<u>Physics 255F – Oscillations and Waves</u>
Department of Physics
University of Toronto

Term Test – 24 October 2003

STUDENT-SUPPLIED AID SHEET (one side 8 1/2" x 11")
NON-PROGRAMMABLE CALCULATORS

THIS TEST HAS THREE PAGES
DO ALL FOUR QUESTIONS
DURATION OF TEST: 2 HOURS

[25] 1) Explain as succinctly as possible (*i.e.,* three sentences or less) the meaning *and* significance of each of the following, in the context of waves and oscillations:

i) simple harmonic oscillator
ii) quality factor Q
iii) phasor notation
iv) Lissajous figure
v) beat frequency, for linear superposition of oscillators

[25] 2) *Multiple choice:* For each item (i)–(iv) below, choose the answer A–F which best responds to the question. Answer questions in order, writing your response in your examination booklet. You may give a brief explanation also, if you think no answer is quite appropriate. Part marks may be given for certain wrong answers which have merit.

i) What is the formula for the oscillation frequency of a weakly damped simple harmonic?

A. $\sqrt{\frac{s}{m}}$

B. $\sqrt{\frac{s}{m} - \frac{r^2}{4m^2}}$

C. $\sqrt{\frac{r^2}{4m^2} - \frac{s}{m}}$

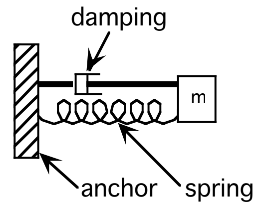
D. $e^{-\frac{rt}{2m}}$

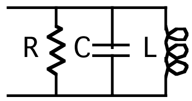
E. $F_o \cos(\omega t)$

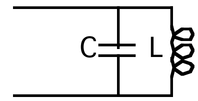
ii) What is a *Lissajous figure*?

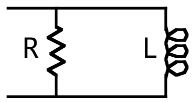
A. a graphical representation of two orthogonal simple harmonic oscillations
B. a the resultant for the addition of two simple harmonic oscillations in one dimension
C. the oscillation sometimes described as a ‘beat frequency’
D. more than one of the above
E. none of the above

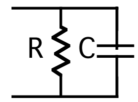
iii) At right is a figure representing a mass on a spring with viscous damping. Which of the electrical circuits below can produce the same sort of oscillation behaviour, given appropriate initial conditions and appropriate values for R, C, L, etc.?

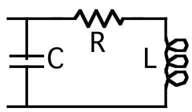


A. 

B. 

C. 

D. 

E. 

F. none, or more than one, is analogous

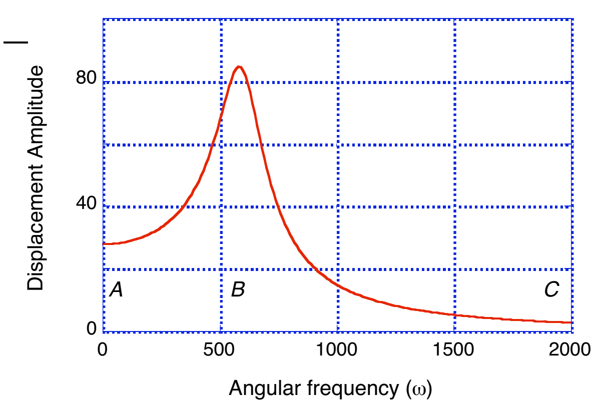
iv) The last note of the piece Josef chose for his piano Conservatory exam was a sustained middle C (256 Hz). While he listened for the last note to fade away to a ghost of its initial amplitude, he had time to calculate the quality factor Q of that oscillation. If the energy died away to 1% of its initial value after 15 seconds, what was the Q?

A. 133
B. 834
C. 3840
D. 5239
E. none of the above

v) Three similar oscillators, A, B, C have the same small damping constant r, but different natural frequencies $\omega_o = (s/m)^{1/2}$: 1200 Hz, 1800 Hz, 2400 Hz. If all three are driven by the same source at 1800 Hz, which statement is correct for the phases of the velocities of the three?

A. $\phi_A = \phi_B = \phi_C$
B. $\phi_A < \phi_B = 0 < \phi_C$
C. $\phi_A > \phi_B = 0 > \phi_C$
D. $\phi_A > \phi_B > 0 > \phi_C$
E. none of the above is correct

[25] 3) *Force terms in forced harmonic oscillator* — The figure at right shows a plot of the displacement amplitude of a driven, damped simple harmonic oscillator.



i) write the equation of motion for a driven, damped simple harmonic oscillator

ii) derive the formula for the displacement amplitude, starting with a general driving force $F_o \cos(\omega t)$.

iii) by simplifying (i), show that the amplitude in the limit of small frequencies (region *A* in the figure) is determined by the relationship between the driving force and the spring force.
iv) similarly, in the limit of large frequencies (region *C* in the figure) what relationship determines the displacement amplitude? Give the form of the function for very large frequencies.

[25] 4) *Critical damped simple harmonic oscillator*

A pendulum is constructed of a long massless rod of length L with a mass m attached to the end. The acceleration due to gravity is $g = 9.8 \text{ m s}^{-2}$.

i) Derive the equation of motion for oscillation around equilibrium, and show for that small-amplitude displacements the pendulum will oscillate with simple harmonic motion in x, where x represents the sideways displacement of the mass.

The mass and rod hang from a pivot point which experiences friction. A damping force $-r\dot{x}$ proportional to the velocity of the mass results.

ii) Write the new equation of motion.

iii) Take $m = 0.1$ kg, $L = 1$ m, and $r = 0.626 \text{ kg s}^{-1}$. Take an initial displacement $x(0) = 1$ cm and initial velocity $\dot{x}(0) = -5 \text{ cm s}^{-1}$. What is the formula for the displacement $x(t)$ subsequently?

[100] TOTAL