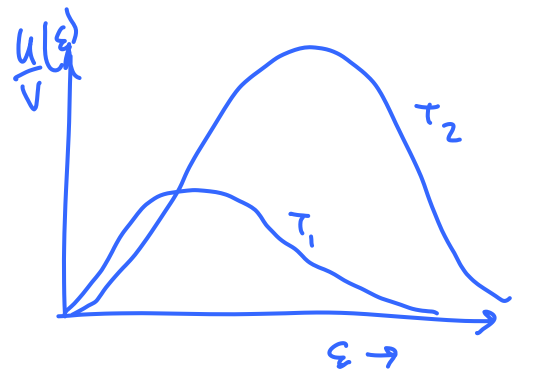
Review

Blackbody radiation - quantum statistical description needed

- Planck distribution $\bar{n}_{Pl} = \dfrac{1}{e^{hf/kT} - 1}$ ← number of photons of given frequency
- Planck spectrum $\dfrac{8\pi}{(hc)^3} \dfrac{\varepsilon^3}{e^{\varepsilon/kT} - 1}$

energy density per photon energy → $\frac{U}{V}(\varepsilon)$ ↑ photon energy $\varepsilon = hf$



$T_2 > T_1$

Heat Capacity and Entropy

- Total energy density found by integrating over all energy units

$$\frac{U}{V} = \frac{8\pi}{(hc)^3} \int_0^\infty \frac{\varepsilon^3}{e^{\varepsilon/kT} - 1} d\varepsilon \qquad x \equiv \frac{\varepsilon}{kT}$$

$$\frac{U}{V} = \frac{8\pi}{(hc)^3} (kT)^4 \underbrace{\int_0^\infty \frac{x^3}{e^x - 1} dx}_{} \rightarrow \pi^4/15$$

$$U = \underbrace{\frac{8\pi^5 k^4 V}{15 \cdot h^3 \cdot c^3}}_{a} T^4 = aT^4$$

Heat capacity

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = 4aT^3 \quad \propto VT^3$$

absolute entropy

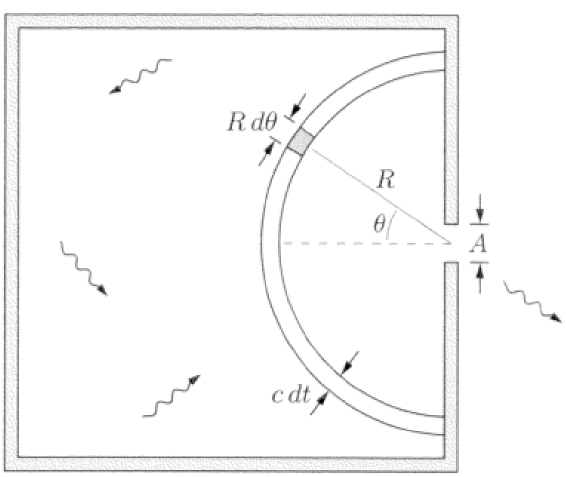
$$S(T) = \int_0^T \frac{C_V(T')}{T'}\,dT' = \frac{4}{3}aT^3$$

Stefan's Law

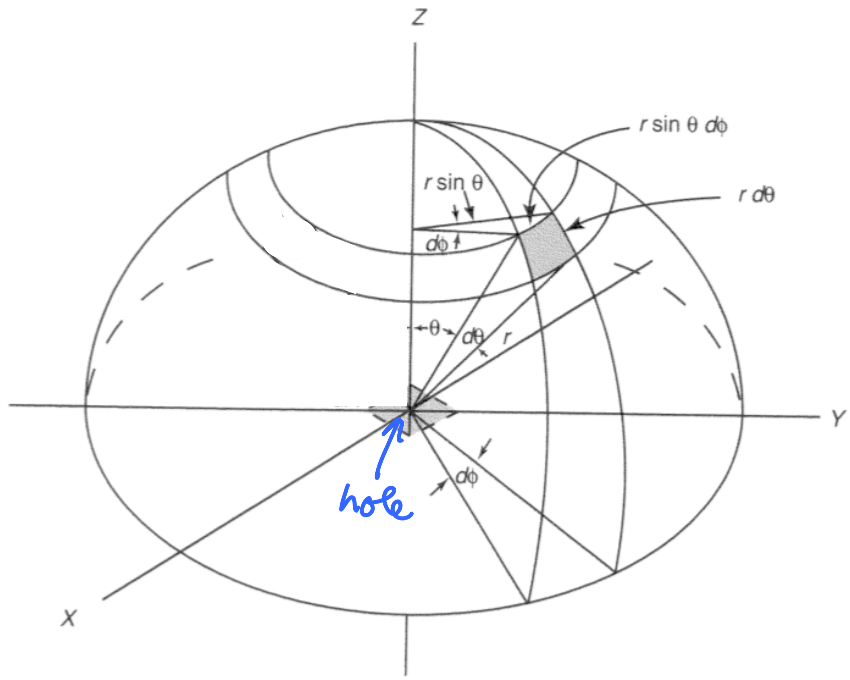
- Relates radiated power to temperature

  ↳ rate of energy transfer

- Consider photons escaping from the box from small hole

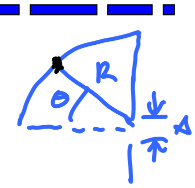


Hemispherical shell



Adapted from Liou, An Introduction to Atmospheric Radiation, 2e

Radiation Escaping



- Volume of radiation $(R\,d\theta)(R\sin\theta\,d\phi)(c\,dt)$
  - has energy $= \dfrac{U}{V}\,c\,dt\,R^2\sin\theta\,d\theta\,d\phi$
- Photons able to escape = $\dfrac{A\cos\theta}{4\pi R^2}$
  - Energy escaping = $\dfrac{A\cos\theta}{4\pi R^2}\dfrac{U}{V}\,c\,dt\,R^2\sin\theta\,d\theta\,d\phi$
- Total energy escaping

$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta\, \frac{A\cos\theta}{4\pi}\frac{U}{V}\,c\,dt\sin\theta$$

$$= \frac{A}{4}\cdot\frac{U}{V}\cdot c\,dt$$

power per unit area

$$= \frac{c}{4}\cdot\frac{U}{V}$$

Stefan's Law

- Power per unit area $= \frac{c}{4}\frac{U}{V}$

$\frac{U}{V} = \frac{8\pi^5}{15}\frac{(kT)^4}{(hc)^3}$

- Knowing total energy, can calculate

- Power per unit area $= \frac{2\pi^5}{15}\frac{(kT)^4}{h^3c^2} = \sigma T^4$

where $\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2} = 5.67\text{x}10^{-8}\,\frac{\text{W}}{\text{m}^2\text{K}^4}$

Stefan-Boltzmann constant

Radiating Objects

- For perfect blackbodies, emit and absorb all frequencies
  - Emissivity and absorptivity are both 1
  - If was perfect reflector, emissivity would be 0

- For "greybodies",
  - Emissivity (ε) is less than one and often depends on frequency
  - Power / unit area = $\varepsilon\,\sigma\,T^4$

- Examples of uses:
  - Infrared thermometry
  - Colour temperature

Image from http://us.fluke.com/usen/Products/Fluke+568+566.htm

Colour Temperature

- When buying compact fluorescent light bulbs, they usually quote a “colour temperature” to describe the range of colours

- This value describes the peak in the lamp spectrum by giving an equivalent blackbody temperature

~ 2000 K: Incandescent bulbs

~ 3000 K: Halogen bulbs

~ 5000-6000 K: Summer sun

from Natural Resources Canada

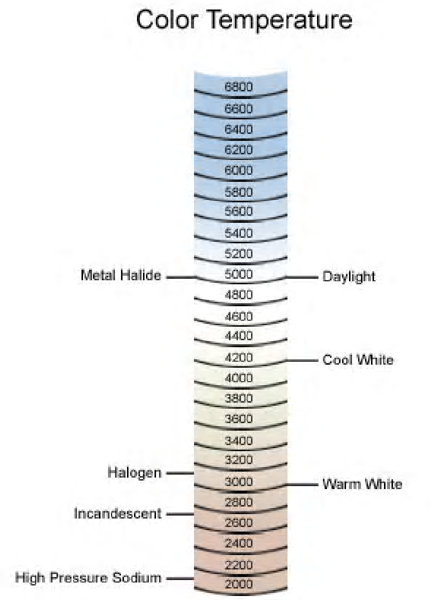


Chart from: http://www.lightbulbsdirect.com/page/001/CTGY/ColorTemp

Radiation from Sun and Earth

Sun can be approximated as a blackbody at ~5800 K

- Solar constant 1370 W/m^2 ← radiation at top of atmosphere

radiation from Sun incepted by Earth → solar constant $\cdot \pi R^2$

radiation emitted by Earth in all directions → $4\pi R^2 \cdot \sigma T^4$

equate

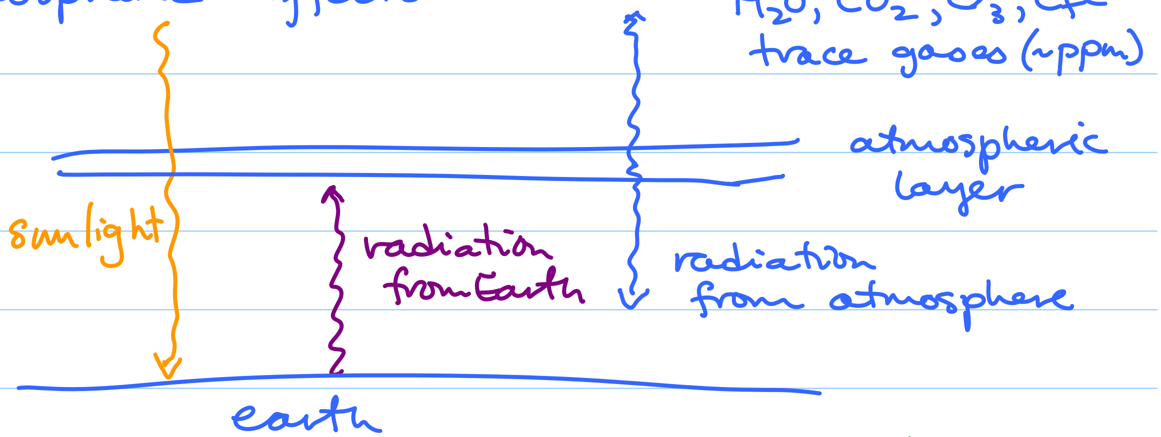
$$1370\ \mathrm{W/m^2} \cdot \pi R^2 = 4\pi R^2 \cdot \sigma T^4$$

$$T = 279\ \mathrm{K}$$

Some of Sun's radiation is reflected by Earth → 30% albedo

$$1370(1-0.3)\pi R^2 = 4\pi R^2 \cdot \sigma T^4$$
$$T = 255\,K$$

atmospheric effects

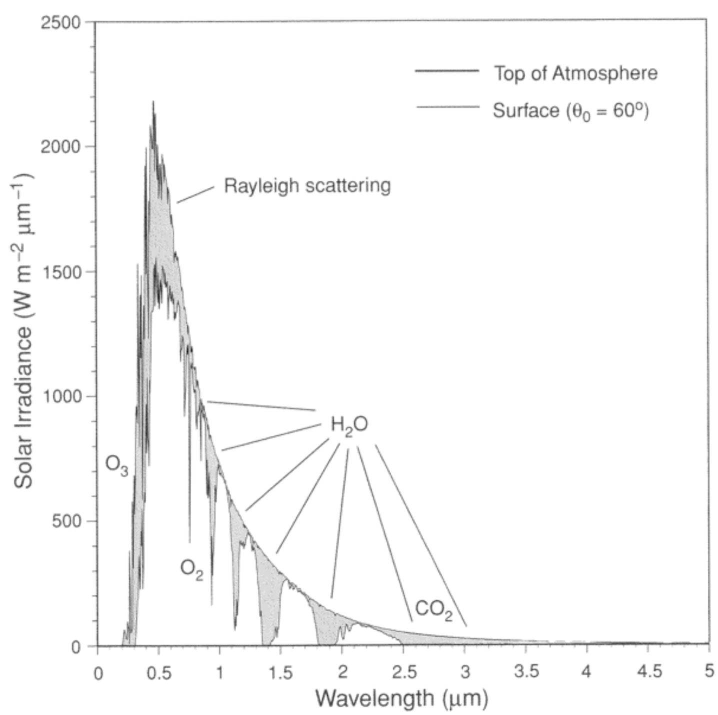


→ if radiation from atmosphere doubles radiation absorbed by Earth

$$2\left(1370(1-0.3)\right)\pi R^2 = 4\pi R^2 \cdot \sigma T^4$$
$$T = 303\,K$$

bit higher than average but shows impact

Absorption by the Atmosphere



← E

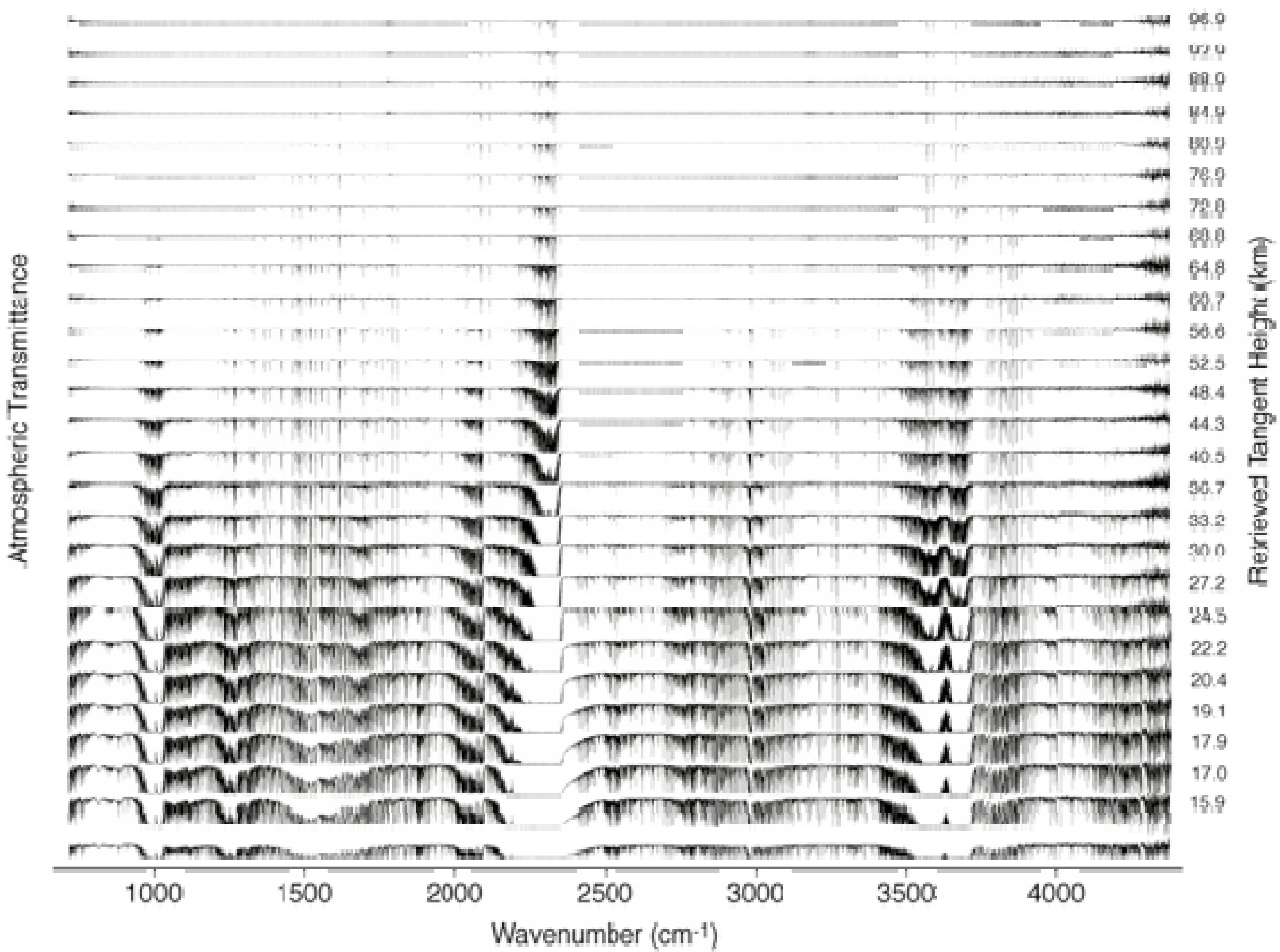
Sequence of Spectra of Atmosphere

Sunset 2245

Jan. 12, 2004
 9:50:23UTC

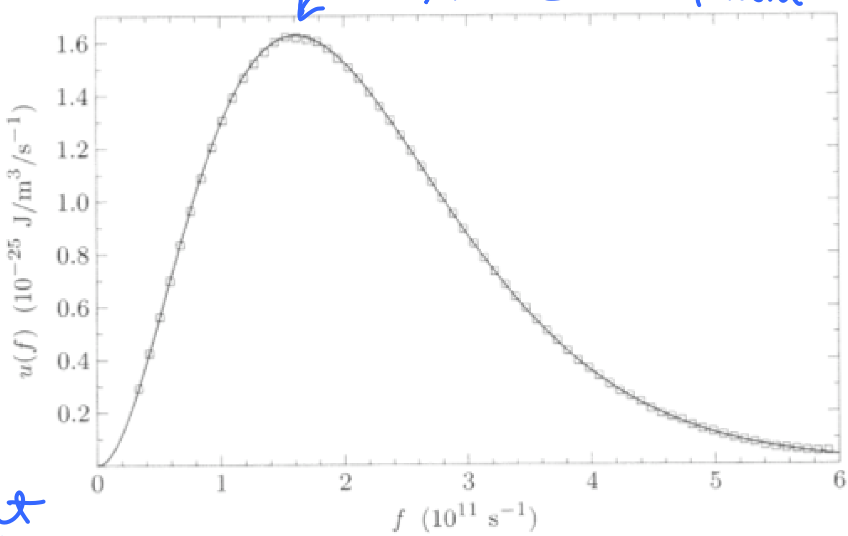
Lat: 67°S

Lon: 168°W



Cosmic Background Radiation

6.6×10^{-4} eV ~1 mm



peak at ~2.7K

Cosmic Background Explorer (COBE) Satellite provided data pts → fit to blackbody curve to 1 in 10^5