deformed at some point, causing a kink to appear in it, as shown in the diagram. Show that this could produce a distortion that remains stationary in the laboratory frame, regardless of the particular values of M, ω, and R. But is this the whole story? [Remember that pulses on a string may travel both ways.]

TOTAL MARKS = [100]